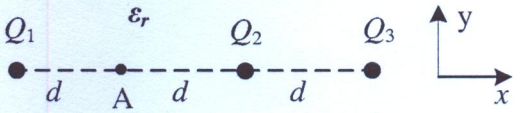
Elektrotehnika

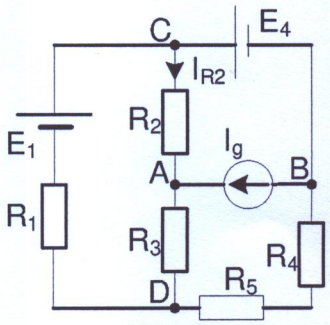
31. januar 2024.

1. Na Slici 1 su prikazana tri tačkasta naelektisanja $Q_1=Q$, $Q_2=-2Q$ i $Q_3=3Q$ koja se nalaze u dielektriku relativne dielektrične konstante ε_r. Odrediti i nacrtati vektor električnog polja u tački A. **(15 poena)**

2. U kolu na Slici 2 poznate su vrednosti $R_1=2R=10\Omega$, $R_2=3R=15\Omega$, $R_3=R_4=R_5=R=5\Omega$, $I_g=1\text{A}$, $E_1=10\text{V}$, $E_4=40\text{V}$. Primenom Tevenenove teoreme odrediti intenzitet struje kroz otpornik R_2 u smeru označenom na slici, kao i snagu na ovom otporniku. **(25 poena)**



Slika 1



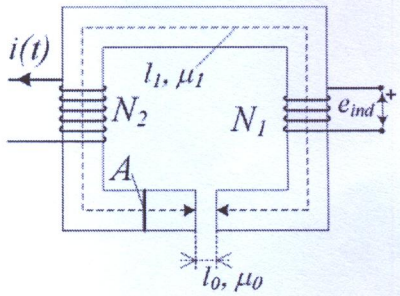
Slika 2

3. Na Slici 3 je prikazano magnetno kolo, sa poznatim parametrima jezgra: dužina srednje linije l_1, površina poprečnog preseka A, magnetna permeabilnost μ_1, i debljina vazdušnog procepa l_0. Na jezgro su namotana dva namotaja, jedan sa N_2 navojaka kroz koji protiče struja konstantnog intenziteta I, i drugi otvorenih krajeva, sa N_1 navojaka.

a) Odrediti intenzitet vektora magnetne indukcije u vazdušnom procepu. **(8 poena)**

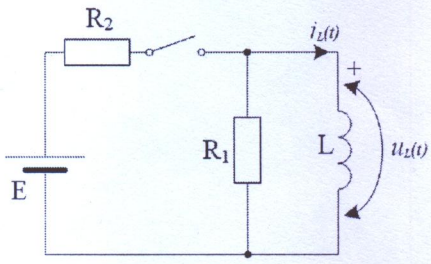
b) Odrediti međusobnu induktivnost namotaja. **(6 poena)**

c) Odrediti indukovanu elektromotornu silu na krajevima namotaja čiji su krajevi otvoreni. **(6 poena)**



Slika 3

4. U kolu na Slici 4 poznate su vrednosti elemenata: $E=30\text{V}$, $R_1=R=5\Omega$, $R_2=2R$ i $L=20\mu\text{H}$. Prekidač Π je zatvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t=0$, prekidač se otvara. Odrediti izraz za struju i napon kalema nakon otvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame. Odrediti snagu otpornika R_1 u trenutku t_1=12μs. **(20 poena)**



Slika 4

5. Na potrošaču nepoznate impedanse poznate su trenutne vrednosti napona $u(t)=20\sqrt{2}\sin(1000t+\pi)\,\text{V}$ i struje $i(t)=10\sin(1000t+3\pi/4)\,\text{A}$.

a) Odretiti kompleksnu impedansu potrošača. **(5 poena)**

b) Odrediti aktivnu, reaktivnu i prividnu snagu potrošača. **(5 poena)**

c) Nacrtati fazorski dijagram struje i napona na impedansi. **(5 poena)**

d) Ako se paralelno potrošaču priključi kondenzator kapacitivnosti $C=250\mu\text{F}$ odrediti faktor snage celokupnog potrošača. **(5 poena)**

