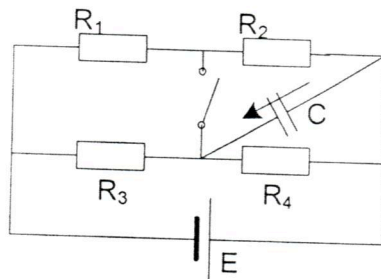


PRVI KOLOKVIJUM IZ ELEKTROTEHNIKE
7. decembar 2015.

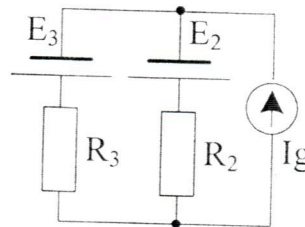
GRUPA 1

1. a) U kolu prikazanom na Slici 1 prekidač je zatvoren i uspostavljeno je stacionarno stanje. Odrediti količinu naelektrisanja na kondenzatoru u naznačenom smeru. Poznato je: $C = 100 \text{ pF}$, $R_1 = 10\Omega$, $R_2 = 20\Omega$, $R_3 = 40\Omega$, $R_4 = 30\Omega$, $E = 20\text{V}$. (5 poena)

b) Ako su ploče kondenzatora površine $S = 10\text{cm}^2$, a rastojanje između njih iznosi $d = 1,2\text{mm}$, odrediti energiju kondenzatora (1 poen), dielektričnu konstantu dielektrika između ploča (2 poena) i intenzitet vektora električnog polja u dielektriku (2 poena).



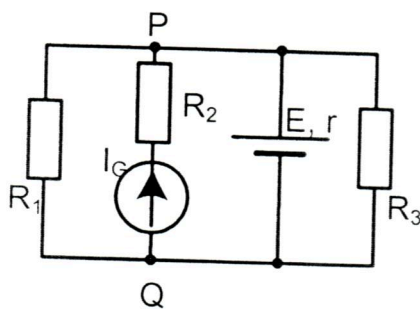
Slika 1



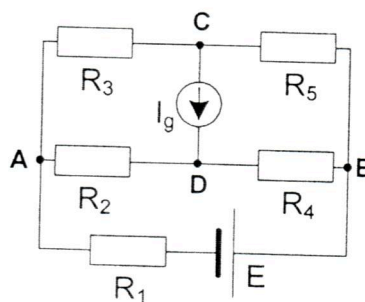
Slika 2

2. U kolu na Slici 2 poznato je: $R_1 = R_2 = 10\Omega$, $E_2 = 120\text{V}$, $E_3 = 40\text{V}$, $I_g = 10\text{A}$. Primenom metode napona između čvorova ili metode konturnih struja odrediti struju kroz granu sa naponskim generatorom E_3 , a zatim i snagu otpornika R_3 . (6 poena)

3. U kolu na Slici 3 poznate su vrednosti: $R_1 = R_2 = R_3 = 12\Omega$, $r = 12\Omega$, $E = 12\text{V}$, $I_G = 5\text{A}$. Primenom metode superpozicije odrediti napon U_{PQ} . (6 poena)



Slika 3

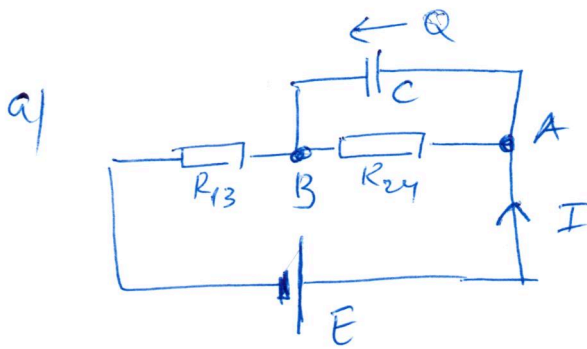
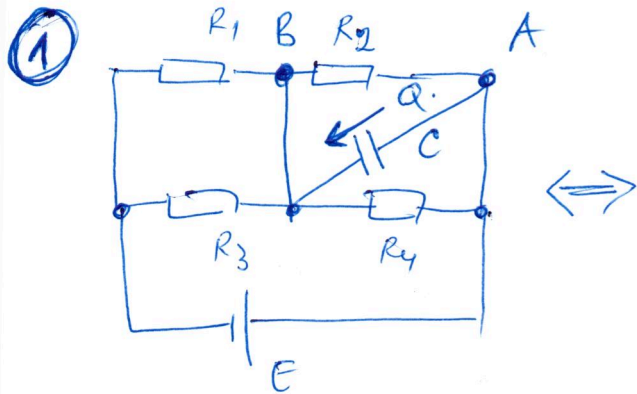


Slika 4

4. Primenom Tevenenove teoreme odrediti intenzitet struje kroz otpornik R_5 u kolu na Slici 4. Poznato je: $E = 30\text{V}$, $R_1 = 20\Omega$, $R_2 = R_3 = R_4 = 10\Omega$, $R_5 = 40\Omega$, $I_g = 1\text{A}$. (8 poena)

Izrada kolokvijuma traje 90 minuta. Nije dozvoljena upotreba digitrona. Na vežbanci napisati broj grupe zadatka. Papir sa tekstom zadatka predaje se u vežbanci tj. ne sme se izneti.

