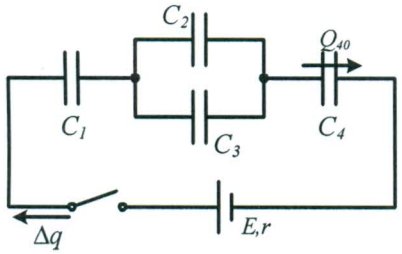
Elektrotehnika

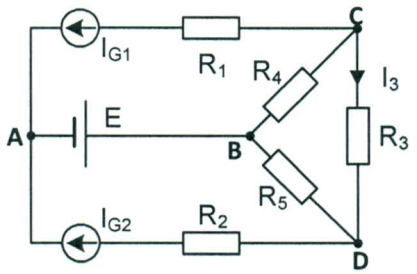
19. februar 2021.

1. Odrediti količinu naelektrisanja Δq koja će proteći, nakon zatvaranja prekidača, kroz granu u kojoj se nalazi naponski generator (Slika 1). Pre povezivanja, kondenzatori C_1, C_2 i C_3 su bili neopterećeni, dok je početno naelektrisanje kondenzatora C_4 poznato i iznosi Q_{40}. Poznate su kapacitivnosti kondenzatora: $C_1 = 4C$, $C_2 = C$, $C_3 = 3C$, $C_4 = 2C$, kao i elektromotorna sila naponskog generatora E. Odrediti napone i količine naelektrisanja na kondenzatorima C_1 i C_4 nakon protoka naelektrisanja Δq. **(20 poena)**

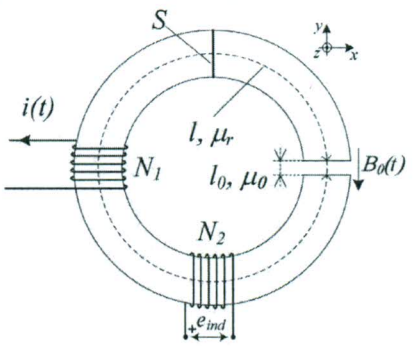
2. U kolu na Slici 2 poznato je: $R_1 = R_2 = R_3 = R_4 = R_5 = R = 10\,\Omega$, $E = 20\text{V}$, $I_{G1} = 3\,\text{A}$, $I_{G2} = 9\,\text{A}$. Primenom metode superpozicije odrediti intenzitet struje I_3 koja protiče kroz otpornik R_3. **(20 poena)**



Slika 1



Slika 2



Slika 3

3. Na Slici 3 prikazano je magnetno kolo, koje se sastoji od jezgra, relativne magnetne permeabilnosti $\mu_r = 400$, dužine srednje linije $l = 40\text{cm}$, sa vazdušnim procepom debljine $l_0 = 1\text{mm}$. Poprečni presek jezgra iznosi $S = 10\text{cm}^2$. Namotaj sa $N_2 = 100$ navojaka je otvorenih krajeva, a kroz namotaj sa $N_1 = 200$ navojaka protiče struja nepoznatog intenziteta. Poznato je da se u vazdušnom procepu intenzitet vektora magnetne indukcije $\vec{B}_0(t)$ menja u vremenu po formuli $B_0(t) = B_{0m}\cos\omega t$, gde je $B_{0m} = 16\pi\,\text{mT}$ i $\omega = 1000\,\text{rad/s}$. ($\mu_0 = 4\pi\cdot 10^{-7}\,\text{H/m}$)

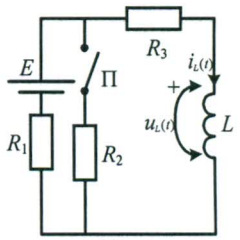
a) Odrediti trenutnu vrednost struje $i(t)$. **(10 poena)**

b) Odrediti efektivnu vrednost indukovane ems (E_{ind}) u namotaju čiji su krajevi otvoreni. **(10 poena)**

4. U kolu na Slici 4 poznati su parametri elemenata: E, R_1=R_2=$2R$, R_3=R i L. Prekidač П je zatvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se otvara.

