

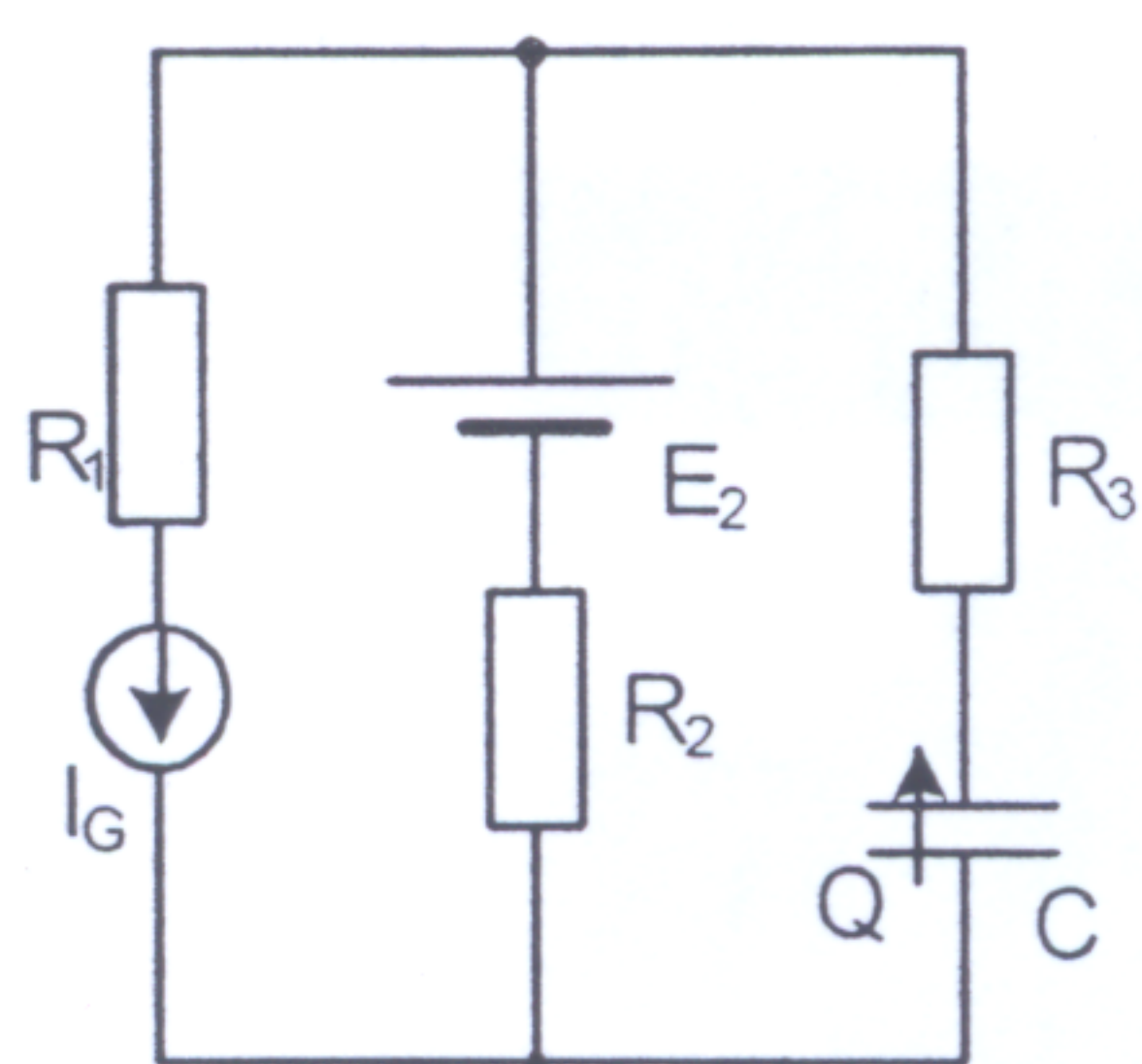
Elektrotehnika

15. jun 2016.

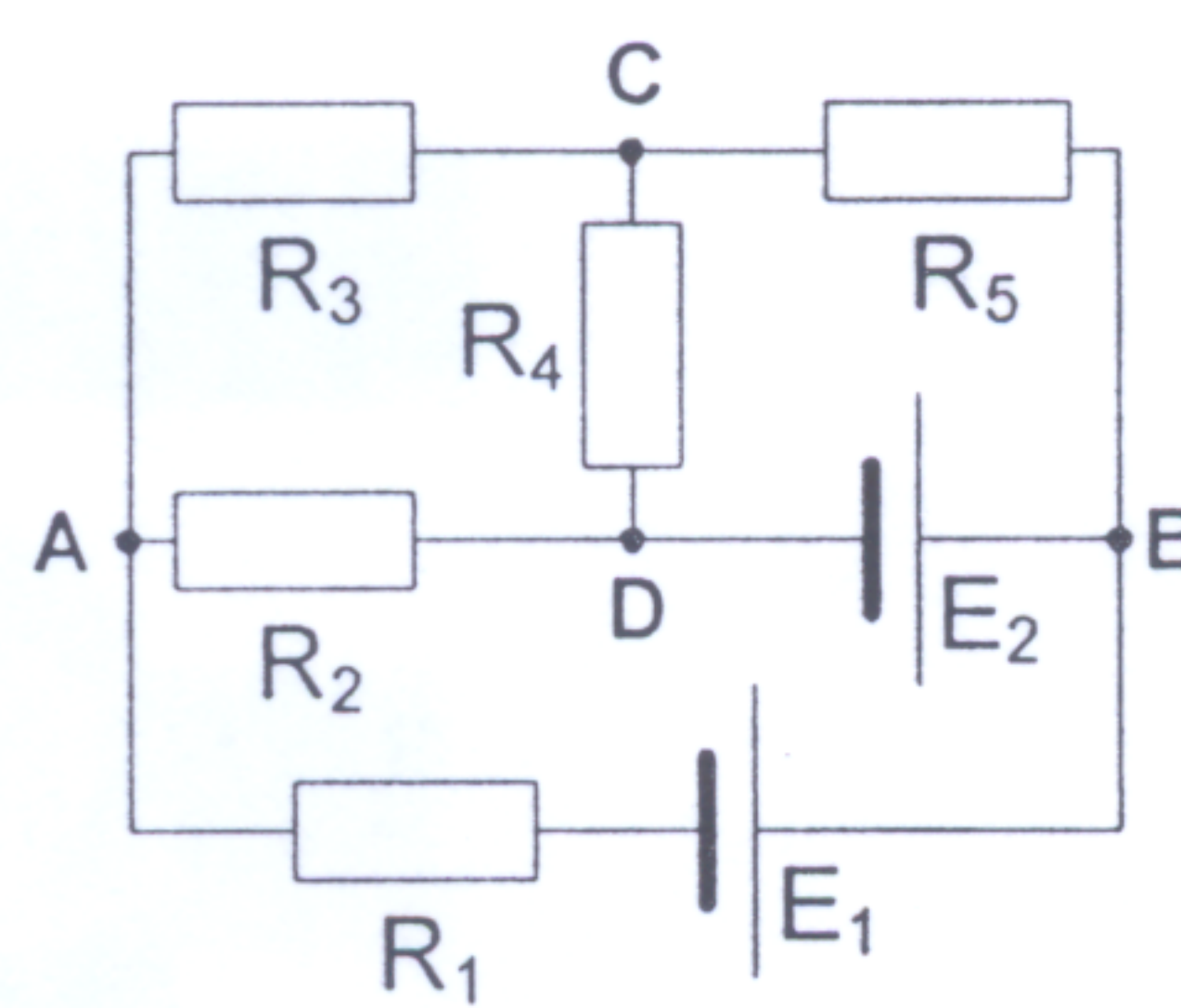
1. U kolu na Slici 1 poznato je: $R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, $C = 200\text{pF}$, $I_G = 2\text{A}$, $E_2 = 2\text{V}$.