GRUPA 1



$$R_{13} = \frac{R_1 R_3}{R_1 + R_3}$$

$$R_{13} = \frac{10 \cdot 40}{10 + 40}$$

$$R_{13} = \frac{400}{50}$$

$R_{13} = 8 \Omega$

$$R_{24} = \frac{R_2 R_4}{R_2 + R_4} = \frac{20 \cdot 30}{20 + 30} = \frac{600}{50}$$

$R_{24} = 12 \Omega$

$$I = \frac{E}{R_{13} + R_{24}} = \frac{20}{12 + 8} = \frac{20}{20} = 1A$$

$U_{AB} = R_{24} I = 12 \cdot 1 = 12V$

$Q = C \cdot U_{AB} = 100p \cdot 12 = 1200pC = 1,2 \mu C$

b)

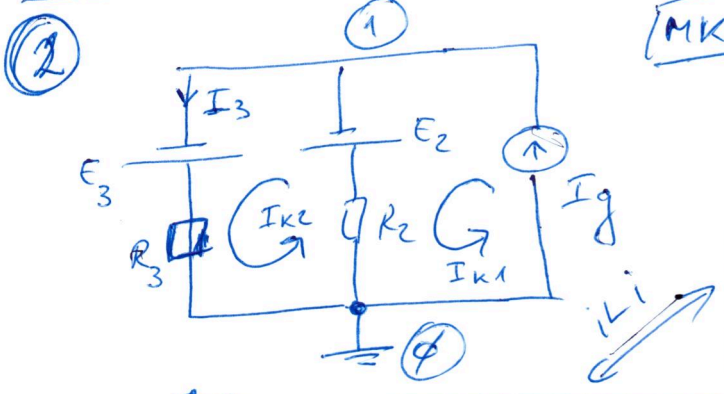
$$W = \frac{1}{2} Q U_{AB} = \frac{1}{2} \cdot 1,2 \mu \cdot 12$$

$W = 7,2 \mu J$

$K = \frac{U_{AB}}{d} = \frac{12V}{1,2mm} = 10kV/m$

$$C = \epsilon \frac{S}{d} \Rightarrow \epsilon = \frac{cd}{S}$$

$\epsilon = \frac{100pF \cdot 1,2mm}{10 \cdot 10^{-4}m^2} = 12 \cdot 10^{-11} \frac{F}{m}$



MKS: $I_{k1} = I_g = 10A$

$$-R_2 I_{k1} + (R_2 + R_3) I_{k2} = E_3 - E_2$$

$$-10 \cdot 10 + (10 + 10) I_{k2} = 40 - 120$$

$I_{k2} = \frac{20}{20} = 1A$

$I_3 = I_{k2} = 1A$

$P_{R3} = R_3 I_3^2 = 10 \cdot 1^2 = 10W$

MNC: $G_{10} U_{10} = \sum I_{(1)}$

$$\left(\frac{1}{R_3} + \frac{1}{R_2}\right) U_{10} = -\frac{E_3}{R_3} - \frac{E_2}{R_2} + I_g$$

$$\frac{2}{10} U_{10} = -\frac{40}{10} - \frac{120}{10} + 10$$

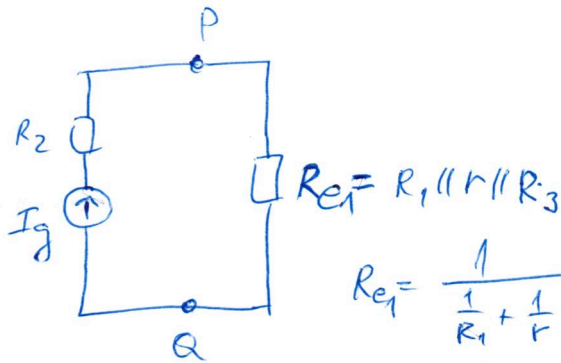
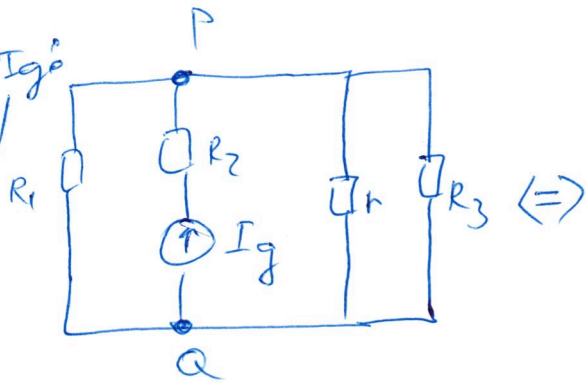
$U_{10} = -30V$

$$U_{10} = +R_3 I_3 - E_3 \Rightarrow I_3 = \frac{U_{10} + E_3}{R_3} = \frac{-30 + 40}{10} = 1A$$

$P_{R3} = R_3 I_3^2 = 10W$

JEDNA OD
DVE DNE
METODE

3
Debye I_g
($E=0$)

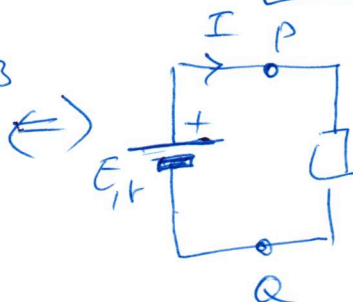
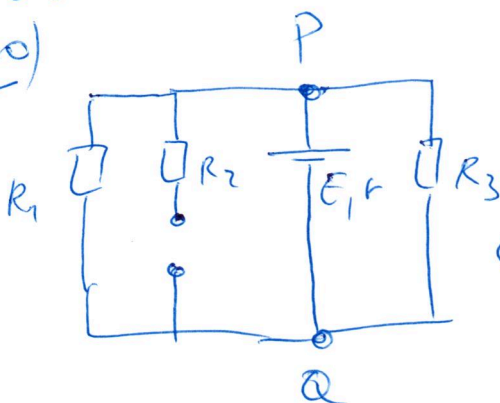


$$R_{e1} = \frac{1}{\frac{1}{R_1} + \frac{1}{r} + \frac{1}{R_3}}$$

$$R_{e1} = \frac{1}{\frac{1}{12} + \frac{1}{12} + \frac{1}{12}} = 4 \Omega$$

$$U_{PQ}(I_g) = R_{e1} I_g = 4 \cdot 5 = 20V$$

Debye E:
($I_g=0$)



