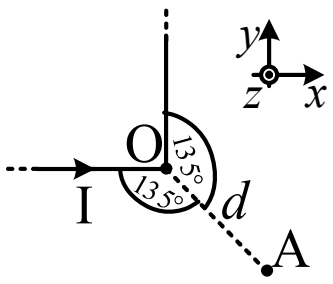
DRUGI KOLOKVIJUM IZ ELEKTROTEHNIKE
31. januar 2018.

1. Kroz dva veoma dugačka provodnika, spojena u tački O pod pravim uglom, protiče struja intenziteta I u smeru označenom na Slici 1. Provodnici se nalaze u vazduhu. Odrediti i nacrtati rezultujući vektor magnetne indukcije u tački **A,** koja se nalazi u istoj ravni sa provodnicima, na rastojanju d od tačke O. **(8 poena)**

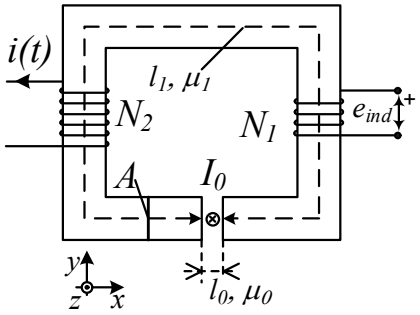
2. Na Slici 2 prikazano je magnetno kolo, koje se sastoji od jezgra, magnetne permeabilnosti μ_1 i dužine srednje linije l_1, na koje je namotan namotaj sa N_2 navojaka kroz koji protiče struja intenziteta ***i(t)***. Poprečni presek jezgra je površine A, oblika kvadrata. Jezgro ima vazdušni procep debljine l_0. Na jezgro je namotan i namotaj sa N_1 navojaka, čiji su krajevi otvoreni.

a) Odrediti indukovanu elektromotornu silu $\boldsymbol{e_{ind}}$, ako je $i(t) = I_m \cos(\omega t + \theta)$. **(6 poena)**

b) Ako se u procep unese pravolinijski provodnik, kroz koji protiče struja intenziteta I_0, odrediti i skicirati **vektor** sile kojom magnetno polje u procepu deluje na provodnik. Uzeti da je $i(t) = I_1 = const$. **(6 poena)**



Slika 1



Slika 2

3. U prostom kolu naizmenične struje, potrošač je priključen na naponski izvor parametara $U = 300\text{V}$, $f = \dfrac{100}{\pi}\text{Hz}$. Reaktivna snaga i faktor snage potrošača iznose $Q = 1.8\text{kVAr}$ i $\cos\varphi = 0.8$.

a) Odrediti efektivnu vrednost struje potrošača i njegovu kompleksnu impedansu. **(7 poena)**

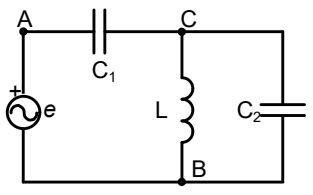
b) Odrediti kapacitivnost kondenzatora, koji je potrebno vezati paralelno potrošaču, tako da se postigne faktor snage jednak 1. (**4 poena)**

4. U kolu na Slici 3 poznato je: $e(t) = 300\sqrt{2}\sin(1000t + \pi/2)\text{V}$, $C_1 = 200\mu\text{F}$, $L = 100\,\text{mH}$, $C_2 = 50\mu\text{F}$.

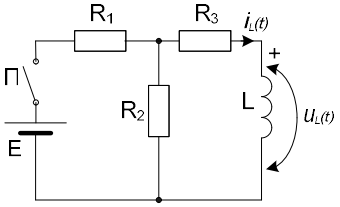
a) Izračunati kompleksnu impedansu $\overline{Z}_e$ i kompleksnu admitansu $\overline{Y}_e$ ekvivalentnog potrošača između tačaka A i B. **(4 poena)**

b) Odrediti kompleksnu vrednost ems i svih struja u kolu i nacrtati odgovarajući fazorski dijagram. **(5 poena)**

d) Odrediti aktivnu, reaktivnu i prividnu snagu ekvivalentnog potrošača. **(3 poena)**



Slika 3



Slika 4

5. Na sistem trofaznog napona $3\times1000\text{V}$, 50Hz priključen je pretežno kapacitivni simetričan trofazni potrošač povezan u trougao. Aktivna snaga trofaznog potrošača je $P = 36\text{ kW}$, a faktor snage $\cos\varphi = 0,6$. Odrediti efektivnu vrednost linijske struje i kompleksnu impedansu potrošača. **(7 poena)**

