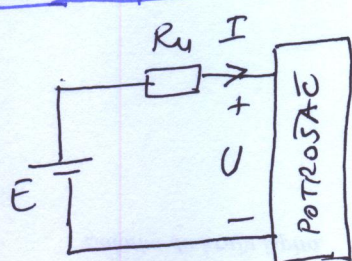


Jednosmerne struje

1

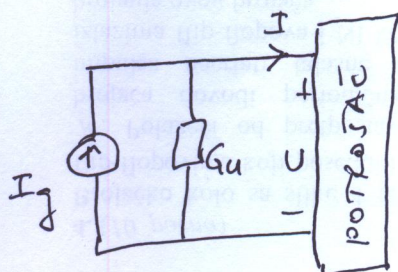
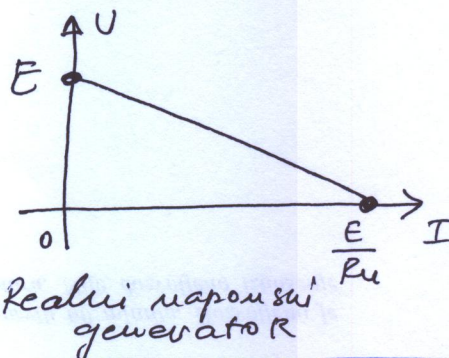
Primer 1



$$U = -R_u I + E$$

$$\text{za } I=0 \Rightarrow U=E$$

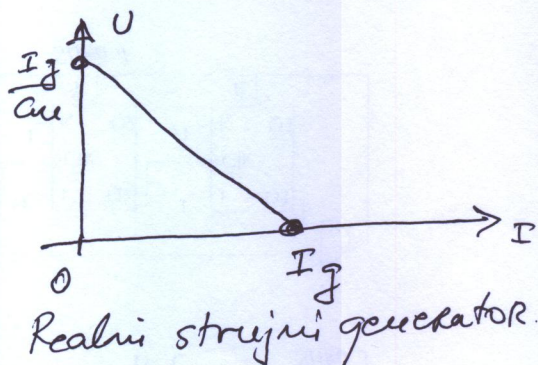
$$\text{za } I = \frac{E}{R_u} \Rightarrow U=0$$



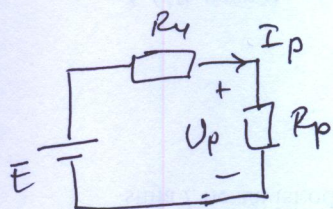
$$I = I_g - G_u U$$

$$\text{za } U=0 \Rightarrow I=I_g$$

$$\text{za } U = \frac{I_g}{G_u} \Rightarrow I=0$$



Primer 2

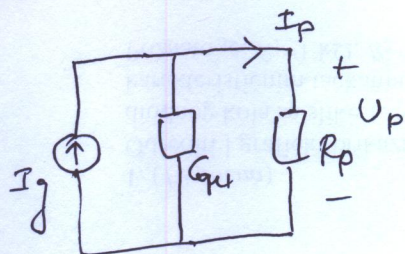


$$P_{R_p} = U_p I_p = R_p I_p^2 = \frac{U_p^2}{R_p} \Leftrightarrow P_{E, R_u} = E I_p - R_u I_p^2$$

$$I_p R_p + I_p R_u - E = 0$$

$$I_p = \frac{E}{R_u + R_p}, \quad U_p = \frac{R_p E}{R_u + R_p} \Rightarrow P_{R_p} = \frac{R_p E^2}{(R_u + R_p)^2}$$

$$P_{E, R_u} = \frac{E^2}{R_u + R_p} - \frac{R_u E^2}{(R_u + R_p)^2}$$



$$P_{R_p} = U_p I_p = R_p I_p^2 = \frac{U_p^2}{R_p} = U_p (I_g - G_u U_p)$$

$$(I_p = I_g - U_p G_u)$$

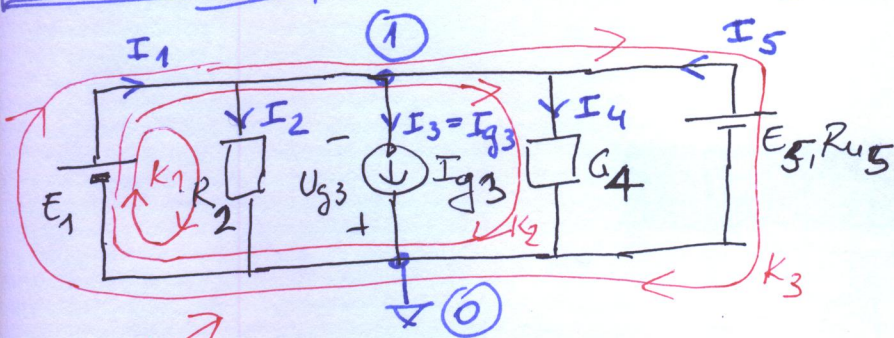
$$U_p = I_g / (G_u + \frac{1}{R_p})$$

$$P_{R_p} = \frac{U_p^2}{R_p} = \frac{I_g^2}{R_p (G_u + \frac{1}{R_p})^2}$$

$$P_{R_p} = U_p I_g - G_u U_p^2$$

$$\Leftrightarrow P_{I_g, G_u}$$

Primer 3



Pogodno je da konture prolaže kroz idealni naponski generator E jer suva jednačina ima samo jednu nepoznatu struju pa se sistem lako rešava!
(Ne mora ovako, ali će onda biti više pisanja....)

