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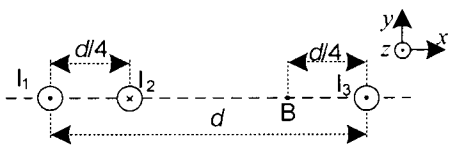
GRUPA2

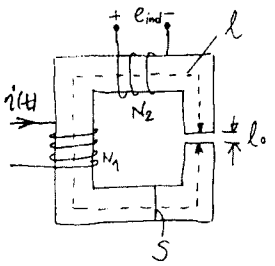
1. Na Slici 1 je prikazan poprečni presek tri beskonačno dugačka paralelna provodinka kroz koje protiču struje intenziteta $I_1 = 3I$, $I_2 = I$ i $I_3 = 2I$ u označenim smerovima. Provodnici se nalaze u vazduhu u istoj ravni. Odrediti i nacrtati rezultujući vektor magnetne indukcije u tački **B**. **(6 poena)**

2. U kolu na Slici 2 prikazano je magnetno kolo sa dva namotaja. Namotaj sa N_2 navojaka je otvorenih krajeva, a kroz namotaj sa N_1 navojaka protiče struja konstantnog intenziteta $i(t) = I$. Jezgro je površine poprečnog preseka S, dužine srednje linije l sa vazdušnim procepom debljine l_0. Relativna permeabilnost jezgra iznosi μ_r.

a) Odrediti intenzitet vektora jačine magnetnog polja u procepu. **(6 poena)**
b) Odrediti međusobnu induktivnost namotaja. **(4 poena)**
c) Odrediti elektromotornu silu indukovanu na krajevima namotaja sa N_2 navojaka. **(4 poena)**



Slika 1

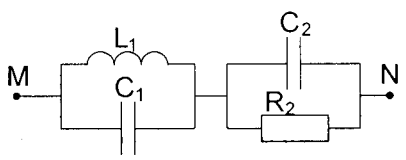


Slika 2

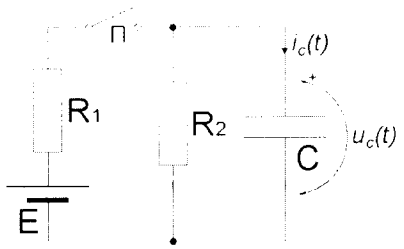
3. Na potrošaču nepoznateimpedanse poznate su trenutne vrednosti napona $u(t) = 20\sqrt{2}\sin(1000t + \pi)$ V i stuje $i(t) = 10\sin(1000t + 3\pi/4)$ A.

a) Odretiti kompleksnu impedansu potrošača. **(4 poena)**
b) Odrediti aktivnu, reaktivnu i prividnu snagu potrošača. **(4 poena)**
c) Nacrtati fazorski dijagram struje i napona na impedansi. **(2 poena)**
d) Ako se paralelno potrošaču priključi kondenzator kapacitivnosti $C = 250\mu F$ odrediti faktor snage celokupnog potrošača. **(5 poena)**

4.Izračunati kompleksnu vrednost impedanse $\overline{Z}_{MN}$ između tačaka M i N u kolu na Slici 3. Da li je impedansa sa Slike 3 pretežno kapacitivna ili induktivna? Poznato je: $L_1 = 1\,\text{mH}$, $C_1 = 5\,\mu\text{F}$, $R_2 = 10\Omega$, $C_2 = 10\mu\text{F}$, $\omega = 10^4$ rad/s. **(5 poena)**



Slika 3



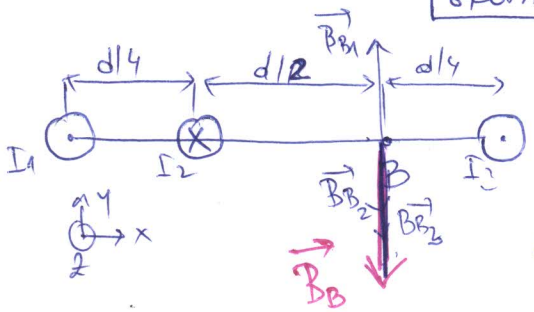
Slika4

5. Na sistem trofaznog napona $3\times1000\text{V}$, 50Hz priključen je simetričan trofazni potrošač povezan u trougao sa kompleksnom impedansom svake faze $\overline{Z} = 100\cdot e^{j\frac{\pi}{4}}\Omega$. Odrediti efektivnu vrednost linijske struje i reaktivnu snagu potrošača. **(8 poena)**

6.U kolu na Slici 4 poznate su vrednosti elemenata: $E = 20\,\text{V}$, $R_1 = R_2 = R = 5\Omega$ i $C = 2\mu\text{F}$. Prekidač Π je zatvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se otvara. Odrediti izraz za napon i struju kondenzatora nakon otvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame.**(12 poena)**

GRUPA 2

1.