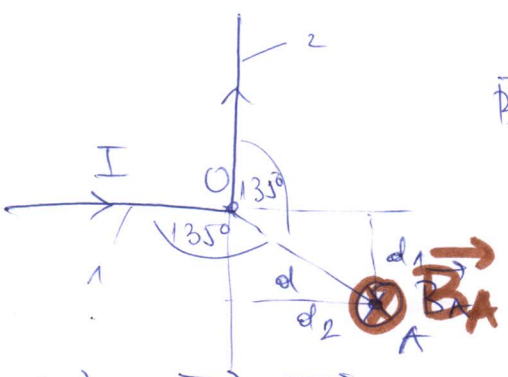
6. U kolu na Slici 4 poznate su vrednosti elemenata: E, $R_1 = R_2 = 2R$, $R_3 = R$ i L. Prekidač Π je otvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se zatvara.

a) Odrediti izraz za struju i napon kalema nakon zatvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame. **(6 poena)**

b) Odrediti vrednost napona na otporniku R_3 u trenutku t_1=2L/R. **(2 poena)**

c) Odrediti magnetnu energiju kalema u trenutku t_1=2L/R. **(2 poena)**

1



$$d_1 = d_2 = \frac{d}{\sqrt{2}}$$

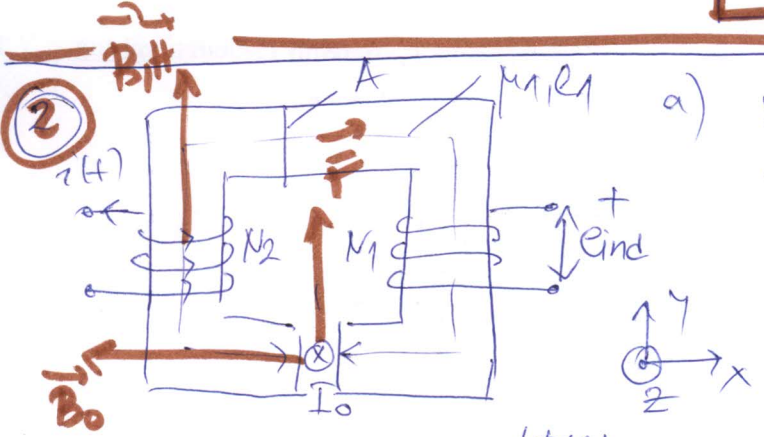
$$\vec{B}_{A1} = \vec{B}_{A2} = \frac{\mu_0 I}{4\pi \frac{d}{\sqrt{2}}}(\cos 0 + \cos 135°)(-\vec{k})$$

$$= \frac{\mu_0 I \sqrt{2}}{4\pi d}\left(1 - \frac{\sqrt{2}}{2}\right)(-\vec{k})$$

$$= -\frac{\mu_0 I}{4\pi d}(\sqrt{2}-1)\vec{k}$$

$$\vec{B}_A = \vec{B}_{A1} + \vec{B}_{A2} = 2\vec{B}_{A1} = \boxed{-\frac{\mu_0 I}{2\pi d}(\sqrt{2}-1)\vec{k}}$$

2



a) $\oint \vec{H}\, d\vec{l} = \sum i(t)$

$$H_1(t) l_1 + H_0(t) l_0 = N_2 i(t)$$

$$\frac{B(t) l_1}{\mu_1} + \frac{B(t) l_0}{\mu_0} = N_2 i(t)$$

$$B(t) = \frac{N_2 i(t)}{\frac{l_1}{\mu_1} + \frac{l_0}{\mu_0}}$$

$$e_{ind} = -N_1 \frac{d\Phi(t)}{dt}$$

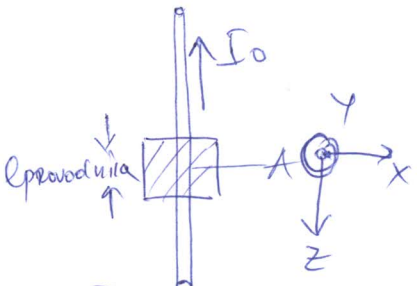
$$\Phi(t) = B(t) \cdot A = \frac{N_2 i(t) A}{\frac{l_1}{\mu_1} + \frac{l_0}{\mu_0}}$$