Nema 5 nepoznatih jer je $I_3 = I_{g3}$!

$$n_g = 5, n_c = 2$$

Ikt: $n_c - 1 = 1$ jednačina

$$(1) I_1 + I_5 = I_2 + I_{g3} + I_4 \quad (1)$$

$$\text{Ikt: } n_k = n_g - (n_c - 1) - n_{I_g}$$

$$n_k = 5 - (1) - 1$$

$$n_k = 3 \text{ konture}$$

(ne uzimati konture kroz I_{g3} !)

$$K_3: -E_1 + E_5 - R_{45} I_5 = 0$$

$$\Rightarrow I_5 = \frac{E_5 - E_1}{R_{45}} \quad (2)$$

$$K_2: -E_1 + \frac{I_4}{G_4} = 0$$

$$\Rightarrow I_4 = G_4 E_1 \quad (3)$$

$$K_1: -E_1 + R_2 I_2 = 0$$

$$I_2 = \frac{E_1}{R_2} \quad (4)$$

$$I_1 = I_2 + I_4 + I_{g3} - I_5$$

$$I_1 = \frac{E_1}{R_2} + G_4 E_1 + I_{g3} - \frac{E_5 - E_1}{R_{45}}$$

Potencijal: $V_0 = 0$. Referentni čvor

$$V_1 = V_{10} = E_1$$

Snage:

$$P_{E1} = E_1 I_1 = \frac{E_1^2}{R_2} + G_4 E_1^2 + E_1 I_{g3} - \frac{E_1 E_5}{R_{45}} + \frac{E_1^2}{R_{45}} \geq 0$$

$$P_{R2} = R_2 I_2^2 = \frac{E_1^2}{R_2}$$

$$P_{I_{g3}} = U_{g3} I_{g3} = -U_{10} I_{g3} = -E_1 I_{g3} < 0$$

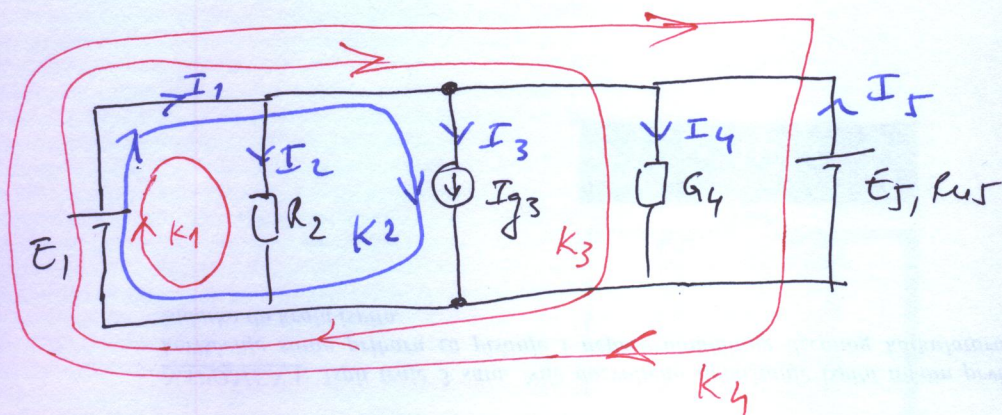
$$P_{G4} = \frac{1}{G_4} I_4^2 = \frac{G_4^2 E_1^2}{G_4} = G_4 E_1^2$$

$$P_{E5, R45} = U_{10} \cdot I_5 = E_1 \left(\frac{E_5 - E_1}{R_{45}} \right) = \frac{E_1 E_5}{R_{45}} - \frac{E_1^2}{R_{45}} \geq 0$$

Generator se ponaša tako samo ako je $E_5 > E_1$ inače se ponaša kao potrošač!

GENERATOR BI MOGAO DA SE PONAŠA I KAO POTROŠAČ AKO JE E_5 MNOGO VEĆI KOD $(E_5 \gg E_1)$

GENERATOR SE PONAŠA KAO POTROŠAČ!



- * $n_k = n_g (n_c - 1) = 5 - 1 = 4$ konture
- * Jedna kontura mora proći SAMA kroz I_{g3} u smjeru I_{g3} .
Ako je to k_2 tada je $I_{k2} = I_{g3}$.
- * Ostale 3 konture se biraju tako da ne prolaze kroz I_{g3}
- * Konture je najbolje birati tako da imaju što manje zajedničkih grana (otpornosti međusobnih)
Ovde sve konture imaju zajednički samo E_1 i
tada su sve međusobne otpornosti $R_{ij} = 0$?

Sistem bi bio 4×4 :

$$K1: R_{11} I_{k1} + R_{12} I_{k2} + R_{13} I_{k3} + R_{14} I_{k4} = \sum_{k1} E$$

$$K2: I_{k2} = I_{g3}$$

$$K3: R_{31} I_{k1} + R_{32} I_{k2} + R_{33} I_{k3} + R_{34} I_{k4} = \sum_{k3} E$$

$$K4: R_{41} I_{k1} + R_{42} I_{k2} + R_{43} I_{k3} + R_{44} I_{k4} = \sum_{k4} E$$

$$K1: R_2 I_{k1} = E_1 \Rightarrow I_{k1} = E_1 / R_2$$

$$K2: I_{k2} = I_{g3}$$

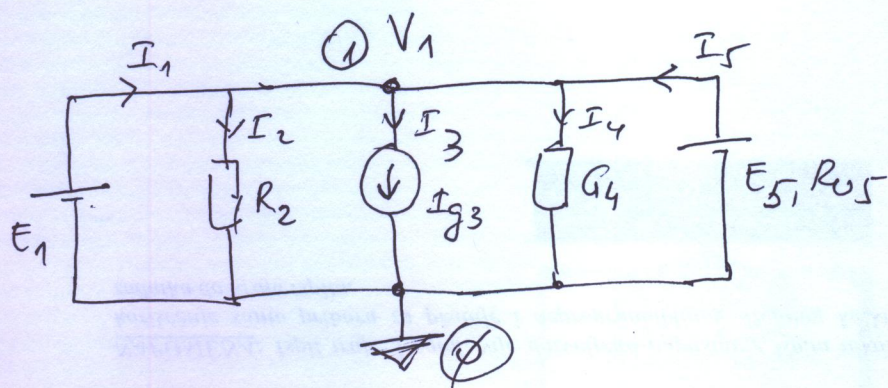
$$K3: \left(\frac{1}{G_4} \right) I_{k3} = E_1 \Rightarrow I_{k3} = G_4 E_1$$

$$K4: R_{u5} I_{k4} = E_1 - E_5 \Rightarrow I_{k4} = \frac{E_1 - E_5}{R_{u5}}$$

$$\begin{aligned} I_1 &= I_{k1} + I_{k2} + I_{k3} + I_{k4} \\ I_1 &= \frac{E_1}{R_2} + I_{g3} + G_4 E_1 + \frac{E_1 - E_5}{R_{u5}} \\ I_2 &= I_{k1} = E_1 / R_2 \\ I_3 &= I_{g3} = I_{k2} \\ I_4 &= I_{k3} = G_4 E_1 \\ I_5 &= -I_{k4} = \frac{E_5 - E_1}{R_{u5}} \end{aligned}$$

Primer 5

4



MNČ/MPC

$$* n_{\bar{c}} - 1 = 1$$

* idealni E_1

$$n_{\bar{c}} - 1 - n_E = 0$$

$\Rightarrow$ nema jednačine po MPC!

