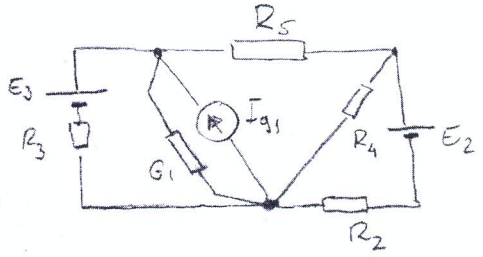
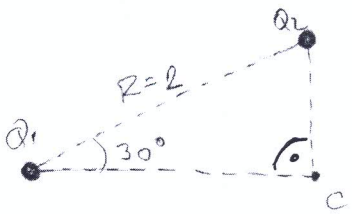
Elektrotehnika, 5.2.2015.
grupa 2

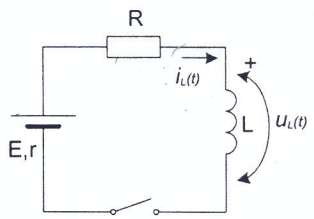
1. a) Složeno električno kolo jednosmerne struje čine otpornici, realni naponski generatori i realni strujni generatori. Kolo ima *Ng*=5 grana i *Nč*=3 čvora. Za ovakav slučaj postaviti opšti sistem jednačina po metodi napona između čvorova. Šta su nepoznate veličine u ovom sistemu i kako se određuju? Šta su koeficijenti uz nepoznate veličine i kako se oni dobijaju? Šta su slobodni koeficijenti u sistemu jednačina i kao se oni određuju?
b) U kolu na slici označiti na pogodan način čvorove i napisati odgovarajući sistem jednačina po metodi napona između čvorova.



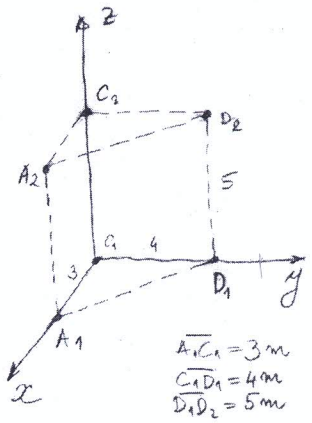
2. Dva tačkasta naelektrisanja Q_1 i Q_2 nalaze se na rastojanju $R = 2\ [cm]$. Potencijal V_C tačke C koja se nalazi u temenu pravouglog trougla čija je jedna hipotenuza duž koja spaja naelektrisanja Q_1 i Q_2 je nula. Odrediti količnik $\frac{Q_1}{Q_2}$.



3. U kolu na slici prelazni proces započinje uključenjem prekidača. Odrediti vremensku konstantu prelaznog procesa i energiju kalema kada se prelazni proces završi. Poznato je: E, r, R, L.



4. Odrediti fluks magnetnog polja kroz svaku stranu trostrane prizme prikazane na slici kao i ukupni fluks kroz zatvorenu površ koju čine bočne strane i osnove prizme. Vektor magnetne indukcije ima intenzitet *B*=1T a usmeren je u pravcu i smeru *y* ose. Podaci o ivicama prizme prikazani su na slici.



5. Kroz namotaj koji ima *N*=20 navojaka magnetni fluks se menja u vremenu po zakonu $\Phi(t) = \sin\left(1000t + \frac{\pi}{4}\right)$ Wb.
Odrediti indukovanu elektromotornu silu na krajevima namotaja.

6. Tri generatora naizmeničnog napona e_a, e_b, e_c imaju istu efektivnu vrednost elektromotorne sile $E_a = E_b = E_c = 100\,[V]$, a faze su $\varphi_a = 0$, $\varphi_b = 120^o$, $\varphi_c = 240^o$. Generatori su spregnuti u zvezdu i napajaju trofazni potrošač koji sačinjavaju tri jednake impedanse $\overline{Z} = 10e^{j\frac{\pi}{6}}\,[\Omega]$, vezane u zvezdu. Odrediti struje i napone u fazama potrošača kao i u linijskim provodnicima koji povezuju trofazni generator sa potrošačem.

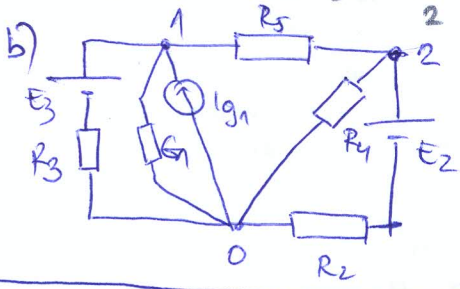
GRUPA 2

1. a) $N_č = 3 \Rightarrow$ Broj napona $N_č - 1 = 2$

$$G_{11}U_{10} - G_{12}U_{20} = \sum_1 I_g$$
$$-G_{21}U_{10} + G_{22}U_{20} = \sum_2 I_g$$

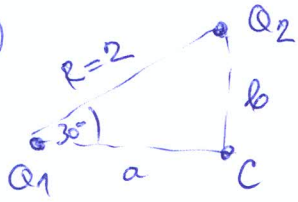
} ZA DETALJE POGLEDATI UDŽBENIK NA STRANI 53.

b)



$$\left(\frac{1}{R_3}+G_1+\frac{1}{R_5}\right)U_{10} - \frac{1}{R_5}U_{20} = \frac{E_3}{R_3} + I_{g1}$$
$$-\frac{1}{R_5}U_{10} + \left(\frac{1}{R_5}+\frac{1}{R_4}+\frac{1}{R_2}\right)U_{20} = \frac{E_2}{R_2}$$

2.