a) Odrediti količinu naelektrisanja Q kojom je opterećen kondenzator C . (9 poena)

b) Odrediti dielektričnu konstantu ϵ i električno polje K u dielektriku kondenzatora, ako je C pločasti kondenzator površine ploča $S = 10\text{cm}^2$ i rastojanja između ploča $d = 0.1\text{mm}$. (6 poena)



Slika 1



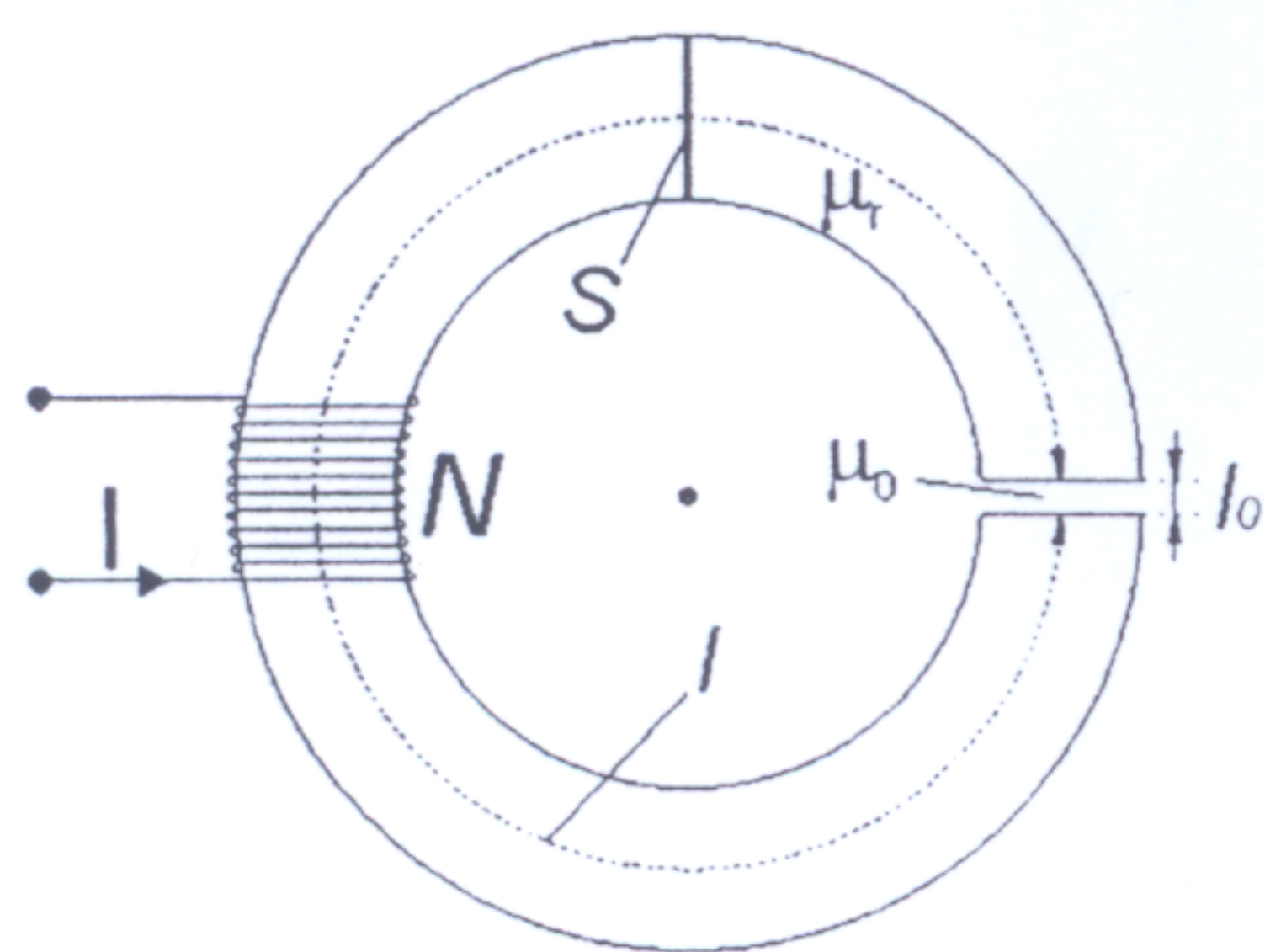
Slika 2

2. U kolu jednosmerne struje sa Slike 2 pimenom Tevenenove teoreme izračunati napon U_{BA} . Poznato je: $E_1 = 27\text{V}$, $E_2 = 10\text{V}$, $R_1 = R_2 = R_3 = R_4 = R_5 = 30\Omega$. (25 poena)