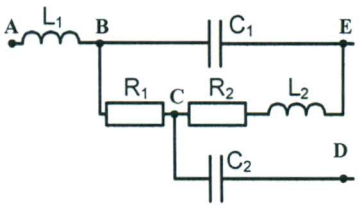
a) Odrediti izraz za struju i napon kalema nakon otvaranja prekidača **(12 p)** i nacrtati odgovarajuće vremenskе dijagrame **(4 p)**.

b) Odrediti snagu generatora E u trenutku t_1=L/R. **(4 p)**



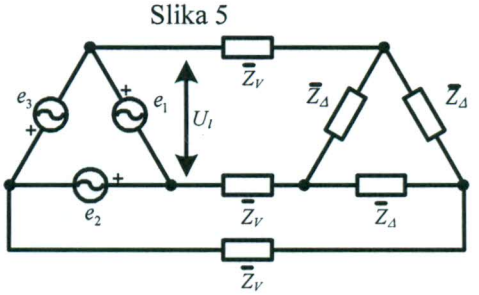
Slika 4

5. Na Slici 5 prikazana je grupa od šest elemenata kola naizmenične struje. Poznato je: $R_1 = R_2 = 10\,\Omega$ $X_{L1} = 20\,\Omega$, $X_{L2} = 10\,\Omega$, $X_{C1} = -10\,\Omega$ i $X_{C2} = -20\,\Omega$. Odrediti ekvivalentnu **kompleksnu** impedansu između tačaka E i D. **(20 poena)**



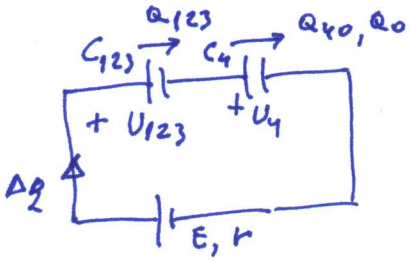
Slika 5

6. Trofazni simetrični generator, spregnut u trougao, napaja trofazni simetrični potrošač, spregnut u trougao, preko električnih vodova čija je impedansa $\overline{Z}_V = 1 + j1\,\Omega$, kao što je prikazano na Slici 6. Poznati su linijski napon generatora $U_l = 450\,\text{V}$ i impedansa faze potrošača $\overline{Z}_\Delta = 9 + j6\,\Omega$. Izračunati kompleksnu prividnu, aktivnu i reaktivnu snagu generatora. **(20 poena)**



Slika 6

(1)



$$C_{123} = \frac{C_1(C_2+C_3)}{C_1+C_2+C_3} = \frac{4C(C+3C)}{4C+C+3C} = 2C$$

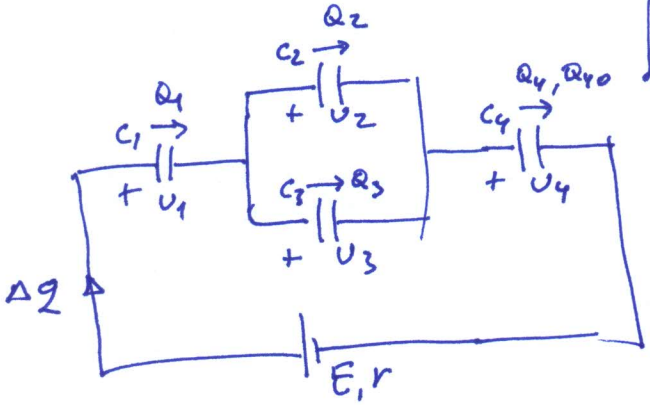
$$U_{123} + U_4 = E$$

$$\frac{Q_{123}}{C_{123}} + \frac{Q_4}{C_4} = E$$

$$\frac{\Delta q}{2C} + \frac{\Delta q + Q_{40}}{2C} = E$$

$$2\Delta q + Q_{40} = 2CE$$

$$\boxed{\Delta q = CE - \frac{1}{2}Q_{40}}$$



$$\boxed{Q_1 = \Delta q = CE - \frac{1}{2}Q_{40}}$$

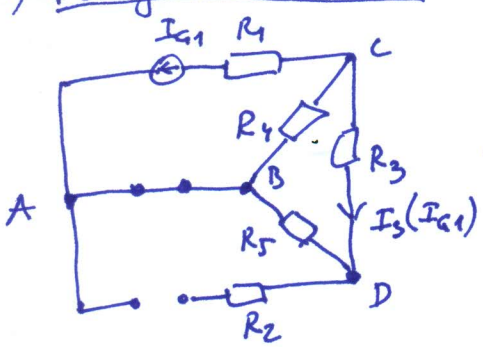
$$U_1 = \frac{Q_1}{C_1} = \frac{CE - \frac{1}{2}Q_{40}}{4C}$$

$$\boxed{U_1 = \frac{E}{4} - \frac{Q_{40}}{8C}}$$

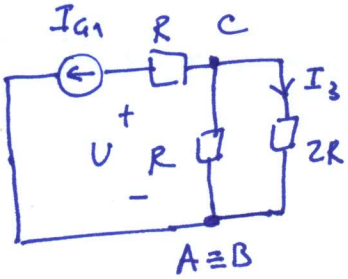
$$\boxed{Q_4 = \Delta q + Q_{40} = CE + \frac{1}{2}Q_{40}}$$

$$U_4 = \frac{Q_4}{C_4} = \frac{CE + \frac{1}{2}Q_{40}}{2C} = \boxed{\frac{E}{2} + \frac{Q_{40}}{4C} = U_4}$$