$$e_{ind} = -\frac{N_1 N_2 A}{\frac{l_1}{\mu_1} + \frac{l_0}{\mu_0}} \frac{d\, i(t)}{dt} = -\frac{N_1 N_2 A}{\frac{l_1}{\mu_1} + \frac{l_0}{\mu_0}} \frac{d}{dt}\left(I_m \cos(\omega t + \theta)\right)$$

$$\boxed{e_{ind} = \frac{N_1 N_2 A I_m \omega}{\frac{l_1}{\mu_1} + \frac{l_0}{\mu_0}} \sin(\omega t + \theta)}$$

b) $$\vec{B}_0 = \frac{\Phi(-\vec{i})}{A} = \frac{N_2 i(t)}{\frac{l_1}{\mu_1} + \frac{l_0}{\mu_0}}(-\vec{i}) = -\frac{N_2 i(t)}{\frac{l_1}{\mu_1} + \frac{l_0}{\mu_0}}\vec{i}$$

$i(t) = I_1 = \text{const}$ $\Rightarrow$ $$\boxed{\vec{B}_0 = -\frac{N_2 I_1}{\frac{l_1}{\mu_1} + \frac{l_0}{\mu_0}}\vec{i}}$$



$$\vec{F} = I_0 \vec{l}_{provodnika} \times \vec{B}_0$$

$$\vec{l}_{provodnika} = \sqrt{A}(-\vec{k})$$

$$\vec{F} = I_0 \sqrt{A}(-\vec{k}) \times \frac{N_2 I_1}{\frac{l_1}{\mu_1} + \frac{l_0}{\mu_0}}(-\vec{i}) = \frac{N_2 I_1 I_0 \sqrt{A}}{\frac{l_1}{\mu_1} + \frac{l_0}{\mu_0}} \vec{k} \times \vec{i}$$

$$\boxed{\vec{F} = \frac{N_2 I_1 I_0 \sqrt{A}}{l_1/\mu_1 + l_0/\mu_0}\vec{j}}$$

(3) a) $Q = 1{,}8\ \text{kVAr}$

$\cos\varphi = 0{,}8 \Rightarrow \sin\varphi = (+)\sqrt{1-\cos^2\varphi} = 0{,}6$ (jer je $Q>0$)

$Q = S\sin\varphi \Rightarrow S = \frac{Q}{\sin\varphi} = 3\ \text{kVA}$

$S = UI \Rightarrow \boxed{I = \frac{S}{U} = \frac{3000}{300} = 10\ \text{A}}$

$Z = \frac{U}{I} = \frac{300}{10} = 30\,\Omega \qquad \bar{Z} = Z\cos\varphi + jZ\sin\varphi = 30(0{,}8+j0{,}6)$

$\boxed{\bar{Z} = (24+j18)\,\Omega}$

b) $\cos\varphi_u = 1 \Rightarrow \sin\varphi_u = 0 \Rightarrow Q_u = 0$

$Q_u = Q + Q_c = Q - \omega C U^2 \Rightarrow C = \frac{Q}{\omega U^2} = \frac{1800}{2\pi\cdot\frac{100}{\pi}\cdot 300^2} = 10^{-4}\,\text{F}$

$\boxed{C = 100\,\mu\text{F}}$

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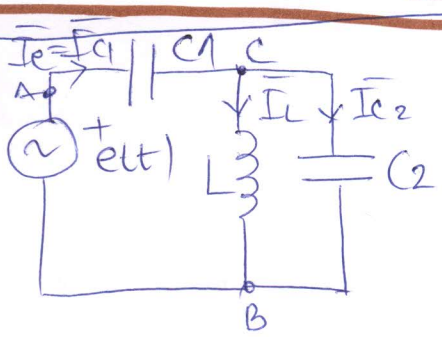
(4)



a) $\bar{Y}_{C_2} = j\omega C_2 = j50\,\text{mS}$

$\bar{Y}_L = \frac{1}{j\omega L} = -j\frac{1}{\omega L} = -j10\,\text{mS}$

$\bar{Y}_{LC_2} = \bar{Y}_L + \bar{Y}_{C_2} = j40\,\text{mS}$

$\bar{Z}_{LC_2} = \frac{1}{\bar{Y}_{LC_2}} = -j25\,\Omega$

$\bar{Z}_{C_1} = \frac{1}{j\omega C_1} = -j\frac{1}{\omega C_1} = -j5\,\Omega$

$\boxed{\bar{Z}_e = \bar{Z}_{AB} = \bar{Z}_{C_1} + \bar{Z}_{LC_2} = -j30\,\Omega}$ $\qquad \boxed{\bar{Y}_e = \frac{1}{\bar{Z}_e} = j\frac{1}{30} = j33{,}3\,\text{mS}}$