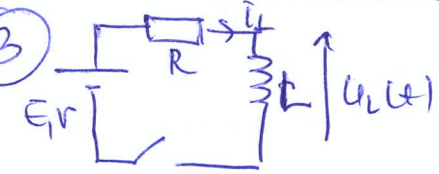
$a = R\cos 30° = \frac{R\sqrt{3}}{2}$, $b = R\sin 30° = \frac{R}{2}$

$V_C = V_{C1} + V_{C2} = 0$

$$V_C = \frac{Q_1}{4\pi\varepsilon_0 a} + \frac{Q_2}{4\pi\varepsilon_0 b} = \frac{Q_1}{2\pi\varepsilon_0 R\sqrt{3}} + \frac{Q_2}{2\pi\varepsilon_0 R} = 0$$

$$\boxed{\frac{Q_1}{Q_2} = -\sqrt{3}}$$

3.

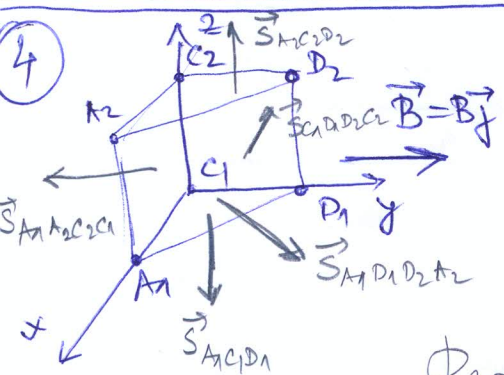


$$i_L(t) = \frac{E}{r+R}\left(1 - e^{-t/\tau}\right), \quad \tau = \frac{L}{R+r}$$

$$W_L(t) = \frac{1}{2}L\,i^2(t)$$

$$W_{L\infty} = \frac{1}{2}L\left(\frac{E}{r+R}\right)^2 = \frac{LE^2}{2(r+R)^2}$$

4.



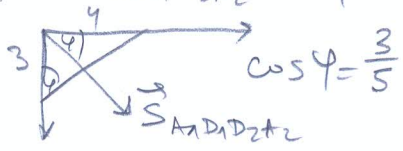
$$\Phi_{A_2D_2C_2} = \vec{B}\vec{S}_{A_2D_2C_2} = BS_{A_2D_2C_2}\cos\sphericalangle(\vec{B}, \vec{S}_{A_2D_2C_2}) = BS_{A_2D_2C_2}\cos 90° = 0$$

$$\Phi_{A_1D_1C_1} = \vec{B}\vec{S}_{A_1D_1C_1} = BS_{A_1D_1C_1}\cos\sphericalangle(\vec{B}, \vec{S}_{A_1D_1C_1}) = BS_{A_1D_1C_1}\cos 90° = 0$$

$$\Phi_{C_1D_1D_2C_2} = \vec{B}\vec{S}_{C_1D_1D_2C_2} = BS_{C_1D_1D_2C_2}\cos\sphericalangle(\vec{B}, \vec{S}_{C_1D_1D_2C_2}) = BS_{C_1D_1D_2C_2}\cos 90° = 0$$

$$\Phi_{A_1A_2C_2C_1} = \vec{B}\vec{S}_{A_1A_2C_2C_1} = BS_{A_1A_2C_2C_1}\cos\sphericalangle(\vec{B}, \vec{S}_{A_1A_2C_2C_1}) = B\cdot\overline{A_1C_1}\cdot\overline{A_1A_2}\cos 180°$$
$$= -15\cdot 10^{-4}\,\text{Wb}$$

$$\Phi_{A_1D_1D_2A_2} = \vec{B}\vec{S}_{A_1D_1D_2A_2} = BS_{A_1D_1D_2A_2}\cos\sphericalangle(\vec{B}, \vec{S}_{A_1D_1D_2A_2}) = BS_{A_1D_1D_2A_2}\cos\varphi$$
$$= B\cdot\overline{A_1D_1}\cdot\overline{D_1D_2}\cos\varphi = 15\cdot 10^{-4}\,\text{Wb}$$



