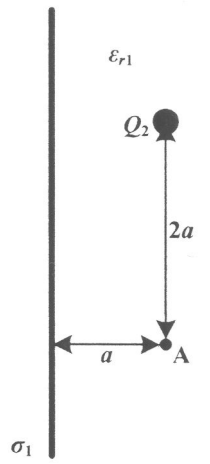
PRVI KOLOKVIJUM IZ ELEKTROTEHNIKE
16. april 2014.

GRUPA 1

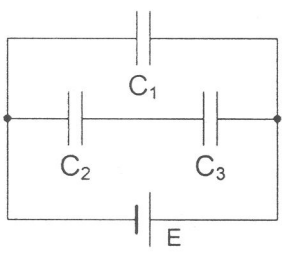
1. Veoma velika, ravnomerno naelektrisana površ, površinske gustine naleketrisanja $\sigma_1 = \sigma > 0$ i tačkasto naelektrisanje $Q_2 = -Q < 0$ nalaze se u dielektriku relativne dielektrične konstante ε_{r1} kao na Slici 1.

a) Odrediti i **skicirati** vektor elektičnog polja u tački A. **(3 poena)**
b) Odrediti i **skicirati** vektor sile kojom površ deluje na tačkasto naelektisanje. **(2 poena)**

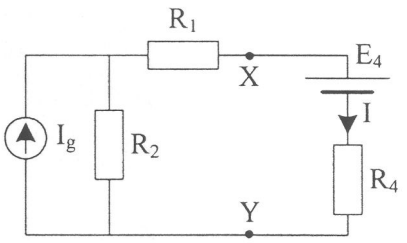


Slika 1

2. Na Slici 2 je prikazana mreža sa kondenzatorima koji su pre povezivanja bili neopterećeni. Odrediti količinu naelektrisanja i napon na svakom kondenzatoru, kao i energiju elektrostatičkog polja kondenzatora C_3. Obavezno naznačiti referentne smerove naelektrisanja i napona svih kondenzatora. Poznato je: $C_1 = 2\,\mu\text{F}$, $C_2 = 4\,\mu\text{F}$, $C_3 = 8\,\mu\text{F}$, $E = 12\,\text{V}$. **(5 poena)**



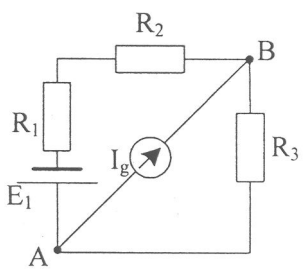
Slika 2



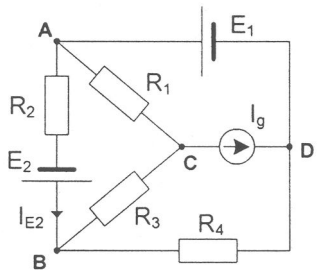
Slika 3

3. U kolu prikazanom na Slici 3, primenom Tevenenove teoreme odrediti intenzitet struje I. Poznato je: $E_4 = 15\,\text{V}$, $I_g = 2\,\text{A}$, $R_1 = 10\,\Omega$, $R_2 = 20\,\Omega$, $R_4 = 20\,\Omega$. **(5 poena)**

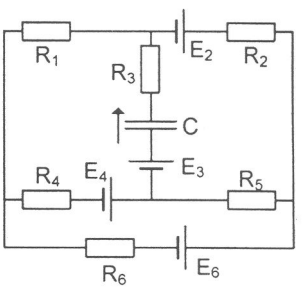
4. Primenom metode superpozicije naći napon U_{AB}. Poznato je: $R_1 = 15\,\Omega$, $R_2 = 25\,\Omega$, $R_3 = 10\,\Omega$, $E_1 = 10\,\text{V}$, $I_g = 1\,\text{A}$. **(5 poena)**