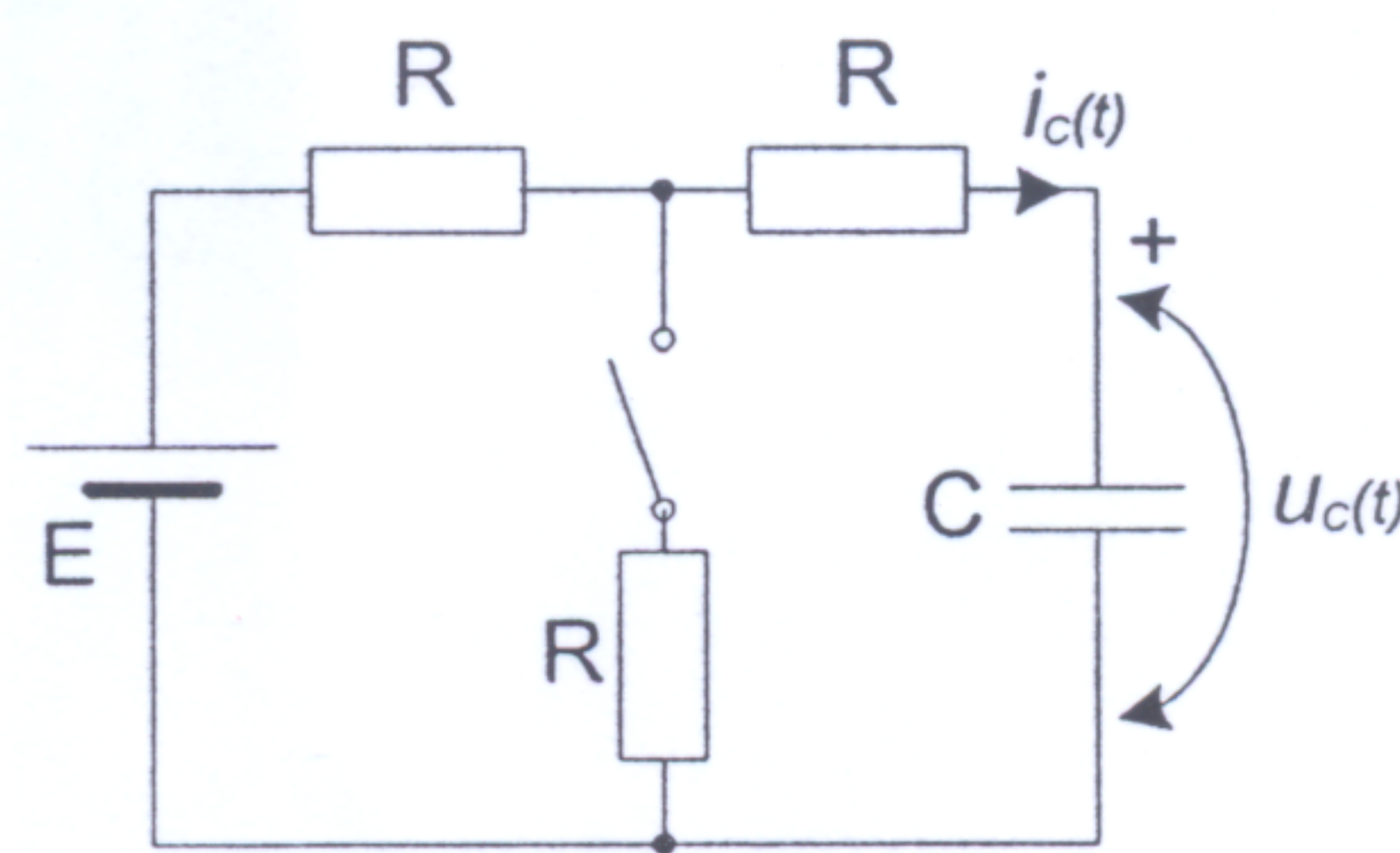
3. Na torus relativnog magnetnog permeabiliteta $\mu_r = 2000$, dužine srednje linije $l = 200\text{cm}$, površine poprečnog preseka $S = 10\text{cm}^2$ i debljine vazdušnog procepa $l_0 = 0.2\text{mm}$, namotan je provodnik sa $N = 300$ navojaka, kao što je prikazano na Slici 3.

a) Kroz namotaj protiče struja intenziteta $I = 1\text{A}$. Odrediti intenzitet vektora jačine magnetnog polja i vektora magnetne indukcije u vazdušnom procepu. (12 poena)

b) Odrediti induktivnost i enrgiju magnetnog polja kalema. (8 poena)



Slika 3



Slika 4

4. U kolu na Slici 4, poznate su vrednosti elemenata: $E = 30\text{V}$, $R = 10\Omega$, $C = 5\text{nF}$. Prekidač je otvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se zatvara.

a) Odrediti izraze za struju i napon kondenzatora nakon zatvaranja prekidača, i nacrtati odgovarajuće vremenske dijagrame. (15 poena)

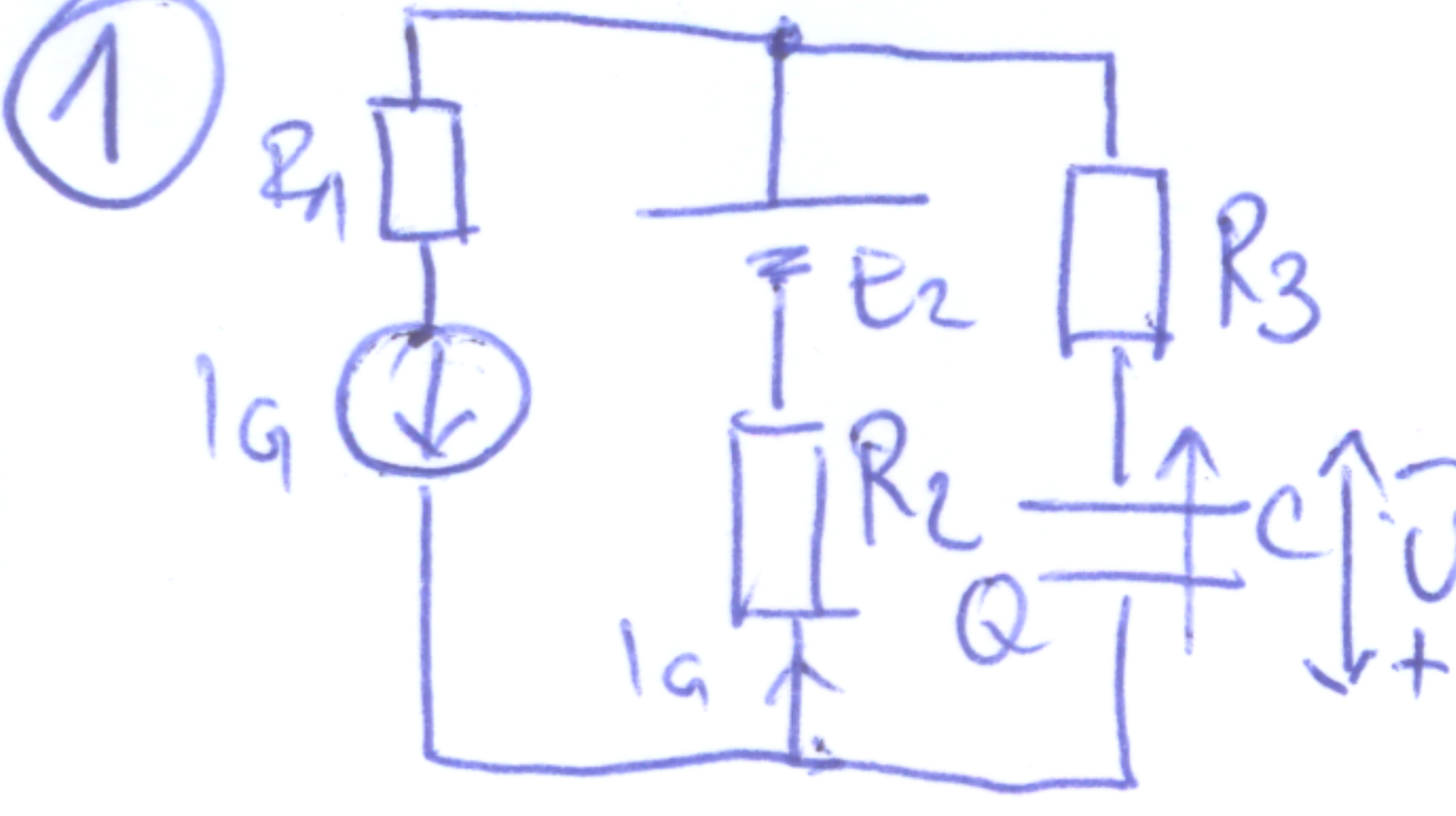
b) Odrediti vrednost struje kondenzatora u trenutku $t_1 = 50\text{ns}$. (5 poena)