(2) 1) Delujc samo I_{G1}



$\Leftrightarrow$

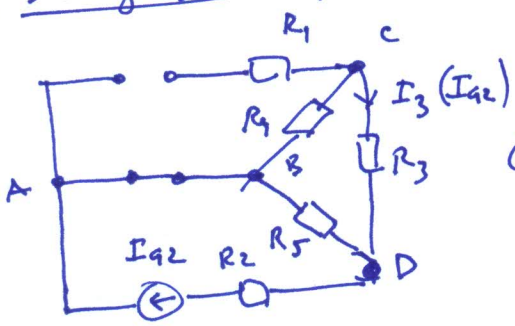


$$U = -I_{G1} \cdot (R \| 2R)$$

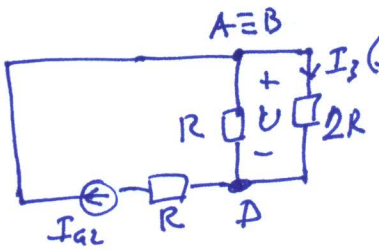
$$U = -I_{G1} \cdot \frac{R \cdot 2R}{R+2R} = -\frac{2R}{3} I_{G1}$$

$$I_3(I_{G1}) = \frac{U}{2R} = -\frac{1}{3} I_{G1} = -1A$$

2) Delujc samo I_{G2}



$\Leftrightarrow$

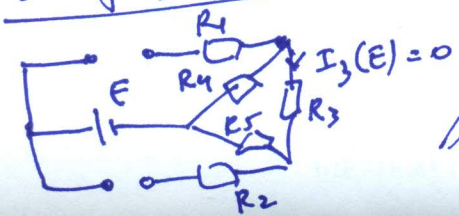


$$U = I_{G2} \cdot (R \| 2R)$$

$$U = I_{G2} \cdot \frac{2R}{3}$$

$$I_3(I_{G2}) = \frac{U}{2R} = \frac{1}{3} I_{G2} = 3A$$

3) Delujc samo E

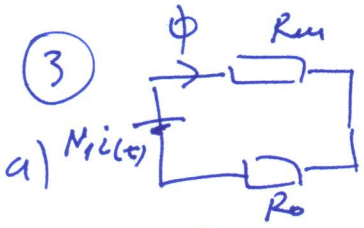


$$I_3 = I_3(I_{G1}) + I_3(I_{G2}) + I_3(E)$$

$$\boxed{I_3 = -1 + 3 + 0 = 2A}$$

(3)

a)



$$R_m = \frac{l}{\mu_0 \mu_r S}$$

$$R_0 = \frac{l_0}{\mu_0 S}$$

$$\Phi(t) = \frac{N_1 i(t)}{R_m + R_0} = \frac{N_1}{\frac{l}{\mu_0 \mu_r S} + \frac{l_0}{\mu_0 S}} i(t)$$

$$B_0(t) = \frac{\Phi(t)}{S} = \frac{\mu_0 N_1}{l/\mu_r + l_0} i(t)$$

$$B_0(t) = B_{0m} \cos(\omega t)$$

$$i(t) = \frac{B_{0m}(l_0 + l/\mu_r)}{\mu_0 N_1} \cos(\omega t)$$

$$\boxed{i(t) = 0{,}4 \cos(1000t)\ A}$$

b)

$$e_{ind}(t) = -\frac{d\Psi_2(t)}{dt} = -\frac{d}{dt}[N_2 \cdot \Phi(t)] = -\frac{d}{dt}[N_2 S \cdot B_0(t)] = -N_2 S B_{0m} \frac{d}{dt}[\cos(\omega t)]$$