$$\vec{B}_B = \vec{B}_{B_1} + \vec{B}_{B_2} + \vec{B}_{B_3}$$

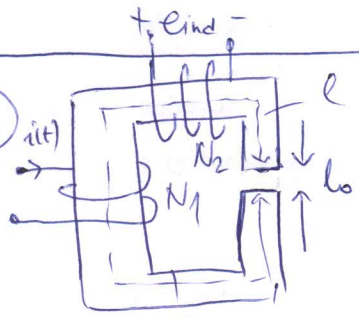
$$\vec{B}_1 = +\frac{\mu_0 I_1}{2\pi d_{B_1}}\vec{j} = \frac{\mu_0 I_1}{2\pi \frac{3d}{4}}\vec{j} = \frac{2\mu_0 I_1}{3\pi d}\vec{j} = \frac{2\mu_0 I}{\pi d}\vec{j}$$

$$\vec{B}_{B_2} = -\frac{\mu_0 I_2}{2\pi d_{B_2}}\vec{j} = -\frac{\mu_0 I_2}{2\pi \frac{d}{2}}\vec{j} = -\frac{\mu_0 I_2}{\pi d}\vec{j} = -\frac{\mu_0 I}{\pi d}\vec{j}$$

$$\vec{B}_{B_3} = -\frac{\mu_0 I_3}{2\pi d_{B_3}}\vec{j} = -\frac{\mu_0 I_3}{2\pi \frac{d}{4}}\vec{j} = -\frac{2\mu_0 I_3}{\pi d}\vec{j} = -\frac{4\mu_0 I}{\pi d}\vec{j}$$

$$\vec{B}_B = \frac{2\mu_0 I}{\pi d}\vec{j} - \frac{\mu_0 I}{\pi d}\vec{j} - \frac{4\mu_0 I}{\pi d}\vec{j} = \boxed{-\frac{3\mu_0 I}{\pi d}\vec{j}\ [T] = \vec{B}_B}$$

2.



a) $\oint \vec{H}d\vec{l} = \sum I$

$$Hl + H_0 l_0 = N_1 i(t)$$

$$\frac{Bl}{\mu_0\mu_r} + \frac{Bl_0}{\mu_0} = N_1 i(t)$$

$$B = \frac{N_1 i(t)}{\frac{l}{\mu_0\mu_r} + \frac{l_0}{\mu_0}}$$

$$\boxed{H_0 = \frac{B}{\mu_0} = \frac{N_1 i(t)}{\frac{l}{\mu_r} + l_0}}$$

b) $\Phi = B \cdot S = \frac{N_1 i(t) S}{\frac{l}{\mu_0\mu_r} + \frac{l_0}{\mu_0}}$

$$\boxed{L_{21} = L_{12} = \frac{N_2\Phi}{i(t)} = \frac{N_1 N_2 S}{\frac{l}{\mu_0\mu_r} + \frac{l_0}{\mu_0}}}$$

c) $e_{ind} = -\frac{N_2 d\Phi}{dt} = -\frac{N_1 N_2 S}{\frac{l}{\mu_0\mu_r} + \frac{l_0}{\mu_0}} \frac{di(t)}{dt}$

$i(t) = I = ct \Rightarrow \frac{di(t)}{dt} = 0$

$$\boxed{e_{ind} = 0}$$

3.

a) $u(t) = 20\sqrt{2}\sin(1000t + \pi) \Rightarrow \bar{U} = 20e^{j\pi}$ V

$i(t) = 10\sin(1000t + \frac{3\pi}{4}) \Rightarrow \bar{I} = \frac{10}{\sqrt{2}}e^{j\frac{3\pi}{4}}$ A

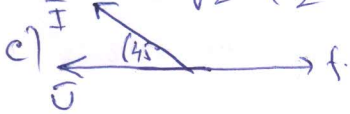
$$\Rightarrow \bar{Z} = \frac{\bar{U}}{\bar{I}} = \frac{20e^{j\pi}}{\frac{10}{\sqrt{2}}e^{j\frac{3\pi}{4}}} = 2\sqrt{2}e^{j(\pi - \frac{3\pi}{4})}$$

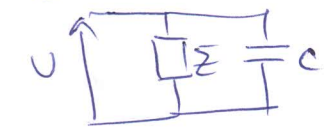
$$\boxed{\bar{Z} = 2\sqrt{2}e^{j\frac{\pi}{4}} = (2 + j2)\Omega}$$

b) $\bar{S} = \bar{U}\bar{I}^* = 20e^{j\pi} \cdot \frac{10}{\sqrt{2}}e^{-j\frac{3\pi}{4}} = \frac{200}{\sqrt{2}}e^{j\pi/4}$

$$\bar{S} = \frac{200}{\sqrt{2}} \cdot \left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right) = (100 + j100)\text{VA} \Rightarrow \boxed{P = 100\text{W},\ Q = 100\text{VAr},\ S = 100\sqrt{2}\text{VA}}$$

c)





d) $Q_c = -C\omega U^2 = -250 \cdot 10^{-6} \cdot 10^3 \cdot 400 = -25 \cdot 4 = -100$ VAr

$Q_{ukupno} = Q + Q_c = 100\text{VAr} - 100\text{VAr} = 0$

$P_{ukupno} = P + \cancel{P_c}^{0} = 100$ W

$S_{ukupno} = \sqrt{P^2_{ukupno} + \cancel{Q^2_{ukupno}}^{0}} = 100$ VA

$$\cos\varphi = \frac{P_{ukupno}}{S_{ukupno}} = 1$$

4.