Slika 4



Slika 5



Slika 6

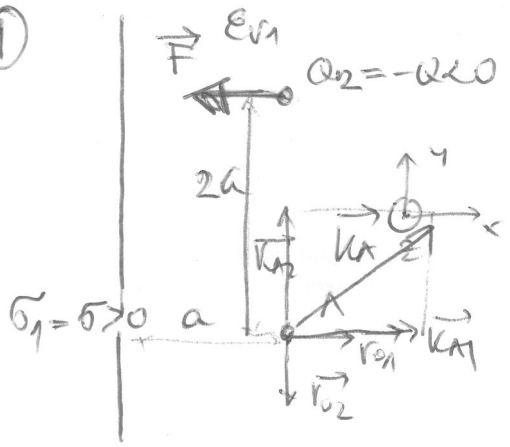
5. U kolu na Slici 5, primenom metode konturnih struja ili napona između čvorova odrediti struju I_{E2} i napon U_{DC}. Poznato je: $E_1 = 40\,\text{V}$, $E_2 = 30\,\text{V}$, $I_g = 2\,\text{A}$, $R_1 = R_2 = R_3 = R_4 = 10\,\Omega$. **(5 poena)**

6. U kolu prikazanom na Slici 6 uspostavljeno je stacionarno stanje. Odrediti količinu naelektrisanja kondenzatora u smeru naznačenom na slici. Poznato je: $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 10\,\Omega$, $E_2 = 10\,\text{V}$, $E_3 = 5\,\text{V}$, $E_4 = 10\,\text{V}$, $E_6 = 50\,\text{V}$, $C = 3\,\mu\text{F}$. **(5 poena)**

Izrada kolokvijuma traje 90 minuta. Na vežbanci napisati broj grupe zadataka. Papir sa tekstom zadataka predaje se u vežbanci tj. ne sme se izneti.

GRUPA 1

1



a) $\vec{E_A} = \vec{E_{A1}} + \vec{E_{A2}}$

$$\vec{E_{A1}} = \frac{\sigma_1}{2\varepsilon_0\varepsilon_{r1}}\vec{r}_{O1} = \frac{\sigma}{2\varepsilon_0\varepsilon_{r1}}\vec{i}$$

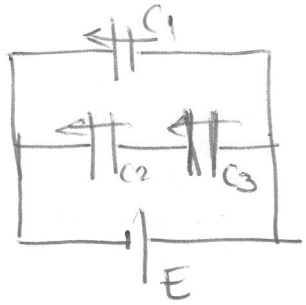
$$\vec{E_{A2}} = \frac{Q_2}{4\pi\varepsilon_0\varepsilon_r(2a)^2}\vec{r}_{O2} = \frac{-Q}{16\pi\varepsilon_0\varepsilon_{r3}a^2}(\vec{j}) = \frac{Q}{16\pi\varepsilon_0\varepsilon_{r3}a^2}\vec{j}$$

$$\vec{E_A} = \frac{\sigma}{2\varepsilon_0\varepsilon_{r1}}\vec{i} + \frac{Q}{16\pi\varepsilon_0\varepsilon_{r3}a^2}\vec{j}$$

b) $\vec{F} = Q_2\vec{E}$

$\vec{E} = \frac{\sigma}{2\varepsilon\varepsilon_{r1}}\vec{i}$ $\Big\}$ $\vec{F} = \frac{-Q\sigma}{2\varepsilon_0\varepsilon_{r1}}\vec{i}$

2



$U_1 = E = 12V$ $\quad Q_1 = U_1 \cdot C_1 = 12V \cdot 2\mu F = 24\mu C$

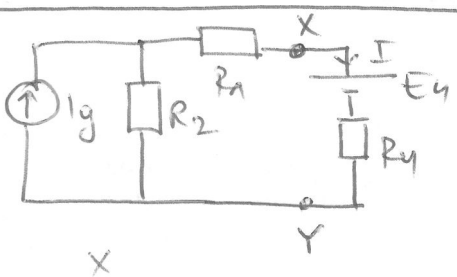
$$Q_2 = Q_3 = Q_{23} = C_{23}\cdot E = \frac{C_2C_3}{C_2+C_3}E = \frac{4\mu F\cdot 8\mu F}{12\mu F}\cdot 12V = 32\mu C$$

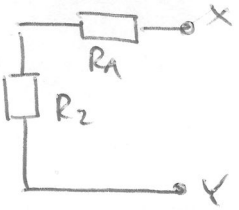
$Q_2 = 32\mu C$ $\quad U_2 = \frac{Q_2}{C_2} = \frac{32\mu C}{4\mu F} = 8V$