1) $Q_1 = Q$ $\quad \varepsilon_r$ $\quad A$ $\quad \vec{K_A}$ $\quad Q_2 = -2Q$ $\quad Q_3 = 3Q$

d $\quad d$ $\quad d$

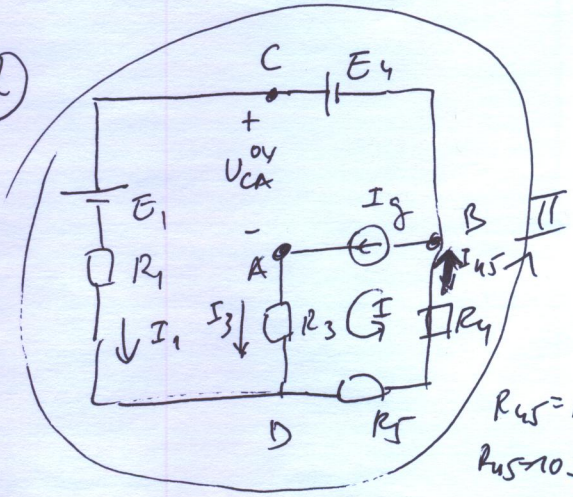
$$\vec{K_A} = \vec{K_{A1}} + \vec{K_{A2}} + \vec{K_{A3}} = \frac{Q_1}{4\pi\varepsilon_0\varepsilon_r d^2}\vec{i} + \frac{Q_2}{4\pi\varepsilon_0\varepsilon_r d^2}(-\vec{i}) + \frac{Q_3}{4\pi\varepsilon_0\varepsilon_r (2d)^2}(-\vec{i})$$

$$\vec{K_A} = \frac{Q}{4\pi\varepsilon_0\varepsilon_r d^2}\vec{i} + \frac{2Q}{4\pi\varepsilon_0\varepsilon_r d^2}\vec{i} - \frac{3Q}{4\pi\varepsilon_0\varepsilon_r 4d^2}\vec{i}$$

$$\vec{K_A} = \frac{Q}{4\pi\varepsilon_0\varepsilon_r d^2}\vec{i}\left(1 + 2 - \frac{3}{4}\right)\vec{i}$$

$$\boxed{\vec{K_A} = \frac{9Q}{16\pi\varepsilon_0\varepsilon_r d^2}\vec{i}}$$

2)



$$I_I = I_g = 1A$$

$$R_{45} I_I + (R_1 + R_{45}) I_{II} = E_4 - E_1$$

$$I_{II} = \frac{E_4 - E_1 - (R_4 + R_5) I_g}{R_1 + R_4 + R_5}$$

$$I_{II} = \frac{40 - 10 - (5+5)\cdot 1}{10 + 5 + 5} = \frac{20}{20} = 1A$$

$R_{45} = R_4 + R_5$

$R_{45} = 10\,\Omega$

$I_{45} = I_I + I_{II} = 2A$

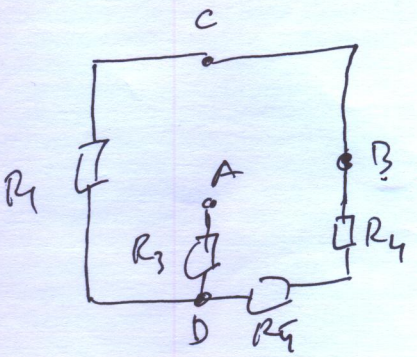
$I_1 = I_{II} = 1A$

$I_3 = I_I = I_g = 1A$

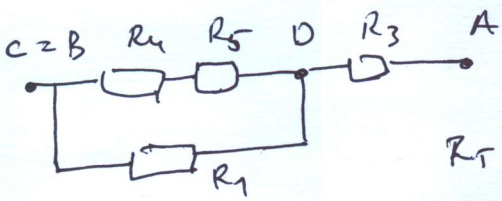
$$\Rightarrow E_T = U_{CA}^{0V} = E_4 - R_{45} I_{45} - R_3 I_3$$