$$\textcircled{1}: V_1 = E_1$$

* Struje se nalaze na isti način kao je izvedeno MPC:

$$V_1 = U_{10} = R_2 I_2 \Rightarrow I_2 = \frac{V_1}{R_2} = \frac{E_1}{R_2}$$

$$I_3 = I_{g3} \text{ (ovo ne zavisi od metode)}$$

$$V_1 = U_{10} = \frac{1}{G_4} \cdot I_4 \Rightarrow I_4 = G_4 V_1 = G_4 E_1$$

$$V_1 = U_{10} = E_5 - R_{55} I_5 \Rightarrow I_5 = \frac{E_5 - V_1}{R_{55}} = \frac{E_5 - E_1}{R_{55}}$$

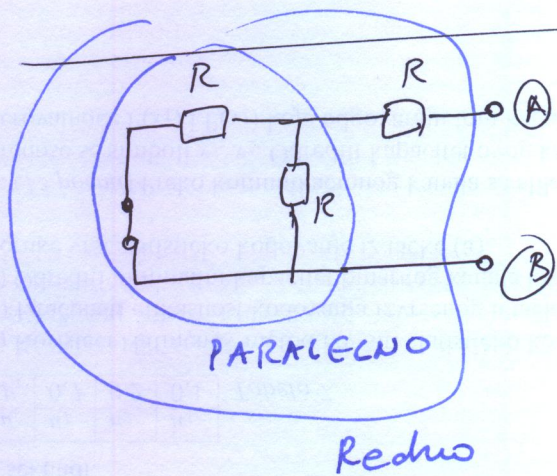
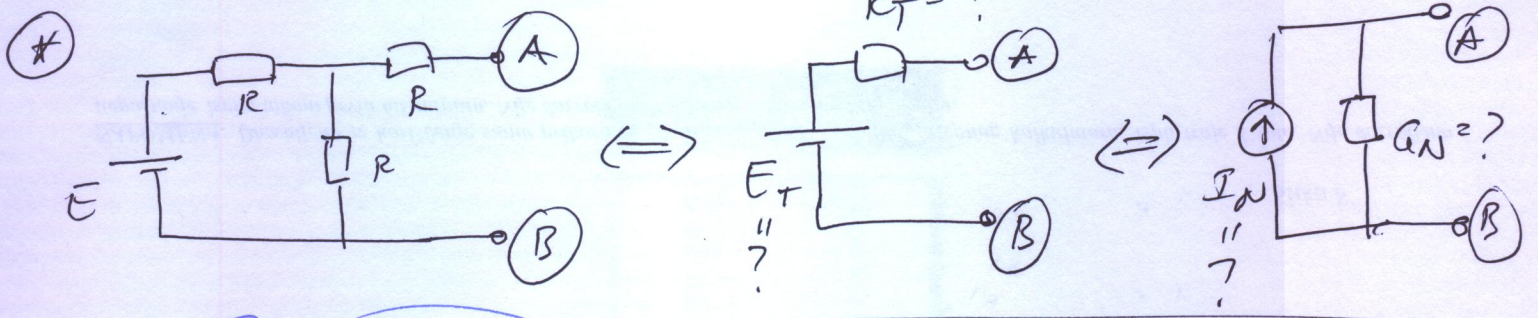
* Kao uobičajeno $I_1 = ???$ uobičajeno idealni E_1 ne može isto,
 $\Rightarrow$ mora se koristiti IKB

$$I_1 = I_2 + I_3 + I_4 - I_5$$

$$I_1 = \frac{E_1}{R_2} + I_{g3} + G_4 E_1 + \frac{E_1}{R_{55}} - \frac{E_5}{R_{55}}$$

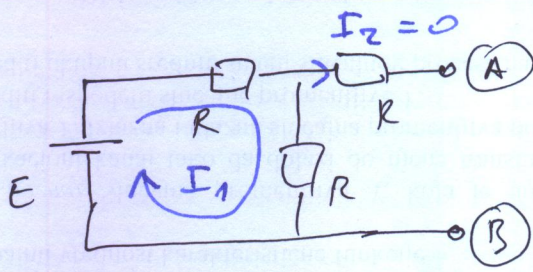
Primer (6)

(5)



$$R_T = R_{AB} = R + \frac{R \cdot R}{R + R} = R + \frac{R}{2} = \frac{3}{2} R$$

$$G_N = \frac{1}{R_T} = \frac{1}{R_{AB}} = \frac{2}{3R}$$

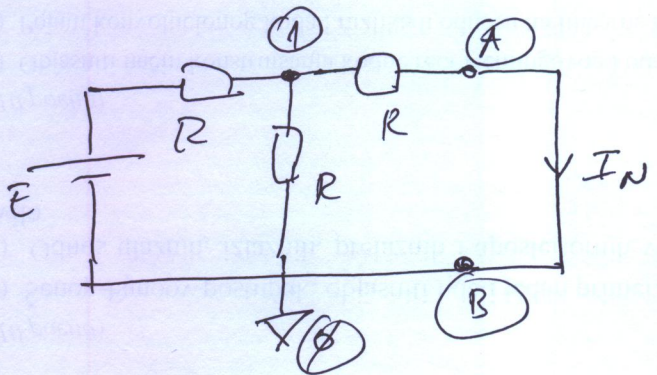


$$-E + RI_1 + RI_1 = 0$$

$$I_1 = E/2R$$

$$E_T = U_{ABO} = RI_1 - RI_2$$

$$E_T = E/2$$

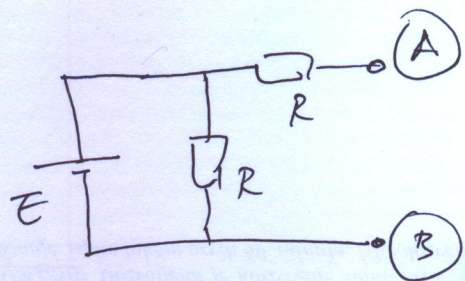


$$\left(\frac{1}{R} + \frac{1}{R} + \frac{1}{R}\right)V_1 = \frac{E}{R}$$

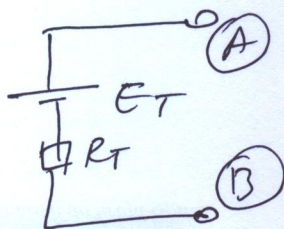
$$V_1 = \frac{E}{3}$$

$$I_N = \frac{V_1}{R} = \frac{E}{3R}$$

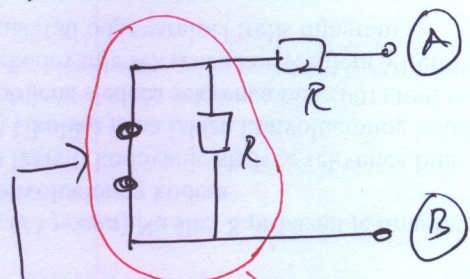
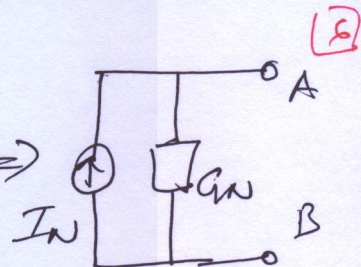
(*)