$$\bar{Z}_{MN} = \bar{Z}_{e1} + \bar{Z}_{e2} = \frac{\bar{Z}_{L_1} \cdot \bar{Z}_{C_1}}{\bar{Z}_{L_1} + \bar{Z}_{C_1}} + \frac{\bar{Z}_{C_2} \cdot R_2}{\bar{Z}_{C_2} + R_2}$$

$\bar{Z}_{L_1} = j\omega L_1 = j \cdot 10^4 \cdot 10^{-3} = j10\Omega$

$\bar{Z}_{C_1} = -j\frac{1}{\omega C_1} = -j\frac{1}{10^4 \cdot 5 \cdot 10^{-6}} = -j20\Omega$

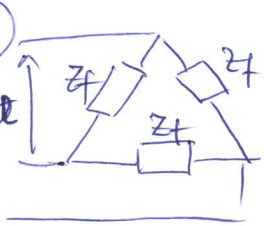
$\bar{Z}_{C_2} = -j\frac{1}{\omega C_2} = -j\frac{1}{10^4 \cdot 10 \cdot 10^{-6}} = -j10\Omega$

$$\bar{Z}_{MN} = \frac{j10 \cdot (-j20)}{j10 - j20} + \frac{-j10 \cdot 10}{10 - j10} = \frac{-j20 \cdot j10}{-j10} + \frac{-j10}{1-j}\frac{1+j}{1+j}$$

$$\bar{Z}_{MN} = j20 + \frac{(-j-j^2)10}{2} = j20 + 5(1-j)$$

$$\boxed{\bar{Z}_{MN} = (5 + j15)\Omega} \Rightarrow \text{INDUKTIVAN POTROŠAČ } \{\operatorname{Im}\bar{Z}_{MN}\} > 0$$

(5)



$3 \times 1000V \Rightarrow U_l = 1000V$

$\bar{Z}_f = \bar{Z} = 100 e^{j\frac{\pi}{4}} = 100\left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right) = (50\sqrt{2} + j50\sqrt{2})\,\Omega = (R + jX) = Z(\cos\varphi + j\sin\varphi)$

Potrošači u $\Delta \Rightarrow U_f = U_l = 1000V$

$Z = |\bar{Z}| = 100\,\Omega$ $\qquad I_f = \frac{U_f}{Z} = \frac{1000V}{100\Omega} = 10A$

$\sin\varphi = \frac{X}{Z} = \frac{\sqrt{2}}{2}$

$$I_l = I_f\sqrt{3} = 10\sqrt{3}\,A$$

$$Q = 3U_f I_f \sin\varphi = 3 \cdot 1000 \cdot 10 \cdot \frac{\sqrt{2}}{2} = 15\sqrt{2}\ kVAr$$

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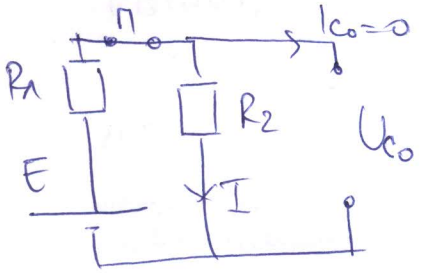
(6) VIDETI REŠENJE ZADATKA IV.5 IZ ZBIRKE ZADATAKA

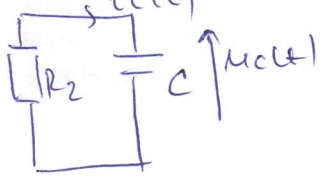
M → ZATVOREN STACIONARNO STANJE



$$I = \frac{E}{R_1+R_2}$$

$$U_{C0} = R_2 I = \frac{R_2}{R_1+R_2}E$$

M OTVOREN



$$R_2 i_C(t) + u_C(t) = 0 \quad , \quad i_C(t) = C\frac{du_C(t)}{dt}$$

$$R_2 C\frac{du_C(t)}{dt} + u_C(t) = 0$$

$$\frac{du_C(t)}{dt} + \frac{u_C(t)}{R_2 C} = 0$$

$$\boxed{\tau = R_2 C = 10\mu s}$$

$$u_C(t) = Ae^{-t/\tau} + B = \frac{R_2}{R_1+R_2}Ee^{-t/\tau} = 10e^{-t/\tau}\,[V]$$

$t = 0$

$B = K\cdot\tau = 0$

$A + B = U_{C0} \Rightarrow A = U_{C0} - B$

$A = U_{C0} = \frac{R_2}{R_1+R_2}E$

$$i_C(t) = C\frac{du_C(t)}{dt} = \frac{R_2 C}{R_1+R_2}E\cdot\frac{-1}{\tau}e^{-t/\tau} = \frac{R_2 C}{R_1+R_2}E\frac{-1}{R_2 C}e^{-t/\tau}$$

$$i_C(t) = -\frac{E}{R_1+R_2}e^{-t/\tau} = -2e^{-t/\tau}\,[A]$$