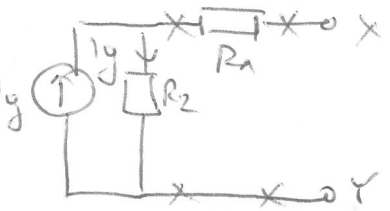
$Q_3 = 32\mu C$ $\quad U_3 = \frac{Q_3}{C_3} = \frac{32\mu C}{8\mu F} = 4V$

$$W_{C_3} = \frac{1}{2}Q_3U_3 = \frac{1}{2}C_3U_3^2 = \frac{1}{2}\frac{Q_3^2}{C_3} = \frac{1}{2}32\mu C\cdot 4V = 64\mu J$$

3

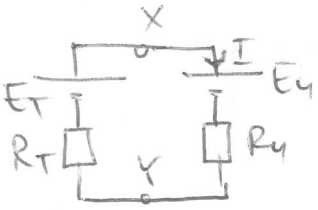






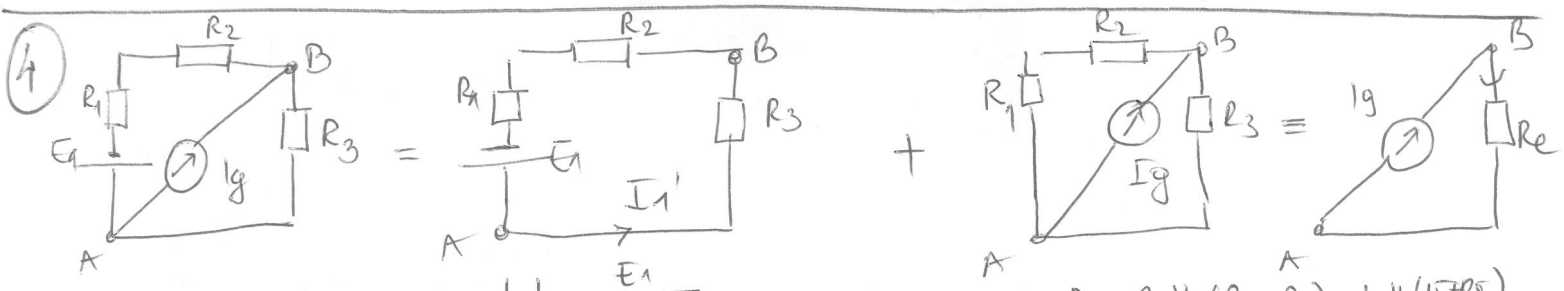
$R_T = R_A + R_2 = 30\Omega$

$E_T = R_2 I_g = 20\Omega \cdot 2A = 40V$



$$I = \frac{E_T - E_4}{R_T + R_4} = \frac{40-15}{30+20} = \frac{25}{50} = 0{,}5A$$

4



$U_{AB} = U_{AB}' + U_{AB}''$

$= 2 - 8$

$\boxed{U_{AB} = -6V}$

$I_1' = \frac{E_1}{R_A + R_2 + R_3}$

$U_{AB}' = R_3 I_1' = \frac{R_3}{R_A+R_2+R_3}E_1$

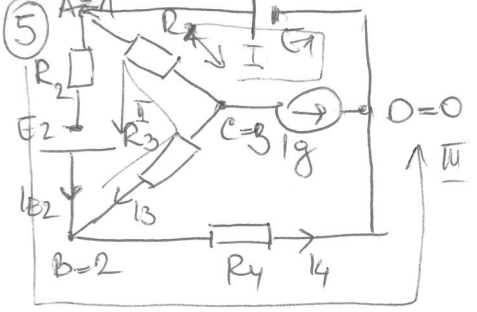
$U_{AB}' = \frac{10}{50}10 = 2V$

$R_e = R_3 \| (R_A + R_2) = 10 \| (15+25)$

$= \frac{10\cdot 40}{50} = 8\Omega$

$U_{AB}'' = -R_e I_g = -8\cdot 1 = -8V$

⑤



KONTURNIM STRUJAMA

$I_I = I_g$

$-R_1 I_I + (R_2 + R_3 + R_1) I_{II} + R_2 I_{III} = E_2$

$0 \cdot I_I + R_2 I_{II} + (R_2 + R_4) I_{III} = E_1 + E_2$

$3R I_{II} + R_2 I_{III} = E_2 + R I_g$

$R I_{II} + 2R I_{III} = E_1 + E_2$

$30 I_{II} + 10 I_{III} = 30 + 20 = 50$

$10 I_{II} + 20 I_{III} = 40 + 30 = 70$

$3 I_{II} + I_{III} = 5 \quad /-2$

$I_{II} + 2 I_{III} = 7 \quad \downarrow +$

$3 I_{II} + I_{III} = 5$

$-6 I_{II} + I_{II} = -10 + 7$

$I_{III} = 5 - 3 I_{II}$

$-5 I_{II} = -3$

$\Rightarrow \boxed{I_{II} = \frac{3}{5} = 0{,}6\,A}$

$I_{III} = 5 - \frac{3 \cdot 3}{5} = 5 - \frac{9}{5} = \frac{25-9}{5} = \frac{16}{5}$