$$R_{e2} = R_1 \parallel R_3$$

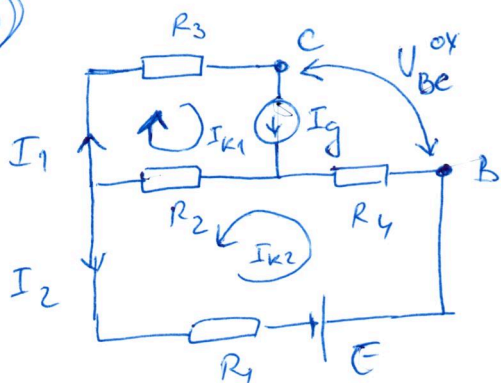
$$R_{e2} = \frac{R_1 R_3}{R_1 + R_3} = 6 \Omega$$

$$I = \frac{E}{R_{e2} + r} = \frac{12}{6 + 12} = \frac{12}{18} = \frac{2}{3}$$

$$U_{PQ}^{(E)} = R_{e2} \cdot I = \frac{2}{3} \cdot 6 = 4V$$

$$U_{PQ} = U_{PQ}(I_g) + U_{PQ}(E) = 24V$$

4



$$I_{k1} = I_g = 1A$$

$$+ R_2 I_{k1} + (R_1 + R_2 + R_4) I_{k2} = E$$

$$10 \cdot 1A + 40 I_{k2} = 30$$

$$40 I_{k2} = 20 \Rightarrow I_{k2} = \frac{1}{2} A$$

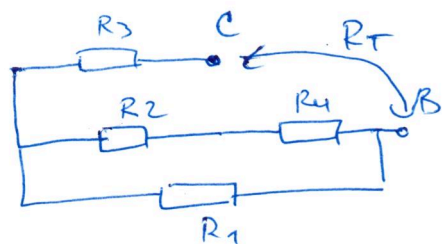
$$I_1 = I_{k1} = 1A$$

$$I_2 = I_{k2} = \frac{1}{2} A$$

$$U_{BC}^{ov} = R_3 I_1 - R_4 I_2 + E$$

$$U_{BC}^{ov} = 10 \cdot 1 - 20 \cdot \frac{1}{2} + 30 = 30V$$

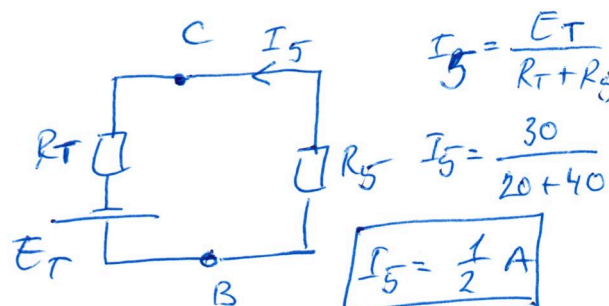
$$E_T = U_{BC}^{ov} = 30V$$



$$R_T = R_{Be} = R_3 + R_1 \parallel (R_2 + R_4)$$

$$R_T = R_3 + \frac{R_1 \cdot (R_2 + R_4)}{R_1 + R_2 + R_4} = 10 + \frac{20 \cdot 20}{40}$$

$$R_T = 20 \Omega$$



$$I_5 = \frac{E_T}{R_T + R_5}$$

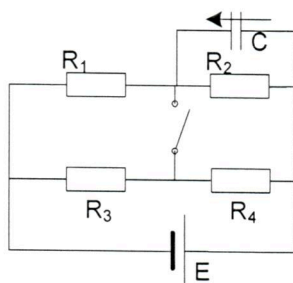
$$I_5 = \frac{30}{20 + 40}$$

$$I_5 = \frac{1}{2} A$$

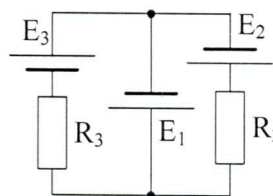
PRVI KOLOKVIJUM IZ ELEKTROTEHNIKE
7. decembar 2015.

GRUPA 2

1. a) U kolu prikazanom na Slici 1 prekidač je otvoren i uspostavljeno je stacionarno stanje. Odrediti količinu naelektrisanja na kondenzatoru u naznačenom smeru. Poznato je: $C = 50 \text{ nF}$, $R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, $R_4 = 4\Omega$, $E = 6\text{V}$. **(5 poena)**
- b) Ako su ploče kondenzatora kvadratnog oblika stranice $a = 5 \text{ cm}$, dielektrična konstanta dielektrika između ploča $\epsilon = 4 \cdot 10^{-8} \text{ F/m}$, odrediti energiju kondenzatora **(1 poen)**, rastojanje između ploča kondenzatora **(2 poena)** i intenzitet vektora električnog polja u dielektriku **(2 poena)**.