$$E_T = 40 - 10\cdot 2 - 5$$

$$\boxed{E_T = 15V}$$



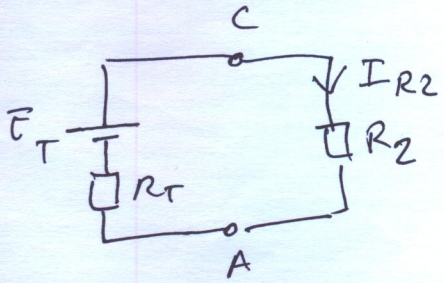
$\Rightarrow$



$$R_T = (R_4 + R_5) \,||\, R_1 + R_3$$

$$R_T = 2R \,||\, 2R + R = R + R = 2R$$

$$\boxed{R_T = 10\,\Omega}$$

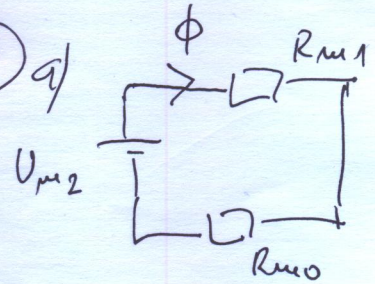


$$I_{R2} = \frac{E_T}{R_T + R_2} = \frac{15}{10 + 15} = \frac{15}{25} = \frac{3}{5} = 0{,}6\,A$$

$$\boxed{I_{R2} = 0{,}6\,A}$$

$$\boxed{P_{R2} = R_2 I_{R2}^2 = 15\cdot 0{,}6^2 = 5{,}4\,W}$$

3) a)



$$U_{m2} = N_2 I$$

$$R_{m1} = \frac{l_1}{\mu_1 A}$$

$$R_{m0} = \frac{l_0}{\mu_0 A}$$

$$\Phi = \frac{U_{m2}}{R_{m1} + R_{m0}} = \frac{N_2 I}{\frac{l_1}{\mu_1 A} + \frac{l_0}{\mu_0 A}}$$

$$(B_1 =) B_0 = \frac{\Phi}{A} = \frac{N_2 I}{\frac{l_1}{\mu_1} + \frac{l_0}{\mu_0}}$$

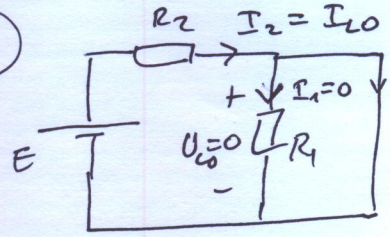
b) $L_{12} = L_{21} = \frac{N_1 N_2}{R_{m1} + R_{m0}} = \frac{N_1 N_2 A}{l_1/\mu_1 + l_0/\mu_0}$

c)

$$\Psi_1 = N_1 \cdot \Phi = \frac{N_1 N_2 I A}{l_1/\mu_1 + l_0/\mu_0} = \text{const}$$

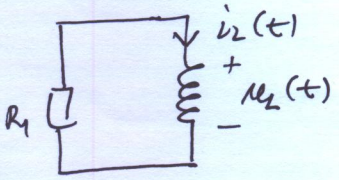
$$e_{ind} = -\frac{d\Psi_1}{dt} = 0\ V$$

(4)



$U_{L0} = 0 \Rightarrow I_1 = 0 \Rightarrow I_2 = I_{L0} = \frac{E}{R_2} = \frac{E}{2R}$

$\boxed{I_{L0} = 3\,A}$



$R i_L + u_L = 0$

$R i_L + L \frac{di_L}{dt} = 0$

$\boxed{\frac{di_L}{dt} + \frac{R}{L} i_L = 0}$

$i_L(t) = A e^{-\frac{t}{\tau}} + B$

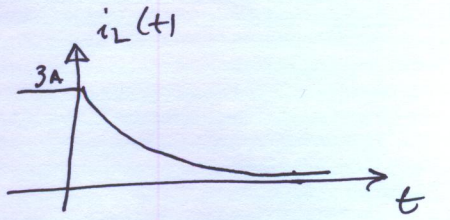
$B = K \cdot 0 = 0$

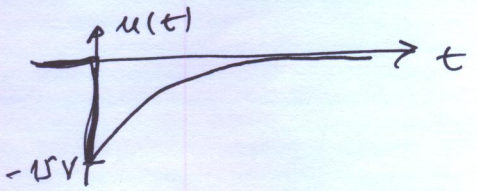
$A = i_L(0) - B = I_{L0} = \frac{E}{2R}$

$\Rightarrow K = 0,\ \alpha = \frac{R}{L},\ \tau = \frac{L}{R} = 4\mu s$

$\boxed{i_L(t) = \frac{E}{2R} e^{-\frac{R}{L} t}}$

$\boxed{i_L(t) = 3\, e^{-\frac{t}{4\mu s}}\ A}$





$u_L(t) = L \frac{di_L}{dt} = L \frac{E}{2R} e^{-\frac{R}{L} t} \cdot \left(-\frac{R}{L}\right)$