$\Leftrightarrow$



$\Leftrightarrow$



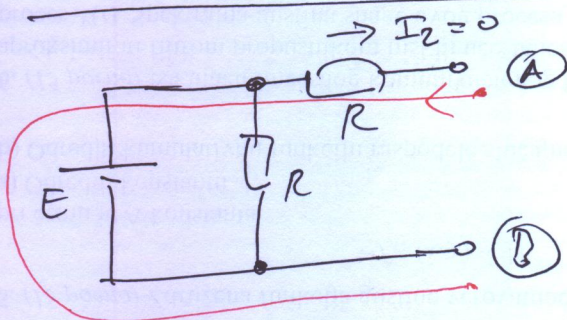
$E=0$
KRAJAN
SPOJ

PARALELNA
VEZA R i K_S
iE K_S!

$$R_{ew} = \frac{0 \cdot R}{0 + R} = 0!$$

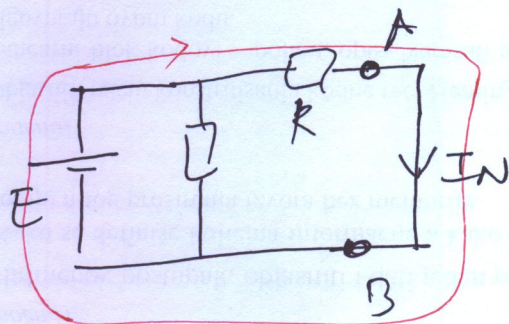
$$R_{ew} = R_T = R + \frac{0 \cdot R}{0 + R} = R = R_T$$

$$G_N = \frac{1}{R}$$



$$E_T = U_{AB0} = -R \cdot I_2 + E = E$$

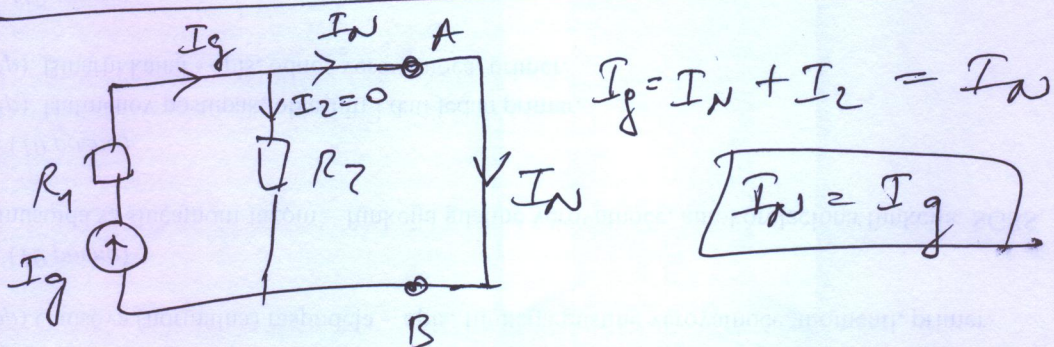
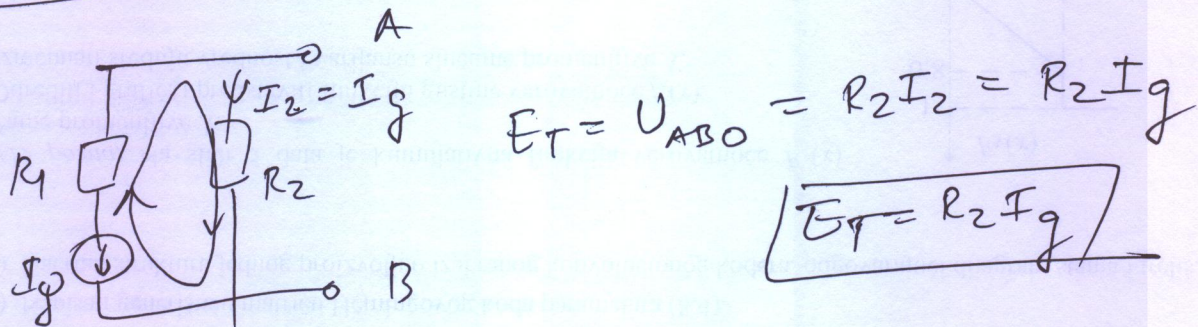
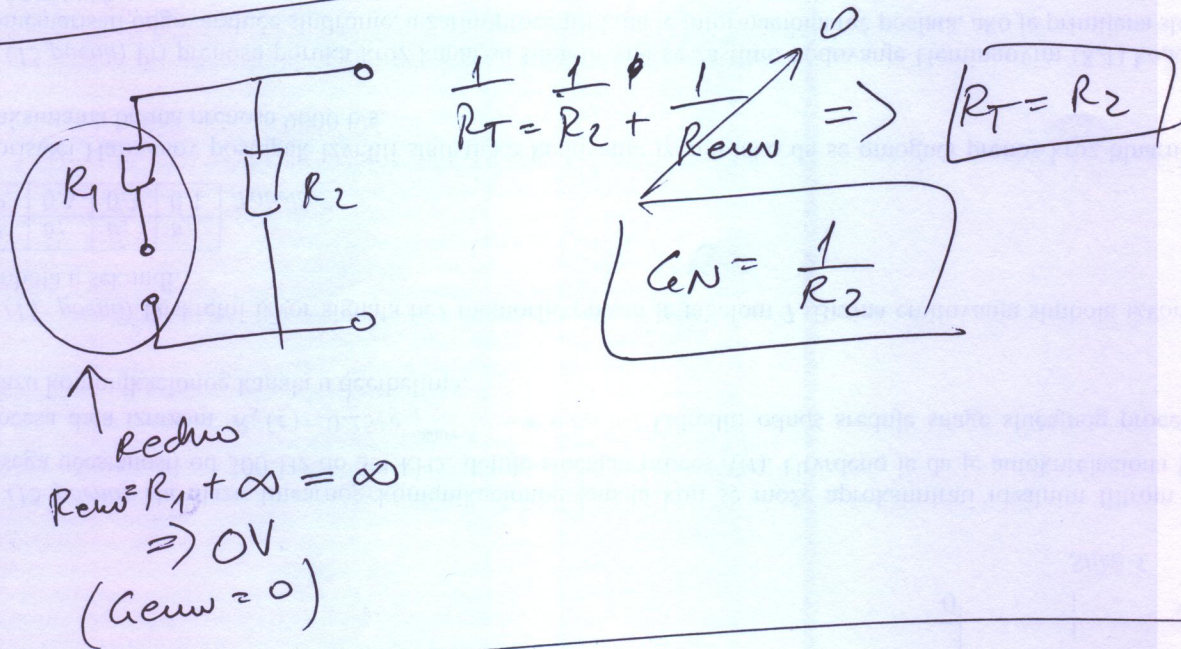
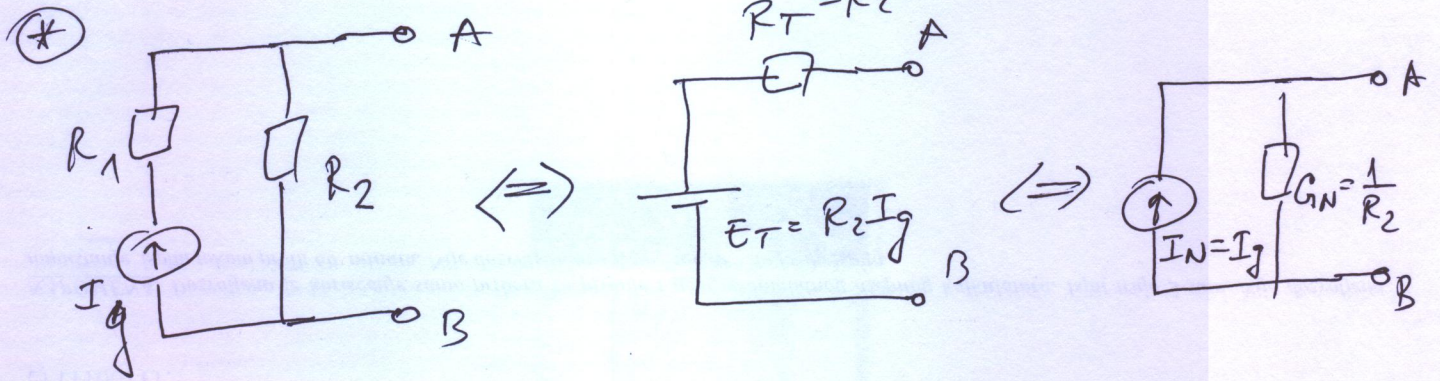
$$E_T = E$$