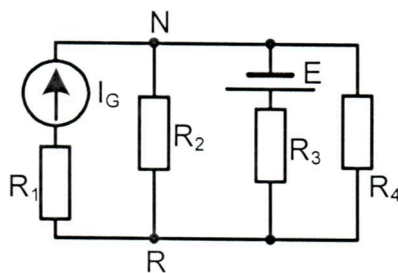


Slika 1

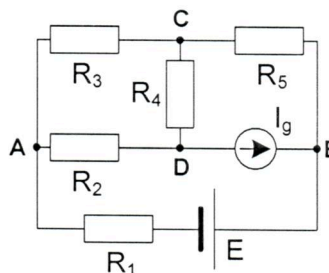


Slika 2

2. U kolu na Slici 2 poznato je: $R_1 = R_2 = 20\Omega$, $E_1 = 80\text{V}$, $E_2 = 120\text{V}$, $E_3 = 40\text{V}$. Primenom metode napona između čvorova ili metode konturnih struja odrediti struju kroz granu sa naponskim generatorom E_2 , a zatim i snagu otpornika R_2 . **(6 poena)**
3. U kolu na Slici 3 poznate su vrednosti: $R_1 = R_2 = R_3 = R_4 = 10\Omega$, $E = 4\text{V}$, $I_G = 1\text{A}$. Primenom metode superpozicije odrediti napon U_{NR} . **(6 poena)**



Slika 3



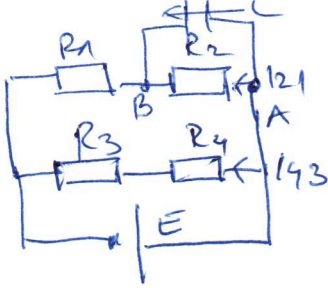
Slika 4

4. Primenom Tevenenove teoreme odrediti intenzitet struje kroz otpornik R_3 u kolu na Slici 4. Poznato je: $E = 60\text{V}$, $R_1 = 6\Omega$, $R_2 = 3\Omega$, $R_3 = 6\Omega$, $R_4 = 3\Omega$, $R_5 = 6\Omega$, $I_g = 1\text{A}$. **(8 poena)**

Izrada kolokvijuma traje 90 minuta. Nije dozvoljena upotreba digitrona. Na vežbanci napisati broj grupe zadatka. Papir sa tekstom zadatka predaje se u vežbanci tj. ne sme se izneti.

ГРУПА 2

1



a) $I_{21} = \frac{E}{R_1 + R_2} = 2A$

$U_C = U_{AB} = R_2 I_{21} = 4V$

$Q = C U_C = 200 \mu C$

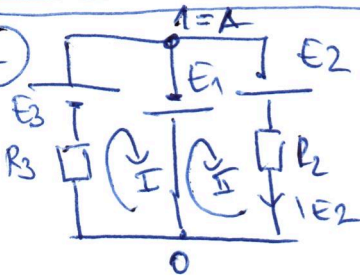
b) $W = \frac{1}{2} C U_C^2 = 400 \mu J$

$S = a^2 = 25 \cdot 10^{-4} m^2$

$\epsilon = \epsilon_0 \frac{S}{d} \Rightarrow d = \frac{\epsilon S}{C} = 2 mm$

$K = \frac{U}{d} = 2 \frac{kV}{m}$

2



МЕТОД НАПОНА УЗМЕЉУ ЧВОРОВА: $U_{10} = -E_1$

$U_{10} = -E_2 + R_2 I_{E2} \Rightarrow I_{E2} = \frac{U_{10} + E_2}{R_2} = 2A$

МЕТОД КОНТИРНИХ СТРУЈА:

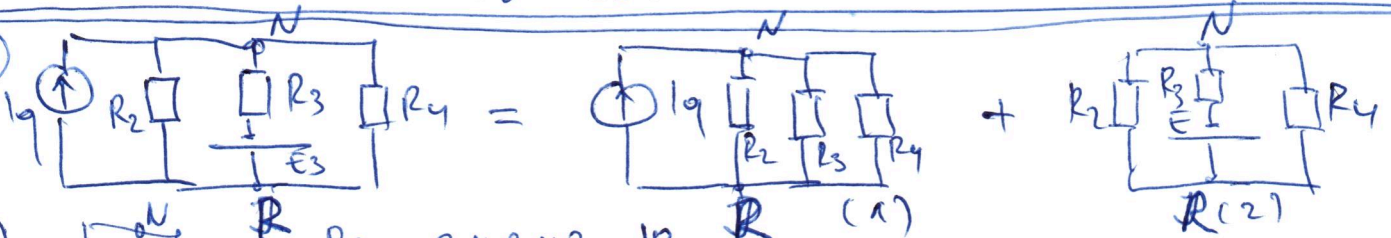
$R_3 I_I = E_3 + E_1 \Rightarrow I_I = \frac{E_1 + E_3}{R_3}$

$R_2 I_{II} = -E_1 + E_2 \Rightarrow I_{II} = \frac{E_2 - E_1}{R_2} = 2A$

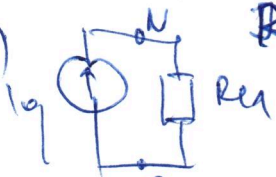
$I_{E2} = I_{II} = 2A$

СИЛА $P_{R2} = R_2 I_{E2}^2 = 80W$

3



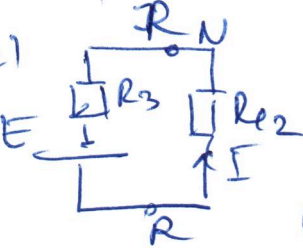
(1)



$R_{eq} = R_2 || R_3 || R_4 = \frac{10}{3} \Omega$

$U_{NR}^{(1)} = R_{eq} \cdot I_q = \frac{10}{3} V$

(2)



$R_{eq} = R_2 || R_4 = 5 \Omega$

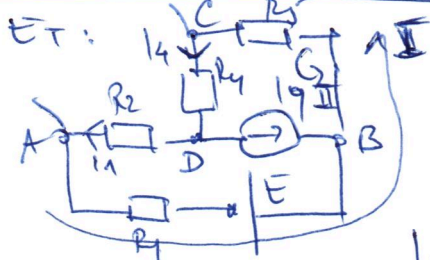
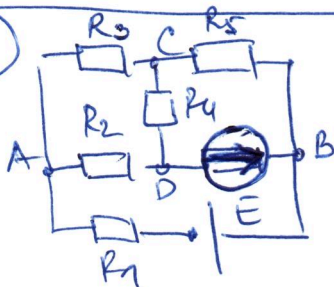
$I = \frac{E}{R_{eq} + R_3} = \frac{4}{15} A$