5. Dva prijemnika vezana su paralelno i priključena na naizmenični napon efektivne vrednosti $U = 10\text{V}$. Kompleksna impedansa prvog prijemnika iznosi $\bar{Z}_1 = 1 + j\Omega$. Drugi prijemnik ima reaktivnu snagu $Q_2 = -45\text{VAr}$ i prividnu snagu $S_2 = 75\text{VA}$.

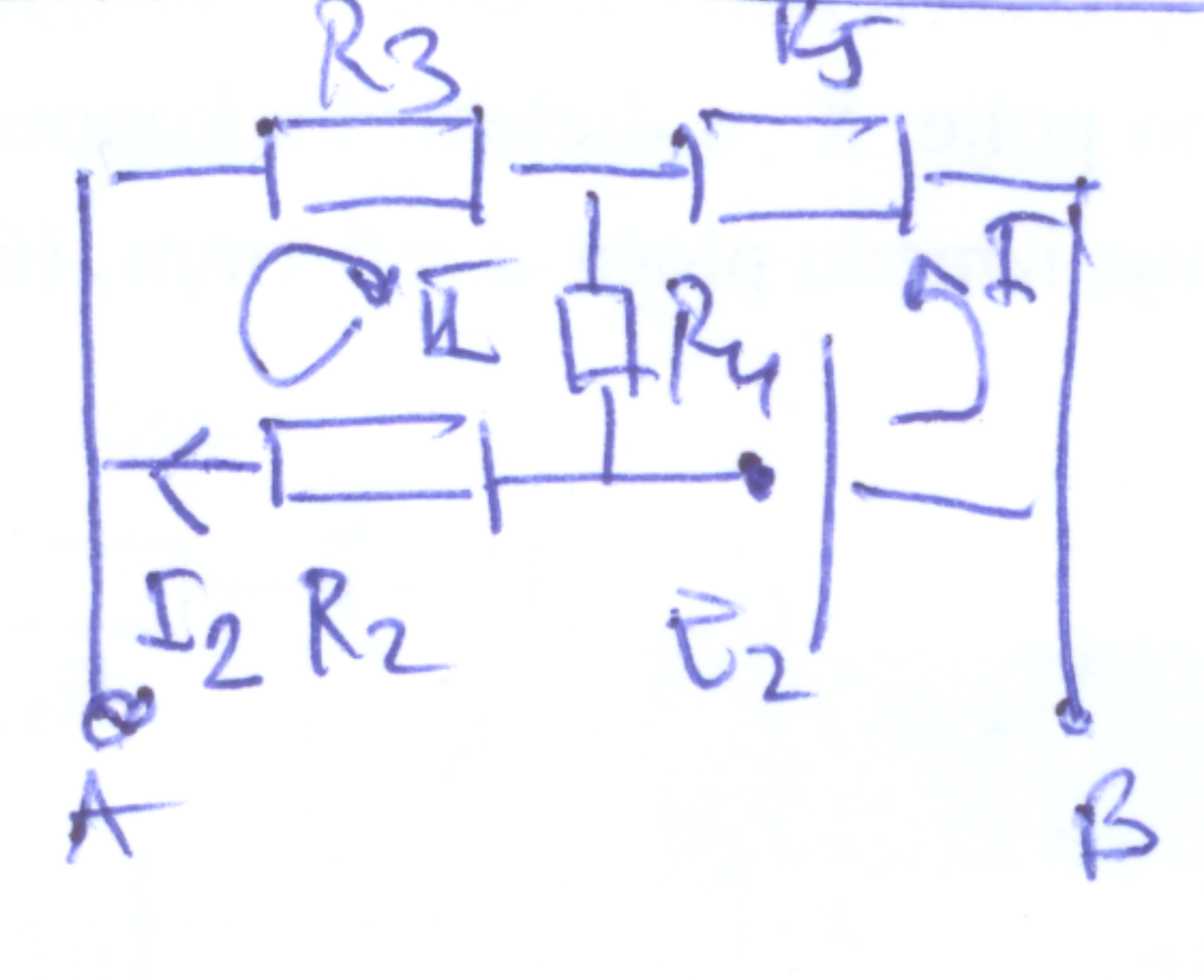
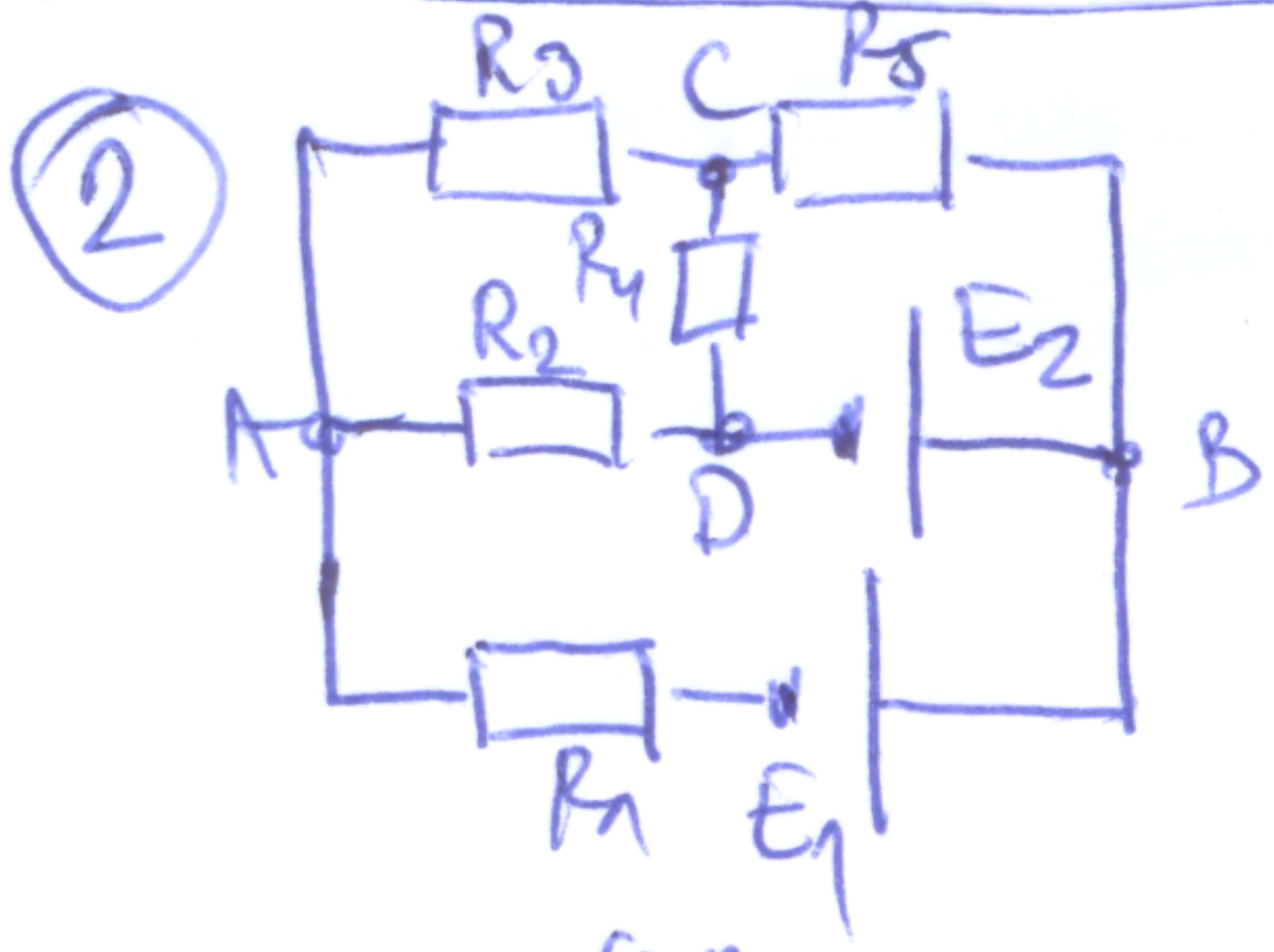
a) Odrediti efektivne vrednosti struja I_1 i I_2 u prijemnicima. (8 poena)