$$-E + R I_N = 0$$

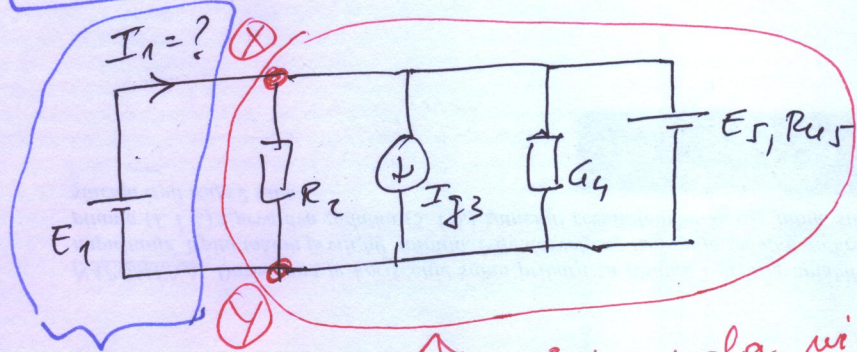
$$I_N = E/R$$

(7)



Primer 7

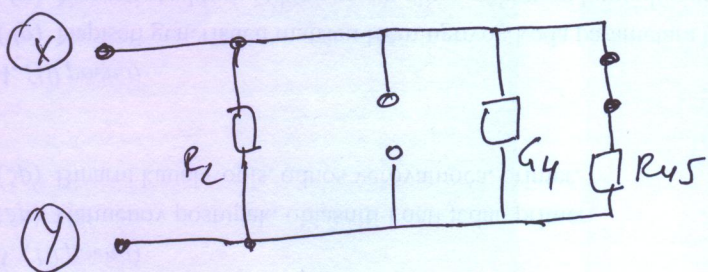
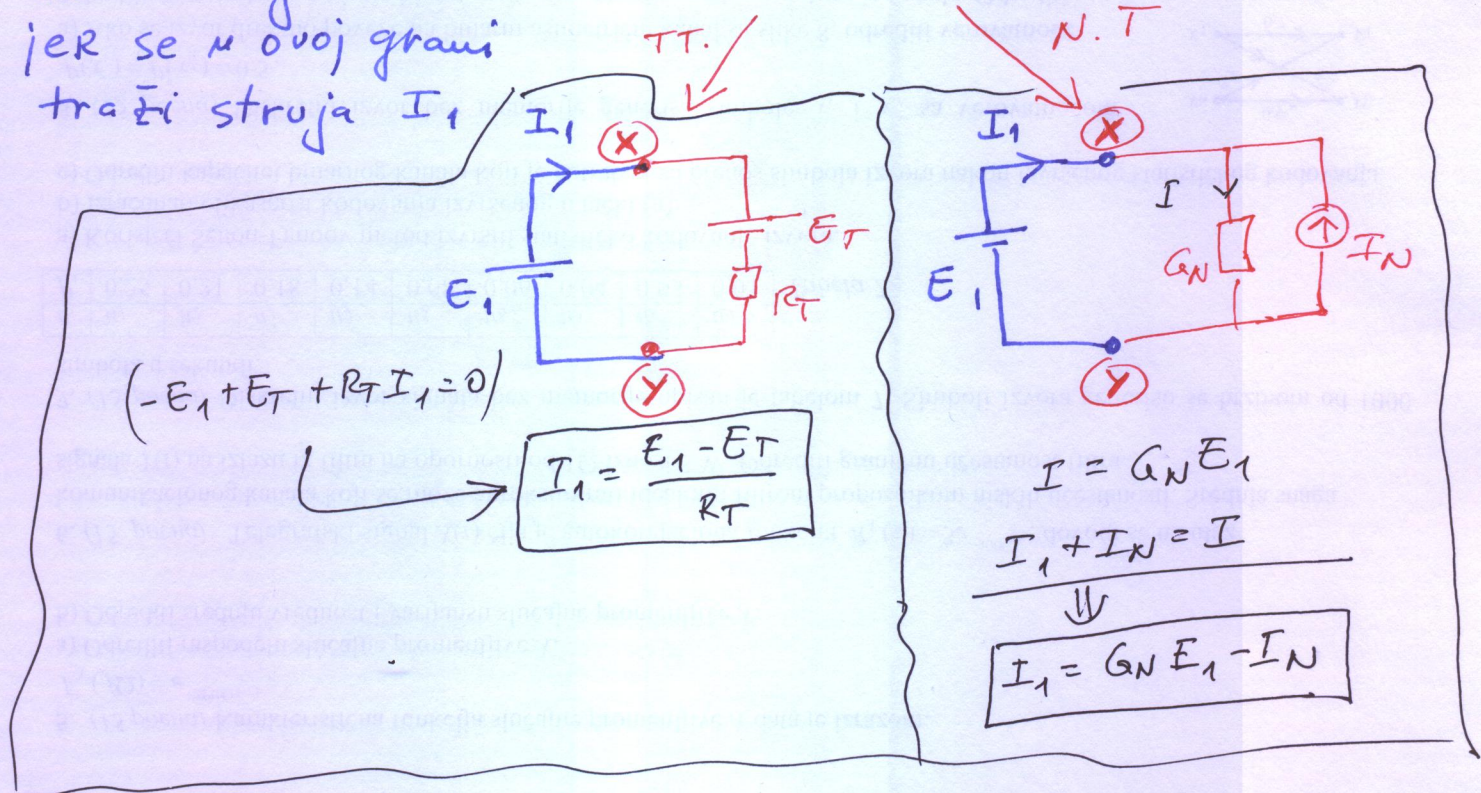
8