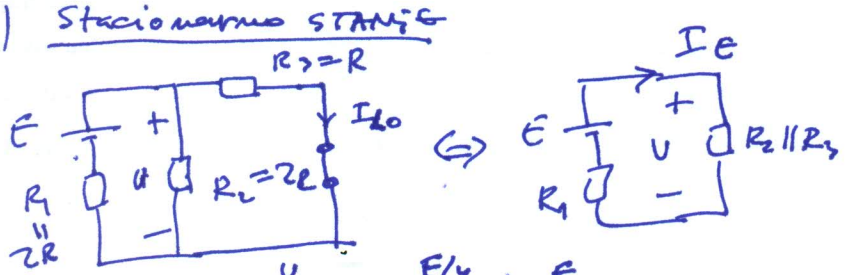
$$e_{ind}(t) = N_2 S B_{0m} \omega \cdot \sin(\omega t) = E_{indm} \sin(\omega t)$$

$$E_{indm} = N_2 S B_{0m} \omega = 1{,}6\pi\ V$$

$$\boxed{E_{ind} = \frac{E_{indm}}{\sqrt{2}} = \frac{1{,}6\pi}{\sqrt{2}}\ V}$$

(4)

a) Stacionarno stanje



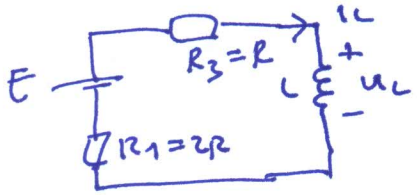
$$I_{L0} = \frac{U}{R_3} = \frac{E/4}{R} = \frac{E}{4R}$$

$$I_E = \frac{E}{R_1 + R_2 \| R_3}$$

$$I_E = \frac{E}{2R + \frac{2R \cdot R}{2R + R}} = \frac{3E}{8R}$$

$$U = I_E \cdot R_2 \| R_3 = \frac{3E}{8R} \cdot \frac{2}{3} R = \frac{E}{4}$$

Prelazni proces



$$E - R_3 i_L - R_1 i_L - u_L = 0$$

$$3R i_L + u_L = E, \quad u_L = L \frac{di_L}{dt}$$

$$3R i_L + L \frac{di_L}{dt} = E \ / : L$$

$$\boxed{\frac{di_L}{dt} + \underbrace{\frac{3R}{L}}_{1/\tau} i_L = \underbrace{\frac{E}{L}}_{K}}$$

POČETNI USLOV: $I_{L0} = E/4R$

$$i_L(t) = A e^{-t/\tau} + B$$

$$\tau = \frac{L}{3R}, \quad B = K\tau = \frac{E}{L} \cdot \frac{L}{3R} = \frac{E}{3R}$$

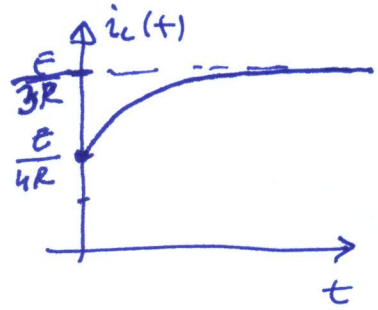
$$I_{L0} = A + B \Rightarrow A = I_{L0} - B = \frac{E}{4R} - \frac{E}{3R} = \frac{3E - 4E}{12R} = -\frac{E}{12R}$$

$$\boxed{i_L(t) = -\frac{E}{12R} e^{-\frac{3R}{L}t} + \frac{E}{3R}}$$

$$u_L(t) = L \frac{di_L}{dt} = L \cdot \left(-\frac{E}{12R}\right) \cdot \left(-\frac{3R}{L}\right) e^{-\frac{3R}{L}t} = \boxed{\frac{E}{4} e^{-\frac{3R}{L}t} = u_L(t)}$$

$i_L(0) = I_{L0} = \frac{E}{4R}$

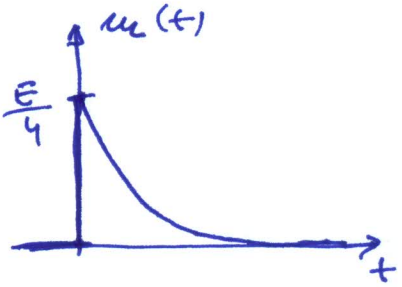
$i_L(\infty) = \frac{E}{3R}$



$u_L(0) = 0$

$u_L(0^+) = \frac{E}{4}$

$u_L(\infty) = 0$

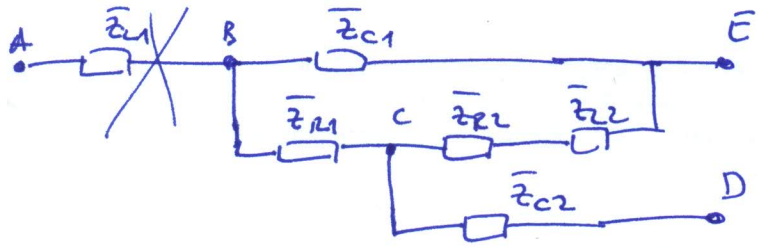


b)