b) Odrediti ukupnu aktivnu, ukupnu reaktivnu i ukupnu prividnu snagu celokupnog potrošača. (8 poena)

c) Odrediti efektivnu vrednost struje koju paralelna veza potrošača uzima iz mreže. (4 poena)



a) $U_c = R_2 I_2 - E_2 = 2 \cdot 2 - 2 = 2V$
 $Q_c = U_c \cdot C = 400 pC$
 b) $C = \epsilon \frac{S}{d} \Rightarrow \epsilon = \frac{Cd}{S} = \frac{200 \cdot 10^{-12} \cdot 10^{-4}}{10 \cdot 10^{-4}} = 20 \cdot 10^{-12} \frac{F}{m}$
 $K = \frac{U_c}{d} = \frac{2V}{10^{-4}m} = 20 \frac{kV}{m}$



$$(R_4 + R_5) I_I + R_4 I_{II} = E_2$$

$$R_4 I_I + (R_2 + R_3 + R_4) I_{II} = 0$$

$$2R I_I + R I_{II} = E_2$$

$$R I_I + 3R I_{II} = 0$$

$$I_I = -3 I_{II}$$

$$-6R I_{II} + R I_{II} = E_2$$

$$I_I = -3 I_{II}$$

$$-5R I_{II} = E_2 \Rightarrow I_{II} = \frac{-E_2}{5R}$$

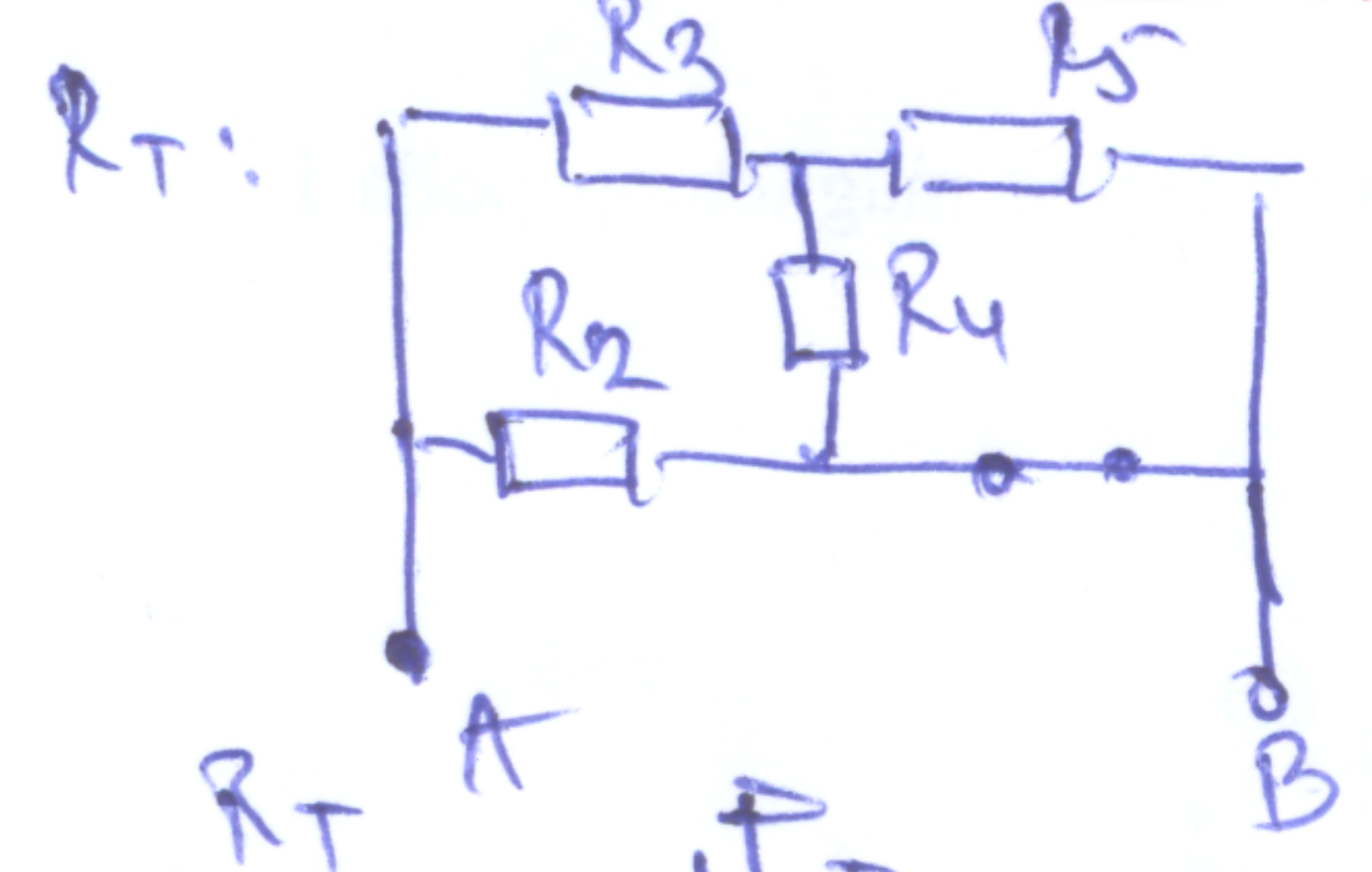
$$I_I = \frac{3E_2}{5R}$$

$$I_2 = I_{II} = \frac{-E_2}{5R}$$

$$E_T = U_{BA}^{(ov)} = E_2 + R_2 I_2$$

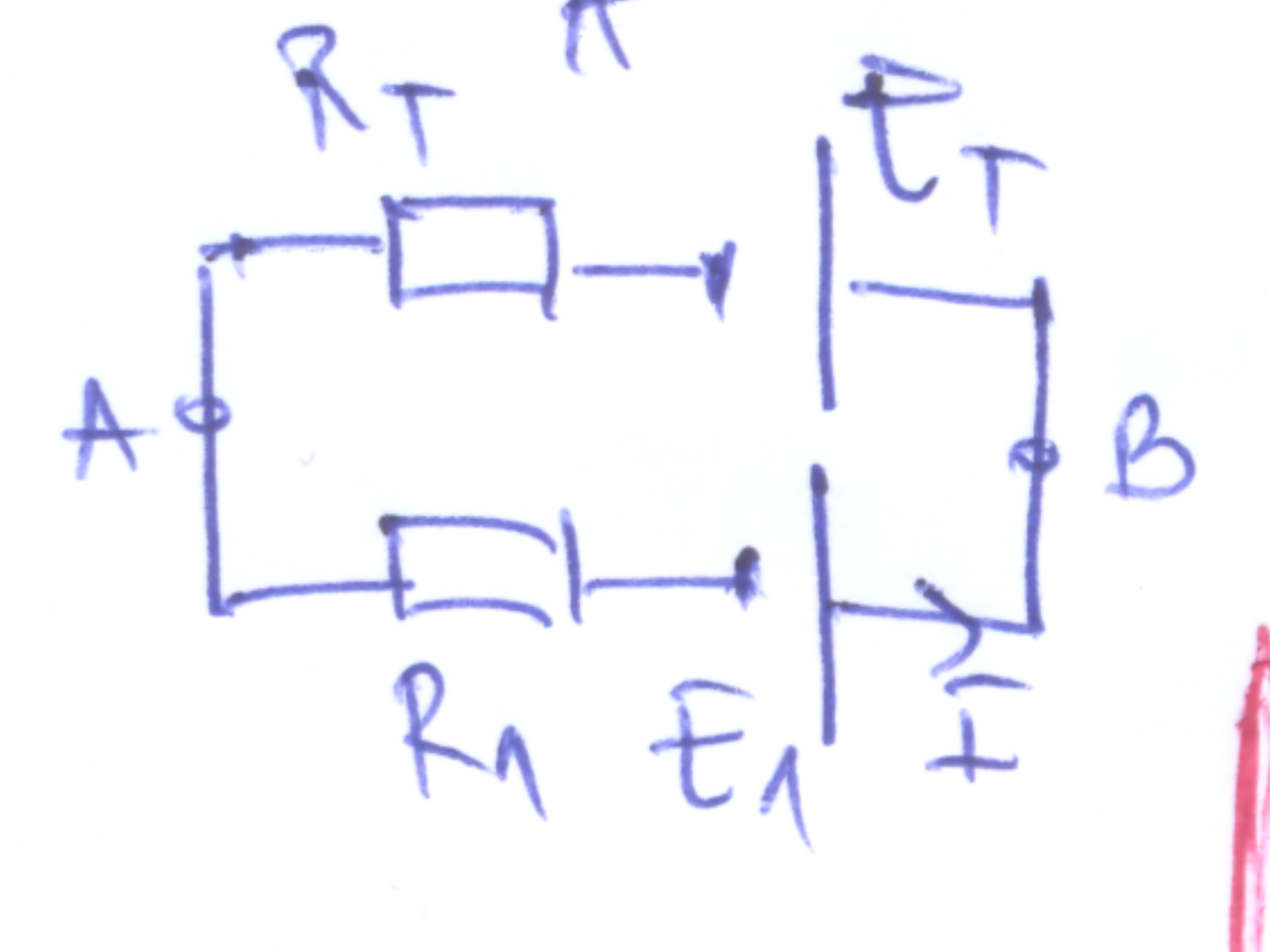
$$E_T = E_2 - R_2 \cdot \frac{E_2}{5R} = E_2 - R \frac{E_2}{5R}$$

$$E_T = E_2 - \frac{E_2}{5} = \frac{4}{5} E_2 = 8V$$



$$R_T = ((R_4 || R_5) + R_3) || R_2 = (R || R + R) || R = (\frac{R}{2} + R) || R$$

$$R_T = \frac{\frac{3R}{2} \cdot R}{\frac{3R}{2} + R} = \frac{\frac{3R^2}{2}}{\frac{5R}{2}} = \frac{3}{5} R = 18 \Omega$$



$$I = \frac{E_1 - E_T}{R_1 + R_T} = \frac{27 - 8}{30 + 18} = \frac{19}{48} A$$

$$U_{BA} = E_1 - R_1 I = 27 - 30 \cdot \frac{19}{48} = \frac{27 \cdot 8 - 19 \cdot 5}{8} = 15,125 V$$