Primenom T.T.
i N.T. maći I_1

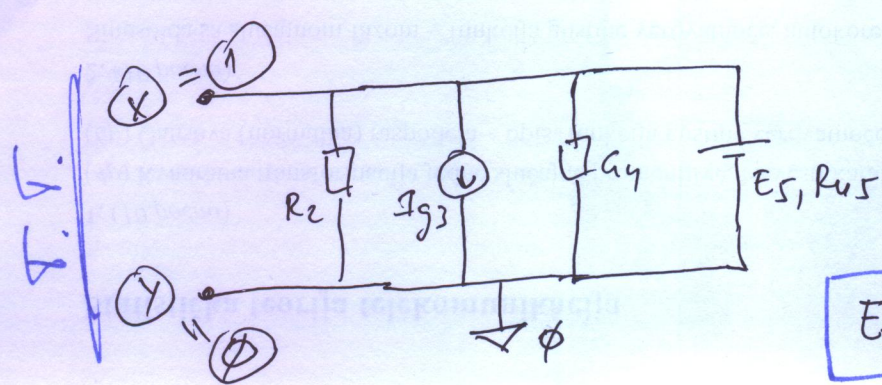
ovaj deo kola ostaje
jer se u ovoj grani
traži struja I_1

ovaj deo kola nije potrebno rešavati
⇒ može se zameniti Theveninovim ili
Nortonovim generatorom?



$$R_T = \frac{1}{\frac{1}{R_2} + G_4 + \frac{1}{R_{55}}}$$

$$G_N = \frac{1}{R_2} + G_4 + \frac{1}{R_{55}}$$



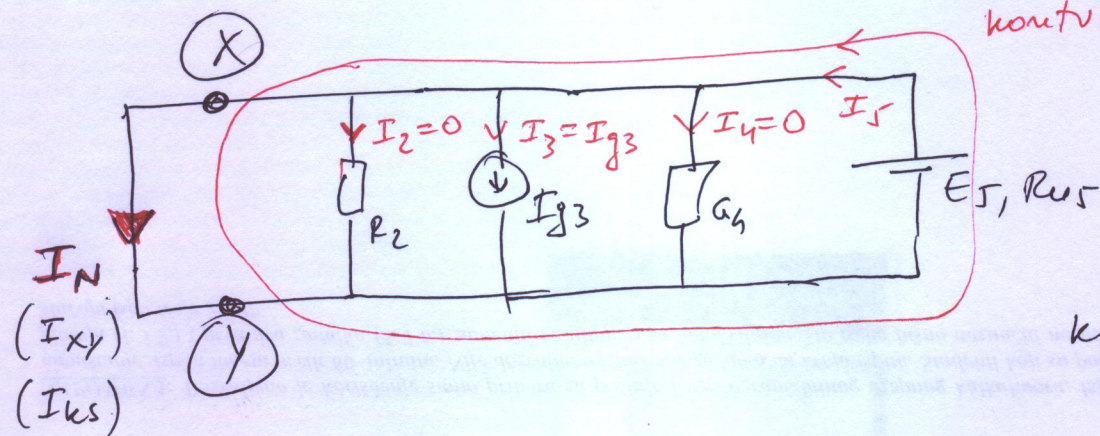
$$E_T = V_{X-Y} = V_1 \leftarrow \text{MPC}$$

$$G_{11} V_1 = \sum I$$

$$\left(\frac{1}{R_2} + G_4 + \frac{1}{R_{55}} \right) V_1 = \frac{E_5}{R_{55}} - I_{g3}$$

$$E_T = V_1 = \frac{E_5 / R_{55} - I_{g3}}{1/R_2 + G_4 + 1/R_{55}}$$

N.T.