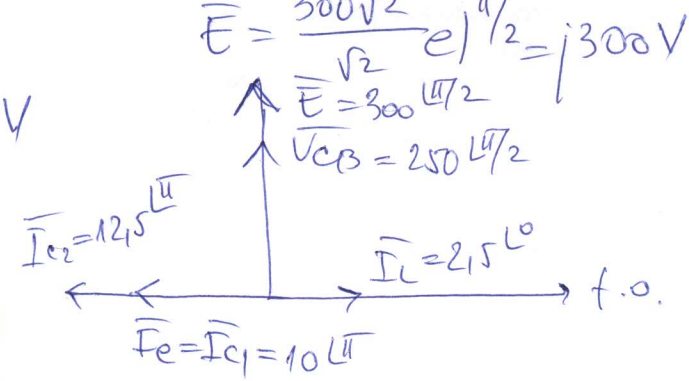
b) $\bar{I}_e = \bar{I}_{C_1} = \frac{\bar{E}}{\bar{Z}_e} = \boxed{-10\,\text{A} = \bar{I}_{C_1}}$

$\bar{E} = \frac{300\sqrt{2}}{\sqrt{2}}e^{j\pi/2} = j300\,\text{V}$

$\bar{U}_{CB} = \bar{U}_{L_2} = \bar{U}_{C_2} = \bar{Z}_{LC_2}\cdot\bar{I}_e = j250\,\text{V}$

$\bar{I}_L = \bar{Y}_L\cdot\bar{U}_L = \boxed{2{,}5\,\text{A} = \bar{I}_L}$

$\bar{I}_{C_2} = \bar{Y}_{C_2}\bar{U}_{C_2} = \boxed{-12{,}5\,\text{A} = \bar{I}_{C_2}}$



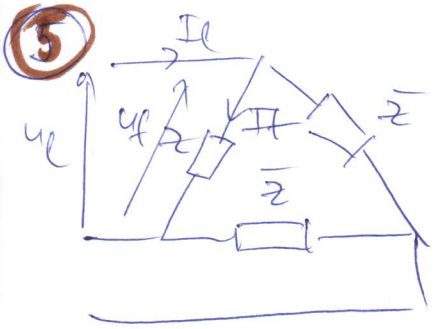
c) $\bar{S} = \bar{E}\bar{I}_e^* = j300\cdot(-10) = -j3000\,\text{VA} = P + jQ$

$\boxed{P = 0\ \text{W}}$

$\boxed{Q = -3000\,\text{VAr} = -3\,\text{kVAr}}$

$\boxed{S = \sqrt{P^2+Q^2} = 3000\,\text{VA} = 3\,\text{kVA}}$

5



$3\times 1000\,V \Rightarrow U_l = 100\,V$

$U_f = U_l = 1000\,V$

$P = 36\,kW \;|\; \cos\varphi = 0{,}6 \Rightarrow S = \frac{P}{\cos\varphi} = 60\,kVA$

$S = 3U_f I_f \Rightarrow I_f = \frac{S}{3U_f} = \frac{60000}{3\cdot 1000} = 20\,A$

$\boxed{I_l = I_f\sqrt{3} = 20\sqrt{3}\,A}$

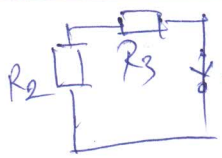
$Z = \frac{U_f}{I_f} = \frac{1000\,V}{20\,A} = 50\,\Omega$ → jer je potrošač kapacitiv.