3) $H_{ole} + H_{l} = NI$

a) $\frac{B l_0}{\mu_0} + \frac{B l}{\mu_{rel} \mu_0} = NI$

$$H_0 = \frac{B}{\mu_0} = \frac{0,1}{4 \cdot 10^{-7}} = \frac{10^6}{4} = 25 \cdot 10^{-4} \frac{A}{m}$$

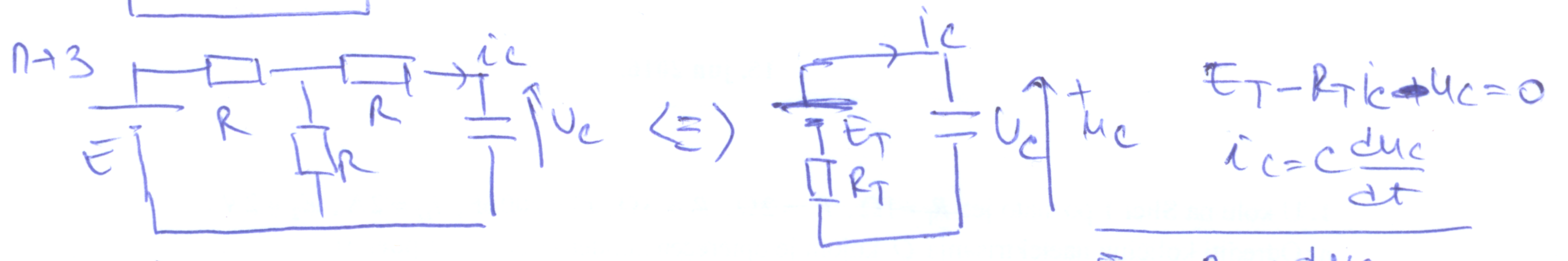
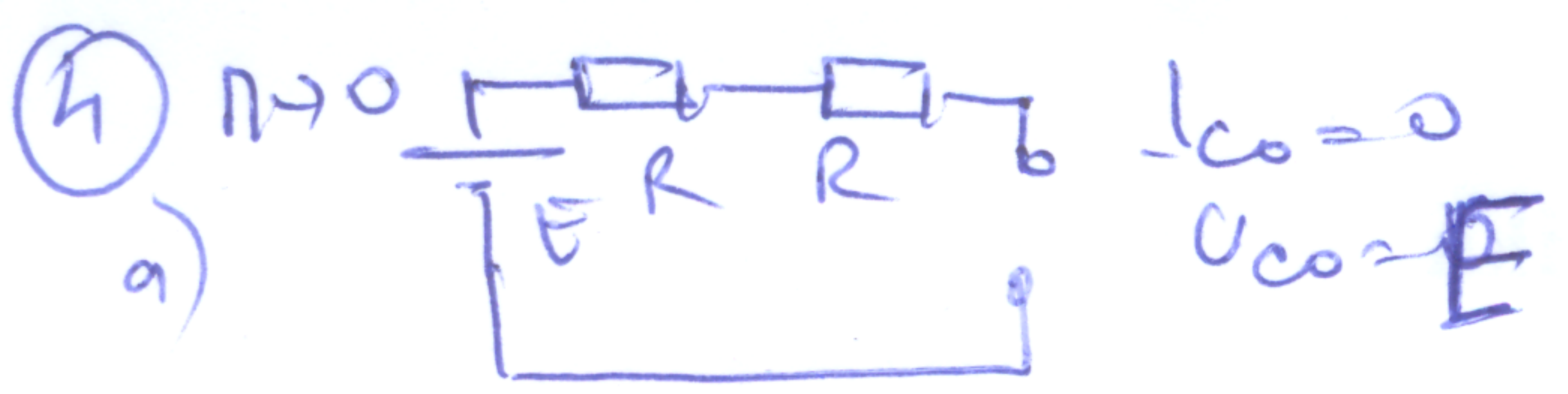
$$B = \frac{\mu_0 NI}{l_0 + l \mu_{rel}}$$

b) $L = \frac{N \Phi}{I} = \frac{NBS}{I} = \frac{300 \cdot 0,1 \cdot 10 \cdot 10^{-4}}{1} = 3 \cdot 10^{-2} H = L$

$$B = \frac{4 \cdot 10^{-7} \cdot 300 \cdot 1}{0,2 \cdot 10^{-3} + \frac{200 \cdot 10^{-2}}{2000}}$$

$$W = \frac{1}{2} L I^2 = \frac{1}{2} \cdot 30 m \cdot 1 = 15 mJ = W$$

$$B = \frac{12 \cdot 10^{-5}}{2 \cdot 10^{-4} + 10^{-3}} = \frac{12 \cdot 10^{-5}}{2 \cdot 10^{-4} + 10 \cdot 10^{-4}} = \frac{12 \cdot 10^{-5}}{12 \cdot 10^{-4}} = 0,1 T = B$$



$E_T = R \cdot \frac{E}{2R} = \frac{E}{2}$
 $R_T = R + R \parallel R = \frac{3R}{2}$