kontura K

9

R_2 i G_4 su
kratko spojeni
 $U_{R2} = U_{G4} = 0$
 $I_{R2} = I_{G4} = 0$

Kontura K : $-E_5 + R_{55} I_5 = 0$

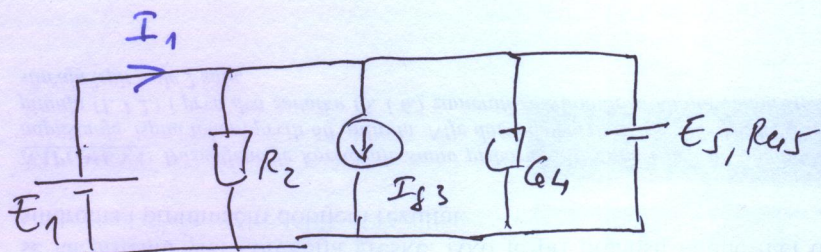
$I_5 = E_5 / R_{55}$

$\Sigma U_{OR}(X) : I_N + \cancel{I_2} + I_3 + \cancel{I_4} = I_5$

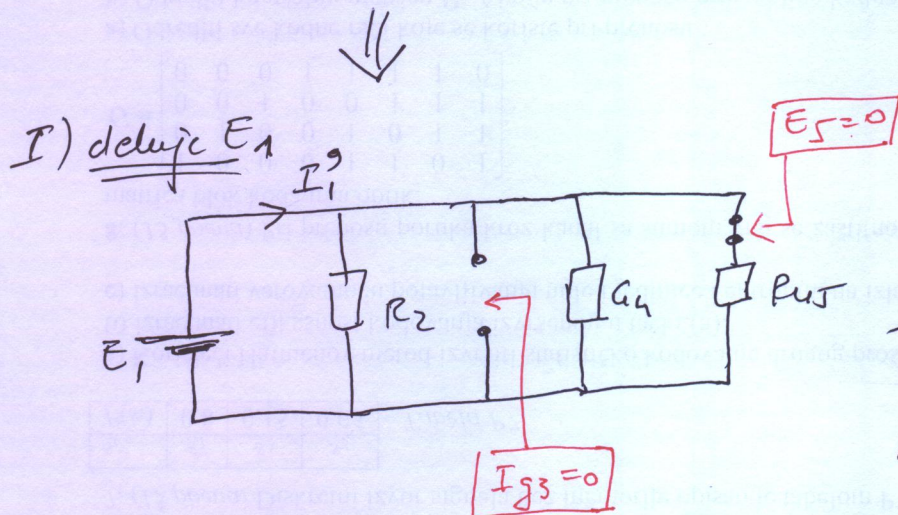
$I_N = I_5 - I_3 = \frac{E_5}{R_{55}} - I_{g3}$

Primer 8

(10)



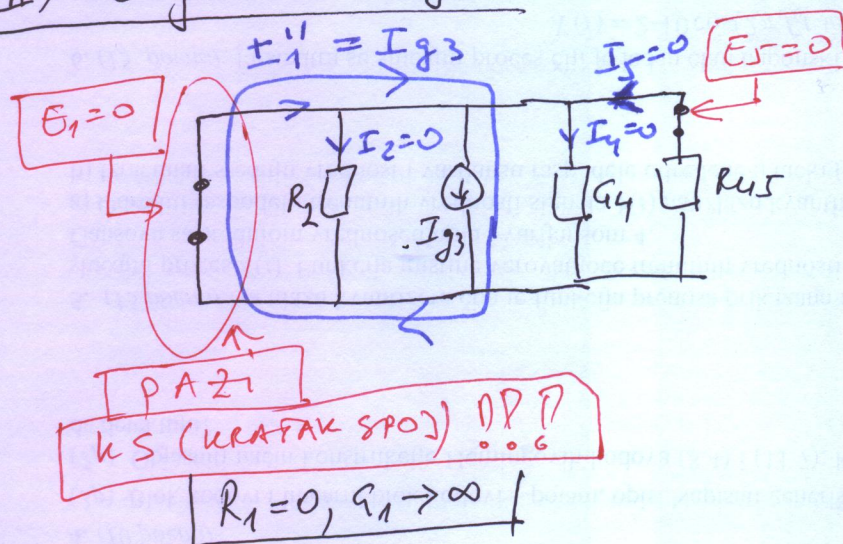
Odrediti I_1
primenom teoreme
superpozicije



$$R_{ekv} = \frac{1}{\frac{1}{R_2} + G_4 + \frac{1}{R_{u5}}}$$

$$I_1' = \frac{E_1}{R_{ekv}} = E_1 \left(\frac{1}{R_2} + G_4 + \frac{1}{R_{u5}} \right)$$

II) deluje samo I_{g3}

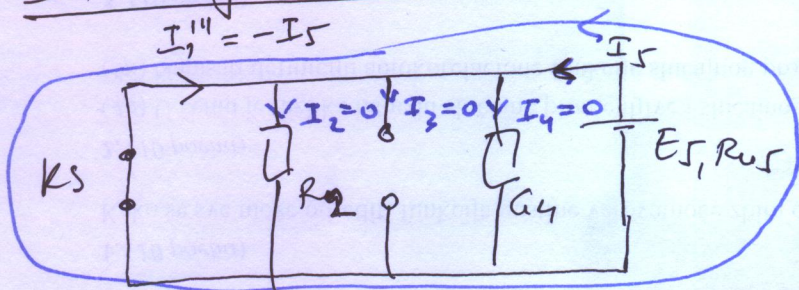


$$R_{ekv} = \frac{1}{\frac{1}{R_2} + G_4 + \frac{1}{R_{u5}} + \left(\frac{1}{R_1} \right)} = 0 \rightarrow \infty$$

OVA STRUJA I_{g3}
PROLAZI KROZ KS
(gde je bio E_1)

$$I_1'' = I_{g3}$$

III) deluje samo E_5



$$E_5 - R_{u5} I_5 = 0$$

$$I_5 = \frac{E_5}{R_{u5}}$$

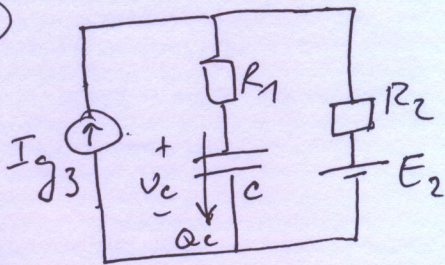
$$I_1''' = -I_5 = -\frac{E_5}{R_{u5}}$$

SUPERPOZICIJA: $I_1 = I_1' + I_1'' + I_1''' = \frac{E_1}{R_2} + G_4 E_1 + \frac{E_1}{R_{u5}} + I_{g3} - \frac{E_5}{R_{u5}}$