$U_{NR}^{(2)} = -R_2 I = -\frac{4}{3} V$

$U_{NR} = U_{NR}^{(1)} + U_{NR}^{(2)}$

$U_{NR} = 2V$

4



$I_{II} = I_q$

$(R_1 + R_2 + R_4 + R_5) I_{II} + (R_4 + R_5) I_I = E$

$I_I = \frac{E - (R_4 + R_5) I_q}{R_1 + R_2 + R_4 + R_5} = \frac{17}{6} A$

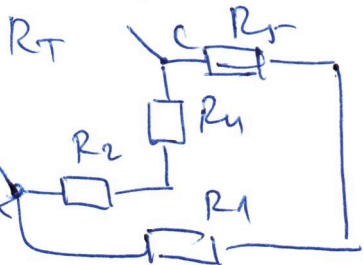
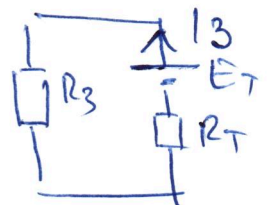
$14 = I_I + I_{II} = \frac{23}{6}$

$I_I = I_{II} = \frac{17}{6} A$

$E_T = U_{AB}^{(ov)} = R_2 I_1 + R_4 I_4 = 20V$

КАК СЕ БРАТУ:

$I_3 = \frac{E_T}{R_T + R_3} = 2A$

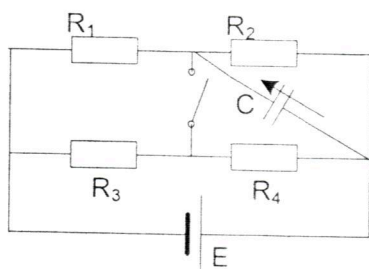


$R_T = R_{AC} = (R_2 + R_4) || (R_1 + R_5) = 4 \Omega$

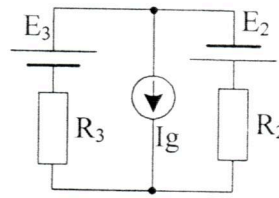
GRUPA 3

1. a) U kolu prikazanom na Slici 1 prekidač je otvoren i uspostavljeno je stacionarno stanje. Odrediti količinu naelektrisanja na kondenzatoru u naznačenom smeru. Poznato je: $C = 15 \text{ pF}$, $R_1 = 5\Omega$, $R_2 = 15\Omega$, $R_3 = 10\Omega$, $R_4 = 20\Omega$, $E = 4\text{V}$. (5 poena)

b) Ako su ploče kondenzatora površine $S = 15\text{mm}^2$, a rastojanje između njih iznosi $d = 0,1\text{mm}$, odrediti energiju kondenzatora (1 poen), dielektričnu konstantu dielektrika između ploča (2 poena) i intenzitet vektora električnog polja u dielektriku (2 poena).



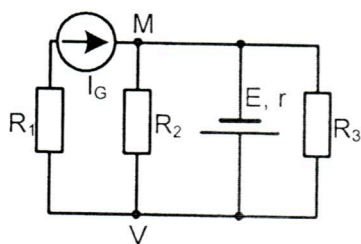
Slika 1



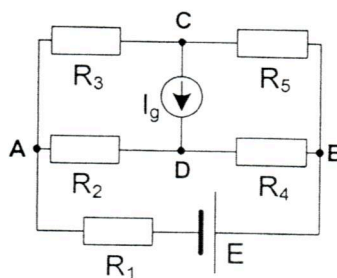
Slika 2

2. U kolu na Slici 2 poznato je: $R_1 = R_2 = 10\Omega$, $E_2 = 40\text{V}$, $E_3 = 120\text{V}$, $I_g = 4\text{A}$. Primenom metode napona između čvorova ili metode konturnih struja odrediti struju kroz granu sa naponskim generatorom E_3 , a zatim i snagu otpornika R_3 . (6 poena)

3. U kolu na Slici 3 poznate su vrednosti: $R_1 = R_2 = R_3 = 15\Omega$, $r = 15\Omega$, $E = 12\text{V}$, $I_G = 3\text{A}$. Primenom metode superpozicije odrediti napon U_{MV} . (6 poena)



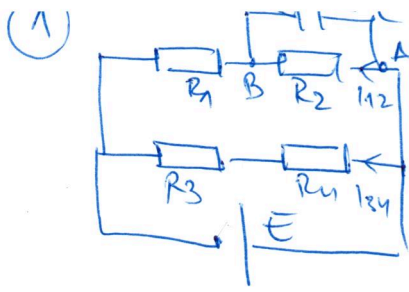
Slika 3



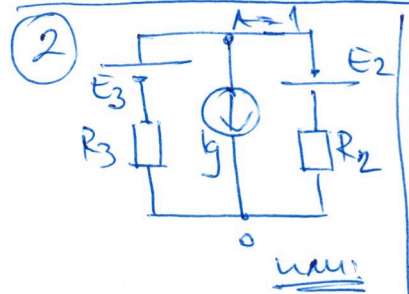
Slika 4

4. Primenom Tevenenove teoreme odrediti intenzitet struje kroz otpornik R_3 u kolu na Slici 4. Poznato je: $E = 40\text{V}$, $R_1 = 40\Omega$, $R_2 = 20\Omega$, $R_3 = 40\Omega$, $R_4 = R_5 = 20\Omega$, $I_g = 1\text{A}$. (8 poena)

Izrada kolokvijuma traje 90 minuta. Nije dozvoljena upotreba digitrona. Na vežbanci napisati broj grupe zadataka. Papir sa tekstom zadataka predaje se u vežbanci tj. ne sme se izneti.