$$P_E(t_1) = E \cdot i_E(t_1) = E\, i_L(t_1) = -\frac{E^2}{12R} e^{-\frac{3R}{L} t_1} + \frac{E^2}{3R}$$

$$\boxed{P_E\left(t_1 = \frac{L}{R}\right) = \frac{E^2}{3R} - \frac{E^2}{12Re^3}}$$

5



$\bar{Z}_{C1} = -j10\ \Omega$

$\bar{Z}_{C2} = -j20\ \Omega$

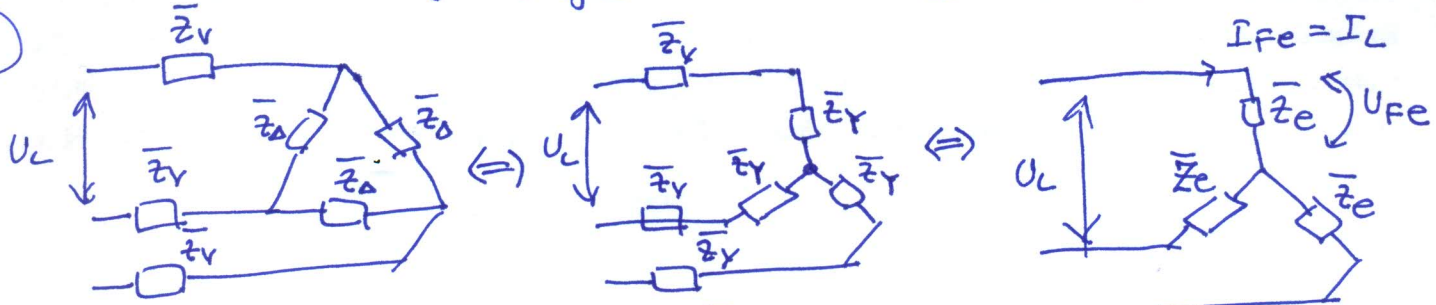
$\bar{Z}_{L2} = j10\ \Omega$

$\bar{Z}_{R1} = \bar{Z}_{R2} = 10\ \Omega$

$$\bar{Z}_{DE} = \bar{Z}_{C2} + (\bar{Z}_{R1} + \bar{Z}_{C1}) \,||\, (\bar{Z}_{R2} + \bar{Z}_{L2}) = -j20 + (10 - j10) \,||\, (10 + j10)$$

$$\bar{Z}_{DE} = -j20 + \frac{(10-j10)(10+j10)}{10-j10+10+j10} = -j20 + \frac{100 - j^2 100}{20} = -j20 + \frac{200}{20} = 10 - j20\ \Omega$$

6



$$\bar{Z}_Y = \frac{\bar{Z}_\Delta}{3} = 3 + j2\ \Omega$$

$$\bar{Z}_e = \bar{Z}_V + \bar{Z}_Y = 4 + j3\ \Omega$$

$$U_{Fe} = \frac{U_L}{\sqrt{3}} = \frac{400}{\sqrt{3}}\ V$$

$$Z_e = \sqrt{4^2 + 3^2} = 5\ \Omega$$

$$I_{Fe} = \frac{U_{Fe}}{Z_e} = \frac{90}{\sqrt{3}}\ A$$

SNAGE GENERATORA:

$$P_e = P_{Fe} = 3 \cdot R_e \cdot I_{Fe}^2 = 3 \cdot 4 \cdot \left(\frac{90}{\sqrt{3}}\right)^2 = 32{,}4\ kW \quad \text{(AKTIVNA)}$$

$$Q_e = Q_{Fe} = 3 X_e \cdot I_{Fe}^2 = 3 \cdot 3 \cdot \left(\frac{90}{\sqrt{3}}\right)^2 = 24{,}3\ kVAr \quad \text{(REAKTIVNA)}$$

$$\bar{S}_e = P_e + jQ_e = (32{,}4 + j24{,}3)\ kVA \quad \text{(KOMPLEKSNA PRIVIDNA)}$$

$$\boxed{\begin{aligned} R_e &= 4\ \Omega \\ X_e &= 3\ \Omega \end{aligned}}$$