$\cos\varphi = \frac{3}{5}$

(5) $N=20 \quad \Phi(t)=\sin\left(1000t+\frac{\pi}{4}\right)$ Wb

$$e_{ind}=-N\frac{d\Phi}{dt}=-20\frac{d}{dt}\left(\sin\left(1000t+\frac{\pi}{4}\right)\right)=-20\cdot\cos\left(1000t+\frac{\pi}{4}\right)\cdot 1000$$

$$e_{ind}=-20000\cos\left(1000t+\frac{\pi}{4}\right)\,[V]=-20\cos\left(1000t+\frac{\pi}{4}\right)\,[kV]$$

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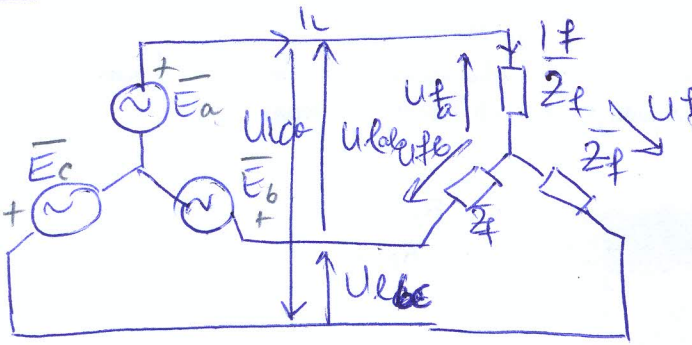
(6)



$E_a=E_b=E_c=100\,V$

$\varphi_a=0$

$\varphi_b=120°=\frac{2\pi}{3}$

$\varphi_c=240°=\frac{4\pi}{3}$

$\overline{Z_f}=10e^{j\frac{\pi}{6}}$

$U_l=\sqrt{3}U_f=E\sqrt{3}=100\sqrt{3}\,V$

$U_f=E=100\,V$

$Z_f=10\,\Omega$

$I_f=\frac{U_f}{Z_f}=\frac{100}{10}=10A$

$I_l=I_f=10A$

$\varphi_a=\varphi_a-\theta=0-\frac{\pi}{6}=-\frac{\pi}{6}\Rightarrow \boxed{\overline{I_{La}}=\overline{I_{Fa}}=10e^{-j\pi/6}}$

$\varphi_b=\varphi_b-\theta=\frac{2\pi}{3}-\frac{\pi}{6}=\frac{3\pi}{6}-\frac{\pi}{2}\Rightarrow \boxed{\overline{I_{Lb}}=\overline{I_{Fb}}=10e^{+j\pi/2}}$

$\varphi_c=\varphi_c-\theta=\frac{4\pi}{3}-\frac{\pi}{6}=\frac{7\pi}{6}\Rightarrow \boxed{\overline{I_{lc}}=\overline{I_{fc}}=10e^{+j7\pi/6}}$

$\overline{E_a}=\boxed{\overline{U_{Fa}}=100e^{j0}}$

$\overline{E_b}=\boxed{\overline{U_{Fb}}=100e^{j2\pi/3}}$

$\overline{E_c}=\boxed{\overline{U_{Fc}}=100e^{j4\pi/3}}$

$$\overline{U_{Lab}}=\overline{U_{fa}}-\overline{U_{fb}}=100e^{j0}-100e^{+j2\pi/3}$$
$$=100e^{j\pi/3}\left(e^{-j\pi/3}-e^{+j\pi/3}\right)=100e^{j\pi/3}\left(-2j\sin\pi/3\right)$$
$$=100e^{j\pi/3}\cdot 2\sin\pi/3\,e^{-j\pi/2}=100\cdot\frac{\sqrt{3}}{2}e^{j(\pi/3-\pi/2)}$$

$$\boxed{\overline{U_{Lab}}=100\sqrt{3}e^{-j\pi/6}}$$

$$\overline{U_{lbc}}=\overline{U_{fb}}-\overline{U_{fc}}=100e^{j2\pi/3}-100e^{j4\pi/3}=100e^{j2\pi/3}\left(1-e^{j2\pi/3}\right)$$
$$=100e^{j2\pi/3}e^{j\pi/3}\left(e^{-j\pi/3}-e^{j\pi/3}\right)=100e^{j\pi}\left(-2j\sin\frac{\pi}{3}\right)$$
$$=100e^{j\pi}\cdot 2e^{-j\pi/2}\cdot\frac{\sqrt{3}}{2}=\boxed{100\sqrt{3}e^{j\pi/2}=\overline{U_{lbc}}}$$

$$\overline{U_{lca}}=\overline{U_{fc}}-\overline{U_{fa}}=100e^{j\frac{4\pi}{3}}-100=100e^{j2\pi/3}\left(e^{j2\pi/3}-e^{-j2\pi/3}\right)=100e^{j\frac{2\pi}{3}}2j\sin\frac{2\pi}{3}$$
$$=100\cdot e^{j\frac{2\pi}{3}}\cdot 2\cdot\frac{\sqrt{3}}{2}e^{j\frac{\pi}{2}}=\boxed{100\sqrt{3}^{7\pi/6}=\overline{U_{lca}}}$$