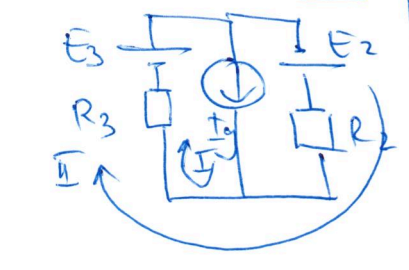


$I_{12} = \frac{E}{R_1 + R_2} = \frac{4}{20} = \frac{1}{5} \text{ A}$ b) $W = \frac{1}{2} Q U_c = 67.5 \text{ J}$
 $U_c = U_{AB} = R_2 I_{12} = 3 \text{ V}$ $\epsilon = \frac{C d}{S} = 10^{-10} \text{ F/m}$
 $Q = U_c \cdot C = 4 \text{ pC}$ $K = \frac{U}{d} = 30 \frac{\text{kV}}{\text{m}}$

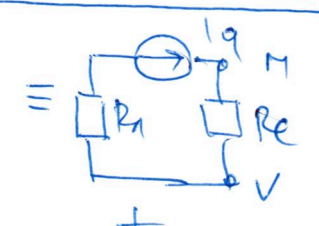
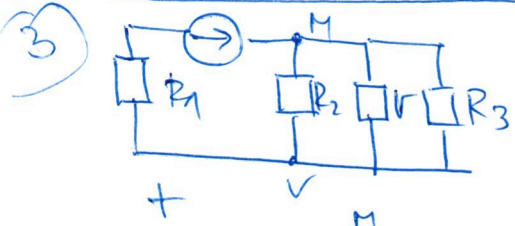


$U_{AO} \left(\frac{1}{R_2} + \frac{1}{R_3} \right) = \frac{E_3}{R_3} - I_q \frac{E_2}{R_2}$
 $\boxed{U_{AO} = 20 \text{ V}}$
 $U_{AO} = E_3 - R_3 I_{E3} \Rightarrow I_{E3} = \frac{E_3 - U_{AO}}{R_3} = 10 \text{ A}$

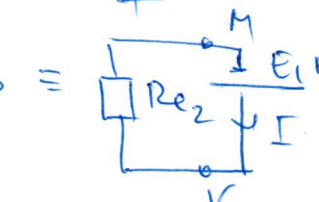
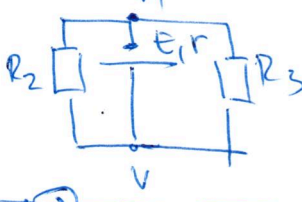
МЕТОДА
УКЛОНА
УЗМОЖНО
ВВОДИТЬ



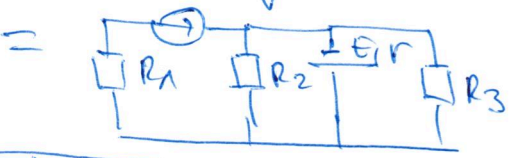
$I I = I_q$ МЕТОДА КОММУТИРУЮЩЕЙ ЦЕРКВИ
 $R_3 I_q + (R_2 + R_3) I_{II} = E_3 + E_2$ $P_{R3} = R_3 I_{E3}^2 = 1000 \text{ W}$
 $I_q = \frac{E_2 + E_3 - R_3 I_q}{R_2 + R_3} = 6 \text{ A}$
 $I_{E3} = I_I + I_{II} = 10 \text{ A}$



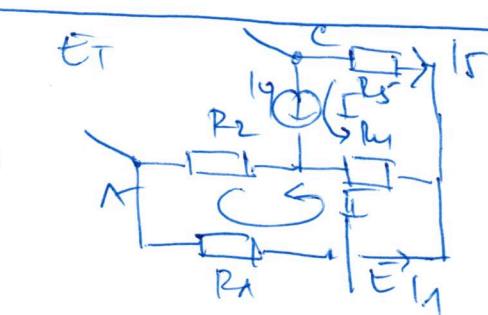
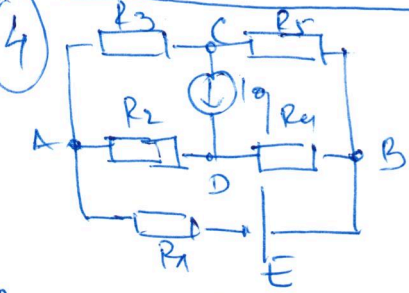
$R_{e1} = R_2 \parallel R_3 = 5 \Omega$
 $U_{Mv}(I_q) = R_{e1} I_q = 15 \text{ V}$



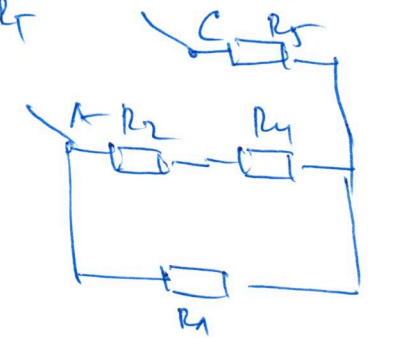
$R_{e2} = R_2 \parallel R_3 = \frac{15}{2} \Omega$
 $I = \frac{E}{r + R_2 \parallel R_3} = \frac{8}{15} \text{ A}$



$U_{Mv}(E) = -R_2 \parallel R_3 \cdot I = -4 \text{ V}$
 $U_{Mv} = U_{Mv}(I_q) + U_{Mv}(E) = 11 \text{ V}$