$\sin\varphi = -\sqrt{1-\cos^2\varphi} = -0{,}8$

$\bar{Z}_f = Z_f\cos\varphi + jZ_f\sin\varphi = 50\cdot 0{,}6 - j50\cdot 0{,}8 = \boxed{(30-j40)\,\Omega = \bar{Z}_f}$

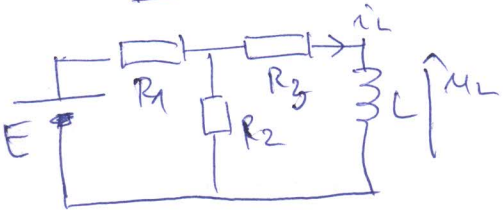
6

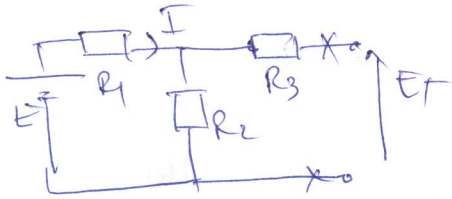
a) $n \to 0$



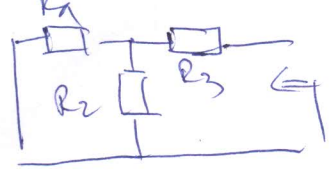
$\boxed{I_{L0} = 0}$ – stac. stanje

$n \to \infty$





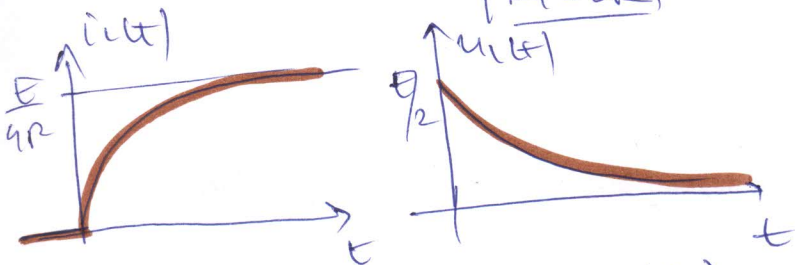
$E_T = R_2\cdot\frac{E}{R_1+R_2} = 2R\frac{E}{4R} = \boxed{\frac{E}{2} = E_T}$



$R_T = R_3 + R_1 \| R_2$

$= R + 2R \| 2R$

$\boxed{R_T = 2R}$

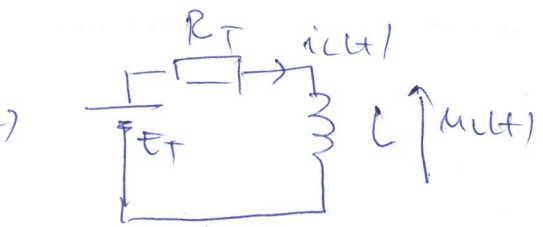


b) $u_{R_3}(t) = R_3 i_L(t) = \frac{E}{4}\left(1-e^{-t/\tau}\right)$

$u_{R_3}\left(t_1 = \frac{2L}{R}\right) = \frac{E}{4}\left(1-e^{-\frac{2L/R}{L/2R}}\right) = \boxed{\frac{E}{4}\left(1-e^{-4}\right)\sqrt{} = u_{R_3}(t_1)}$

c) $W_L(t) = \frac{1}{2}L i_L^2(t) =$

$W_L\left(t_1 = \frac{2L}{R}\right) = \frac{1}{2}L\left(\frac{E}{4R}\right)^2\left(1-e^{-4}\right)^2 = \boxed{\frac{LE^2}{32R^2}\left(1-e^{-4}\right)^2 \quad J = W_L(t_1)}$



$E_T - R_T i_L(t) - u_L(t) = 0$

$u_L(t) = L\frac{di_L(t)}{dt}$

$E_T - R_T i_L(t) - L\frac{di_L(t)}{dt} = 0$

$\frac{di_L(t)}{dt} + \frac{i_L(t)}{\underbrace{L/R_T}_{\tau}} = \frac{E_T}{L} = K$

$i_L(t) = Ae^{-t/\tau} + B$

$\boxed{\tau = \frac{L}{2R}}$

$B = K\cdot\tau = \frac{E_T}{L}\cdot\frac{L}{R_T} = \frac{E_T}{R_T} = \frac{E}{4R}$

$A + B = I_{L0} = 0 \Rightarrow A = -B = \frac{-E}{4R}$

$\boxed{i_L(t) = \frac{E}{4R}\left(1-e^{-t/\tau}\right)}$

$\boxed{u_L(t) = L\frac{di_L(t)}{dt} = \frac{E}{2}e^{-t/\tau}}$