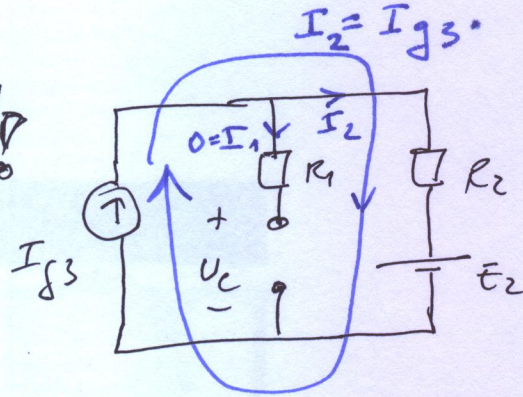
Primer 9

111

⊛



$$I_C = 0 \Rightarrow$$



$$Q_C = C U_C$$

$$Q_C = C (R_2 I_{g3} + E_2)$$

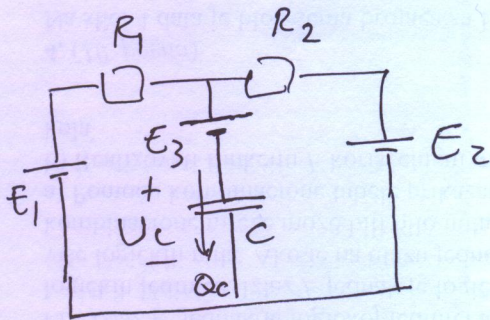
$$I_1 = I_C = 0$$

$$I_2 = I_{g3}$$

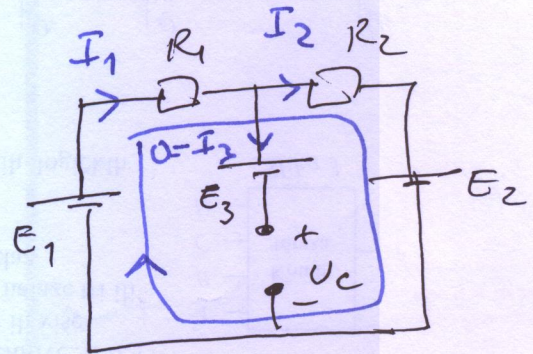
$$U_C = -R_1 I_1 + R_2 I_2 + E_2$$

$$U_C = R_2 I_{g3} + E_2$$

⊛



$$I_C = 0 \Rightarrow$$



$$Q_C = C U_C$$

$$Q_C = C U_C$$

$$Q_C = \frac{R_2}{R_1 + R_2} E_1 C + \frac{R_1}{R_1 + R_2} E_2 C - E_3 C$$

$$I_3 = I_C = 0$$

$$I = I_1 = I_2$$

$$-E_1 + R_1 I_1 + R_2 I_2 + E_2 = 0$$

$$I = I_1 = I_2 = \frac{E_1 - E_2}{R_1 + R_2}$$

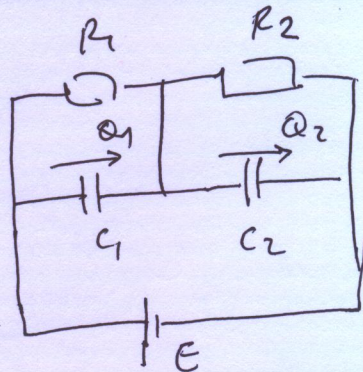
$$U_C = R_2 I_2 + E_2 - E_3$$

PAZI!

$$U_C = \frac{R_2}{R_1 + R_2} (E_1 - E_2) + E_2 - E_3$$

OBIČNO SE ZABORAVI

⑦



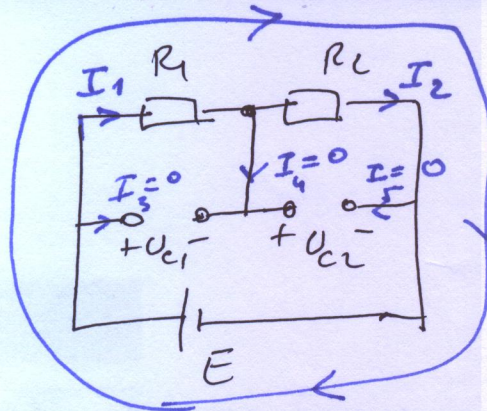
$$I_{C1} = 0$$

$$\Rightarrow$$

$$I_{C2} = 0$$

$$Q_{C1} = C_1 U_{C1} = \frac{R_2 C_1 E}{R_1 + R_2}$$

$$Q_{C2} = C_2 U_{C2} = \frac{R_2 C_2 E}{R_2 + R_1}$$



12

$$I_{C1} = I_{C2} = 0 \Rightarrow I_3 = I_4 = I_5 = 0$$

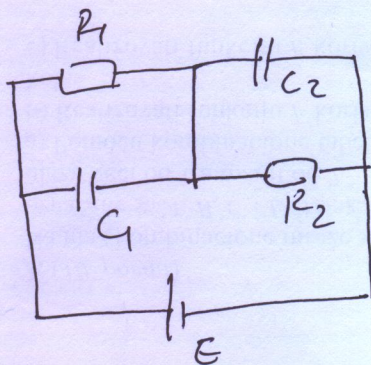
$$I = I_1 = I_2$$

$$I = \frac{E}{R_1 + R_2}$$

$$U_{C1} = R_1 I_1 = \frac{R_1}{R_1 + R_2} E$$

$$U_{C2} = R_2 I_2 = \frac{R_2}{R_1 + R_2} E$$

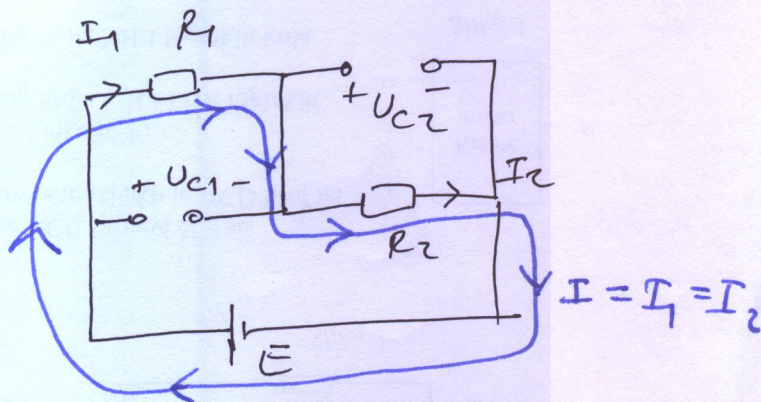
⑧



$$I_{C1} = 0$$

$$I_{C2} = 0$$

$$\Rightarrow$$



isto rešenje kao prethodni primer