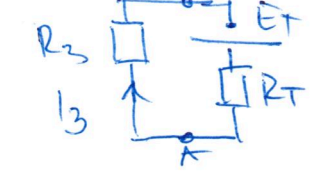
$(R_1 + R_2 + R_4) I_{II} - R_4 I_q = E$
 $I_{II} = I_q$
 $I_{II} = \frac{E + R_4 I_q}{R_1 + R_2 + R_4} = \frac{3}{4} \text{ A}$



$15 = -I I = -I_q = -1 \text{ A}$
 $I_1 = I_{II} = \frac{3}{4} \text{ A}$
 $E_T = U_{AC} = -R_5 I_1 - E + R_4 I_1 = 10 \text{ V}$

$R_T = R_5 + R_4 \parallel (R_2 + R_4) = 40 \Omega$

КАК СЕ ВПЯТНУ:



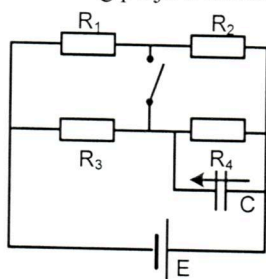
$I_3 = \frac{E_T}{R_T + R_3} = \frac{1}{8} = 0.125 \text{ A}$

PRVI KOLOKVIJUM IZ ELEKTROTEHNIKE
7. decembar 2015.

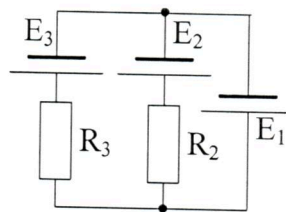
GRUPA 4

1. a) U kolu prikazanom na Slici 1 prekidač je zatvoren i uspostavljeno je stacionarno stanje. Odrediti količinu naelektrisanja na kondenzatoru u naznačenom smeru. Poznato je: $C = 0,1 \mu\text{F}$, $R_1 = 30\Omega$, $R_2 = 40\Omega$, $R_3 = 70\Omega$, $R_4 = 60\Omega$, $E = 45\text{V}$ (5 poena).

b) Ako su ploče kondenzatora kvadratnog oblika stranice $a = 10\text{cm}$, dielektrična konstanta dielektrika između ploča $\epsilon = 5 \cdot 10^{-9} \text{F/m}$, odrediti energiju kondenzatora (1 poen), rastojanje između ploča kondenzatora (2 poena) i intenzitet vektora električnog polja u dielektriku (2 poena).



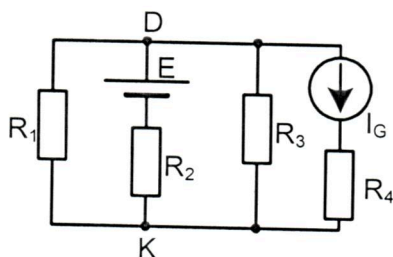
Slika 1



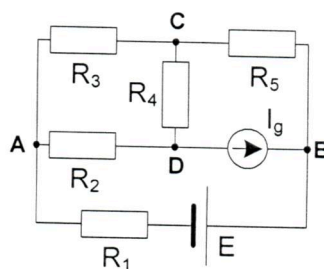
Slika 2

2. U kolu na Slici 2 poznato je: $R_1 = R_2 = 2\Omega$, $E_1 = 8\text{V}$, $E_2 = 12\text{V}$, $E_3 = 4\text{V}$. Primenom metode napona između čvorova ili metode konturnih struja odrediti struju kroz granu sa naponskim generatorom E_2 , a zatim i snagu otpornika R_2 . (6 poena)

3. U kolu na Slici 3 poznate su vrednosti: $R_1 = R_2 = R_3 = R_4 = 10\Omega$, $E = 6\text{V}$, $I_G = 3\text{A}$. Primenom metode superpozicije odrediti napon U_{DK} . (6 poena)



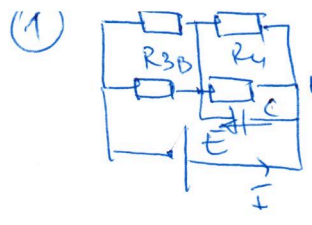
Slika 3



Slika 4

4. Primenom Tevenenove teoreme odrediti intenzitet struje kroz otpornik R_5 u kolu na Slici 4. Poznato je: $E = 10\text{V}$, $R_1 = R_2 = 15\Omega$, $R_3 = 30\Omega$, $R_4 = 15\Omega$, $R_5 = 70\Omega$, $I_g = 4\text{A}$. (8 poena)

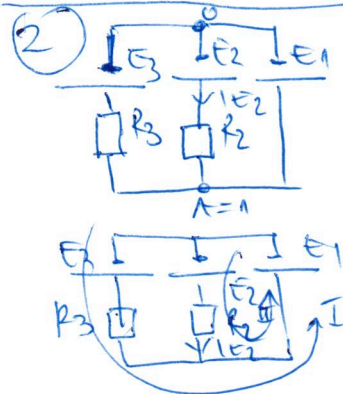
Izrada kolokvijuma traje 90 minuta. Nije dozvoljena upotreba digitrona. Na vežbanci napisati broj grupe zadataka. Papir sa tekstom zadataka predaje se u vežbanci tj. ne sme se izneti.