$\tau = R_T C = \frac{3RC}{2} = 75 \mu s$

$k = \frac{E_T}{R_T C}$

$\frac{dU_c}{dt} + \frac{U_c}{R_T C} = \frac{E_T}{R_T C} = k$

$U_c(t) = \frac{E}{2} (1 + e^{-t/\tau}) = 15 (1 + e^{-t/\tau}) \text{ V}$

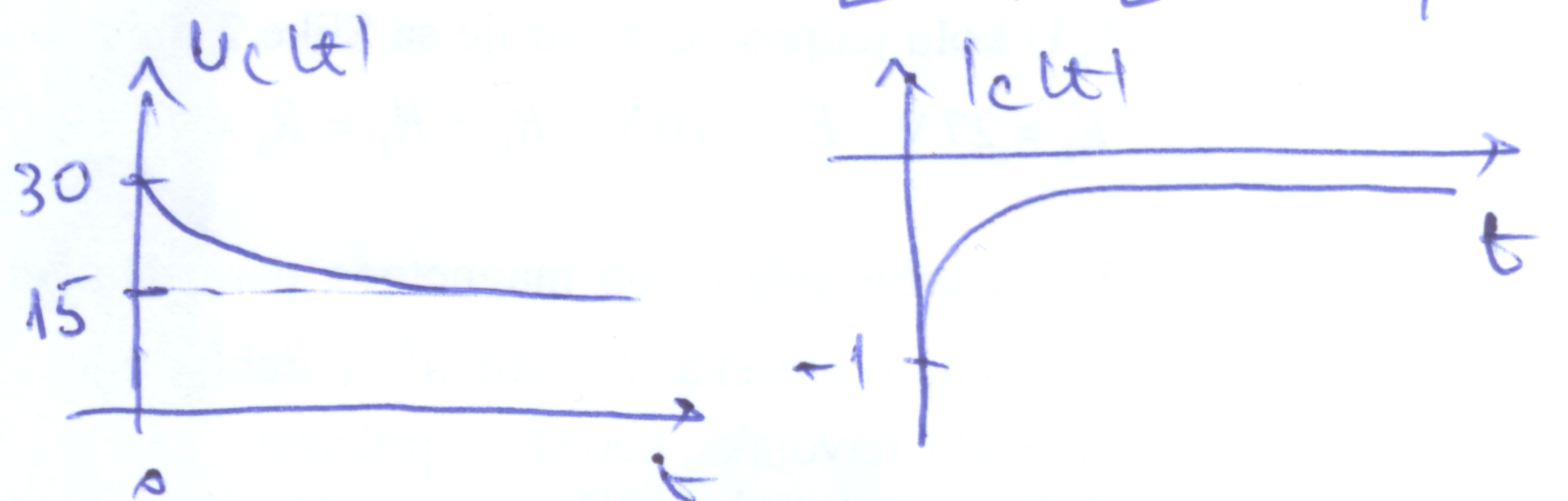
$i_c(t) = C \frac{dU_c}{dt} = \frac{CE}{2} \frac{1}{\tau} e^{-t/\tau}$
 $= \frac{CE}{2} \frac{1}{3RC} e^{-t/\tau} = \frac{-E}{3R} e^{-t/\tau}$

$i_c(t) = -e^{-t/\tau} \text{ A}$

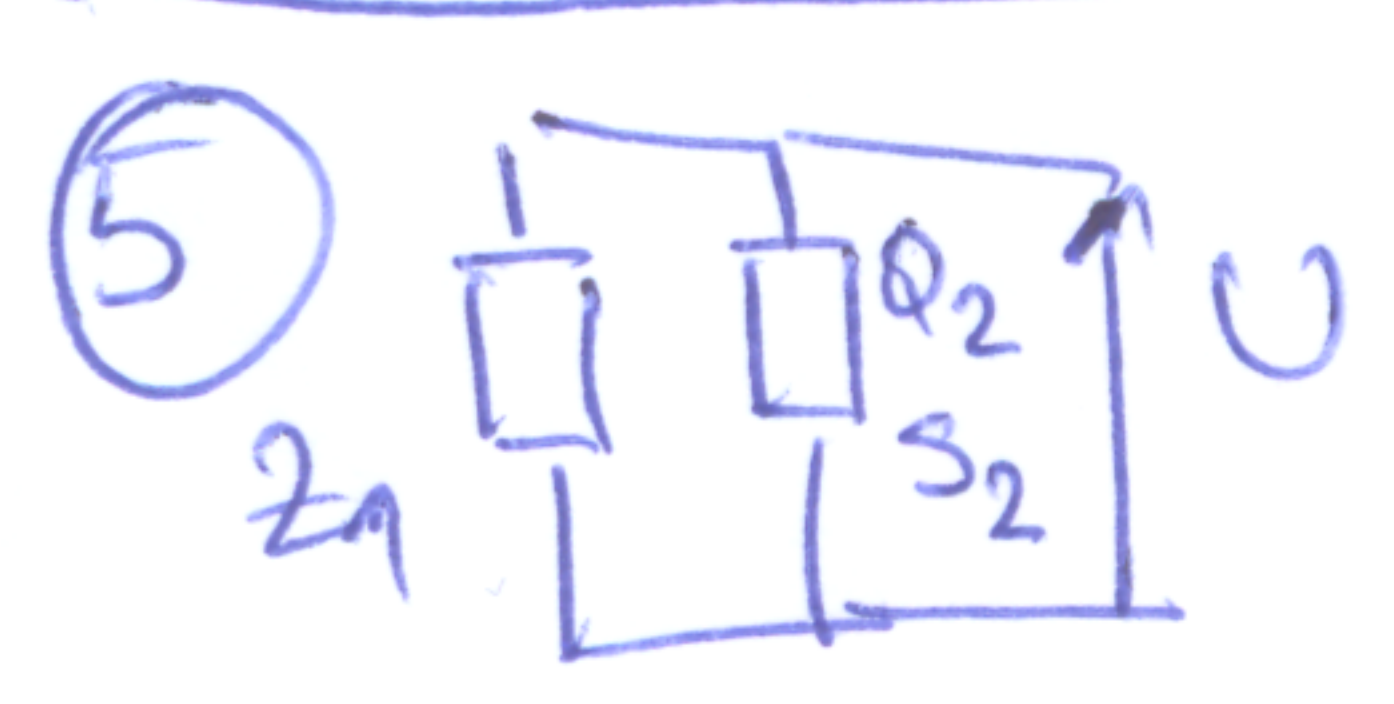
$U_c = Ae^{-t/\tau} + B$

$B = k \cdot \tau = \frac{E_T}{R_T C} \cdot R_T C = E_T = \frac{E}{2}$

$A + B = U_{co} = E \Rightarrow A = E - B = \frac{E}{2}$



b) $i_c(t_1 = 50 \mu s) = -e^{-\frac{t_1}{\tau}} = -e^{-\frac{50 \mu s}{75 \mu s}} = -e^{-2/3} = -0,513 \text{ A} = i_c(t_1)$



a) $Z_1 = \sqrt{1^2 + 1^2} = \sqrt{2} \Omega$

$I_1 = \frac{U}{Z_1} = \frac{10}{\sqrt{2}} = 5\sqrt{2} \text{ A}$

$I_2 = \frac{S_2}{U} = \frac{75}{10} = 7,5 \text{ A}$

b) $P_1 = P_1 I_1^2 = 1 \cdot (5\sqrt{2})^2 = 50 \text{ W}$

$Q_1 = X_1 I_1^2 = 1 \cdot (5\sqrt{2})^2 = 50 \text{ VAR}$

$P_2 = \sqrt{S_2^2 - Q_2^2} = 60 \text{ W}$

$Q_2 = -45 \text{ VAR}$

$P = P_1 + P_2 = 110 \text{ W}$

$Q = Q_1 + Q_2 = 5 \text{ VAR}$

$S = \sqrt{P^2 + Q^2} = \sqrt{110^2 + 5^2} = 5\sqrt{22^2 + 1} = 5\sqrt{485} = 110,13 \text{ VA}$

c) $I = \frac{S}{U} = \frac{5\sqrt{485}}{10} = \frac{\sqrt{485}}{2} = 11,01 \text{ A} = I$