$\boxed{I_{III} = 3{,}2\,A}$

$I_{E2} = I_{II} + I_{III} = 0{,}6 + 3{,}2$

$\boxed{I_{E2} = 3{,}8\,A}$

$I_4 = I_{III} = 3{,}2\,A \qquad I_3 = -I_{II} = -0{,}6\,A$

$U_{CO} = R_3 I_3 + R_4 I_4 = -6 + 32 = 26\,V$

$\boxed{U_{CO} = 26\,V}$

NAPOMENA: $U_{10} = U_{A0} = E_1 = 40$

$-\frac{1}{R_2} U_{10} + U_{20}\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) - \frac{1}{R_3} U_{30} = \frac{E_2}{R_2}$

$-\frac{1}{R_1} U_{10} - \frac{1}{R_3} U_{20} + \left(\frac{1}{R_1} + \frac{1}{R_3}\right) U_{30} = -I_g$

$\frac{3}{R} U_{20} - \frac{1}{R} U_{30} = \frac{E_2}{R} + \frac{E_1}{R}$

$-\frac{1}{R} U_{20} + \frac{2}{R} U_{30} = -I_g + \frac{E_1}{R}$

$3U_{20} - U_{30} = E_1 + E_2 = 70$

$-U_{20} + U_{30} = -IgR + E_1 = 20$

$U_{20} = 32\,V \qquad U_{30} = 26\,V$

$\boxed{U_{CO} = U_{30} = 26\,V}$

$U_{BA} = U_{21} = U_{20} - U_{10} = 32 - 40$

$U_{BA} = -8 = E_2 - R_2 I_{E2}$

$I_{E2} = \frac{E_2 - U_{BA}}{R_2} = \frac{30 - (-8)}{10} = \frac{38}{10}$

$\boxed{I_{E2} = 3{,}8\,A}$

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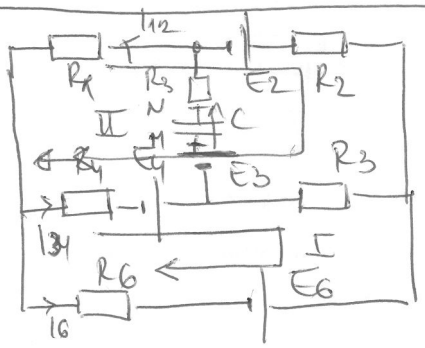
⑥



$(R_4 + R_3 + R_6) I_I - (R_3 + R_4) I_{II} = E_4 - E_6$

$-(R_3 + R_4) I_I + (R_1 + R_2 + R_3 + R_4) I_{II} = -E_4 + E_2$

$3R I_I - 2R I_{II} = 10 - 50 = -40 \qquad R = 1\,\Omega$

$-2R I_I + 4R I_{II} = -10 + 10$

$30 I_I - 20 I_{II} = -40$

$-2 I_I + 4 I_{II} = 0$

$3 I_I - 2 I_{II} = -4$

$-I_I + 2 I_{II} = 0$

$2 I_I = -4 \Rightarrow \boxed{I_I = -2\,A}$

$2 I_{II} = I_I \Rightarrow \boxed{I_{II} = -1\,A}$

$I_{12} = -I_{II} = 1\,A$

$I_{34} = I_I - I_{II} = -2 + 1 = -1\,A$

$U_C = U_{MN} = -R_1 I_{12} - R_4 I_{34} + E_4 + E_3$

$= -10 \cdot 1 - 10 \cdot (-1) + 10 + 5$

$= -10 + 10 + 15 = 15\,V$

$U_C = 15\,V$

$Q_C = U_C \cdot C = 15\,V \cdot 3\,\mu F$

$\boxed{Q_C = 45\,\mu C}$