$\boxed{u_L(t) = -\frac{E}{2} e^{-\frac{R}{L} t} = -15\, e^{-\frac{t}{4\mu s}}\ V}$

$P_{R1}(t_1) = R_1 \cdot i_L^2(t_1) = \frac{E^2}{4R} e^{-2\frac{R}{L} t_1} = 45\, e^{-\frac{2 \cdot 12\mu s}{4\mu s}} = 45\, e^{-6}\ W$

(5)

$u(t) = 20\sqrt{2}\ \sin(1000t + \bar{u}) \Rightarrow \underline{U} = 20\, e^{j\bar{u}}\ V = -20\ V$

a) $i(t) = 10\ \sin(1000t + 3\bar{u}/4) \Rightarrow \underline{I} = \frac{10}{\sqrt{2}} e^{j\frac{3\bar{u}}{4}} A = 5\sqrt{2}\left(\cos\frac{3\bar{u}}{4} + j\sin\frac{3\bar{u}}{4}\right) = -5 + j5\ A$

$\cos\frac{3\bar{u}}{4} = -\frac{1}{\sqrt{2}}$, $\sin\frac{3\bar{u}}{4} = \frac{1}{\sqrt{2}}$

$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{+20 e^{j\bar{u}}}{5\sqrt{2} e^{j3\bar{u}/4}} = \frac{4}{\sqrt{2}} e^{j(\bar{u} - \frac{3\bar{u}}{4})} = 2\sqrt{2}\, e^{j\bar{u}/4} = 2\sqrt{2}\left(\cos\frac{\bar{u}}{4} + j\sin\frac{\bar{u}}{4}\right)$

$\cos\frac{\bar{u}}{4} = \sin\frac{\bar{u}}{4} = \frac{1}{\sqrt{2}}$

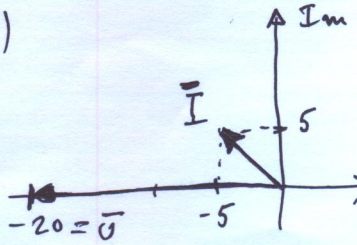
$= 2 + j2\ \Omega$

$\boxed{\underline{Z} = 2\sqrt{2}\, e^{j\bar{u}/4}\ \Omega = 2 + j2\ \Omega}$

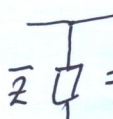
b) $\underline{S} = \underline{U}\,\underline{I}^* = \underline{Z} I^2 = 2\sqrt{2}\, e^{j\bar{u}/4} \cdot (5\sqrt{2})^2 = 100\sqrt{2}\, e^{j\bar{u}/4} = 100\sqrt{2}\left(\cos\bar{u}/4 + j\sin\bar{u}/4\right)$

$\underline{S} = 100\sqrt{2}\, e^{j\bar{u}/4}\ VA = 100 + j100\ VA \Rightarrow$ $P = 100\ W,\ Q = 100\ VAR,\ S = 100\sqrt{2}\ VA$

c)



d)



$\underline{Z}_C = -j\frac{1}{\omega C} = -j\frac{1}{1000 \cdot 250 \cdot 10^{-6}} = -j4\ \Omega$

$\underline{Z}_e = \frac{\underline{Z}_C \cdot \underline{Z}}{\underline{Z}_C + \underline{Z}} = \frac{-j4(2+j2)}{-j4+2+j2} = \frac{8-j8}{2-j^2} = 4\ \Omega$

$\underline{Z}_e = R_e + jX_e = 4 + j0\ \Omega \Rightarrow \boxed{\cos\varphi = \frac{R_e}{z_e} = \frac{4}{4} = 1}$

$z_e = |\underline{Z}_e| = 4\ \Omega$

d) alternativamente si ha:

$$P_e = P_z + \cancel{P_C}^{\,0} = P_z = 100\,W$$

$$Q_C = -\omega C U^2 = -1000 \cdot 250 \cdot 10^{-6} \cdot (20)^2 = -100\ VAR$$

$$Q_e = Q_z + Q_C = 100 - 100 = 0\ VAR$$

$$S_e = \sqrt{P_e^2 + Q_e^2} = 100\ VA$$

$$\boxed{\cos\varphi = \frac{P_e}{S_e} = \frac{100}{100} = 1}$$