$I = \frac{E}{R_1 \parallel R_3 + R_2 \parallel R_4} = 1 \text{ A}$
 $U_C = U_{AB} = R_2 \parallel R_4 I = 24 \text{ V}$
 $Q = U_C C = 2,4 \mu\text{C}$

b) $S = a^2 = 10^{-2} \text{ m}^2$
 $d = \frac{\epsilon S}{C} = 0,5 \mu\text{m}$
 $W = \frac{1}{2} Q U = 28,8 \mu\text{J}$
 $\kappa = \frac{U}{d} = 48 \frac{\text{kV}}{\text{m}}$

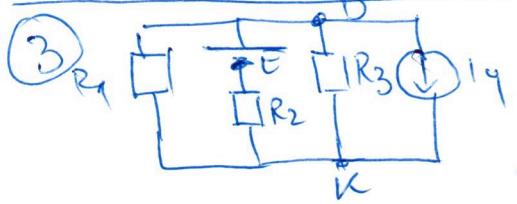
ГРУПА 4



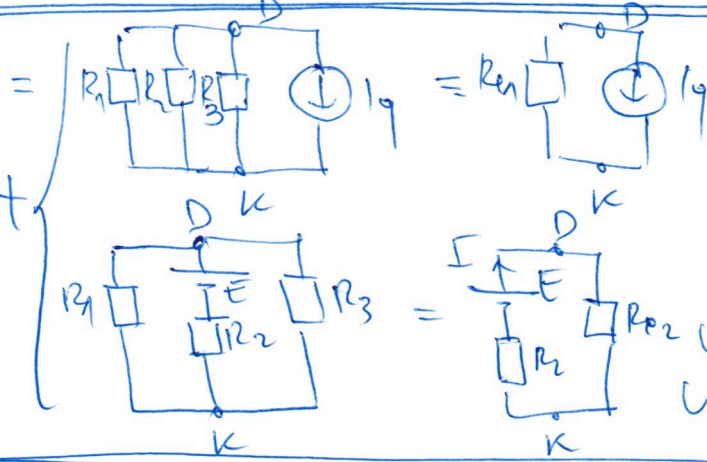
$U_{AO} = E_1 = U_{AO}$
 $U_{AO} = E_2 - R_2 I \Rightarrow I = \frac{E_2 - U_{AO}}{R_2} = 2 \text{ A}$
 или МЕТОДА КОНТУРНЫХ СТРУЙ
 $R_3 I_I = E_3 - E_1 \Rightarrow I_I = -2 \text{ A}$
 $R_2 I_{II} = E_2 - E_1 \Rightarrow I_{II} = 2 \text{ A}$
 $I_{E2} = I_{II} = 2 \text{ A}$

СИЛА
(ОБЕ МЕТОДЕ)
 $P_{R2} = R_2 I_{E2}^2 = 8 \text{ W}$

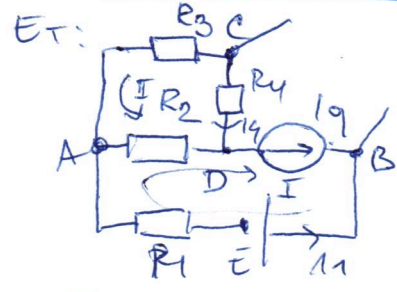
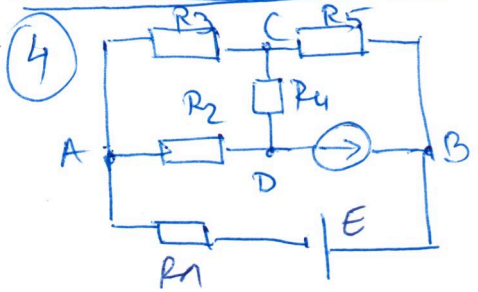
МЕТОДА НАПОНА ИЗМЪЖУ ЧВОРОВА



$U_{DK} = U_{DK}' + U_{DK}''$
 $U_{DK} = -8 \text{ V}$



$R_{eq} = \frac{10}{3} \Omega = R_1 \parallel R_2 \parallel R_3$
 $U_{DK}' = -R_{eq} I_q = -10 \text{ V}$
 $I = \frac{E}{R_2 + R_1 \parallel R_3}$
 $U_{DK}'' = R_2 I = 2 \text{ V}$

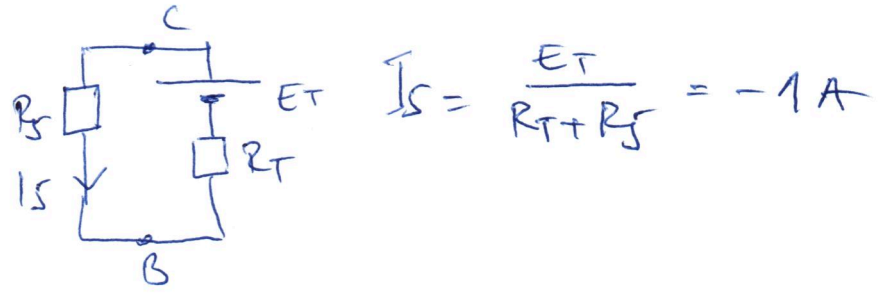
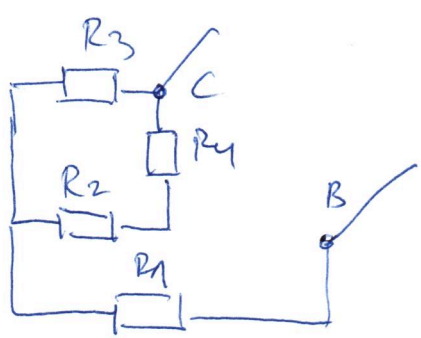


$(R_2 + R_3 + R_4) I_{II} + R_2 I_q = 0$
 $I_{II} = -1 \text{ A} \quad I_I = I_q$
 $I_4 = -I_{II} = 1 \text{ A}$
 $I_1 = -I_I = -I_q = -4 \text{ A}$

$E_T = U_{CB}^{or} = -E + R_1 I_1 - R_3 I_4 = -100 \text{ V}$

$R_T = R_1 \parallel R_3 \parallel (R_2 + R_4) = 30 \Omega$

КАК СЕ ЗАКЛУЧУ:



$I_S = \frac{E_T}{R_T + R_5} = -1 \text{ A}$