1

$z = x\sqrt{y} - x^2 - y + 6x + 3$

$z'_x = \sqrt{y} - 2x + 6$

$z'_y = \frac{x}{2\sqrt{y}} - 1$ 2

$\sqrt{y} - 2x + 6 = 0$

$\frac{x}{2\sqrt{y}} - 1 = 0 \Rightarrow x = 2\sqrt{y}$

$\sqrt{y} - 4\sqrt{y} + 6 = 0$

$3\sqrt{y} = 6$

$\sqrt{y} = 2$

$\boxed{y = 4} \Rightarrow \boxed{x = 4}$ 5

S(4,4) – стац. точка.

$z''_{xx} = -2$

$z''_{xy} = \frac{1}{2\sqrt{y}}$ 3

$z''_{yy} = -\frac{1}{4}\frac{x}{\sqrt{y^3}}$

$r = z''_{xx}(4,4) = -2$

$s = z''_{xy}(4,4) = \frac{1}{4}$ 2

$t = z''_{yy}(4,4) = -\frac{1}{4} \cdot \frac{4}{8} = -\frac{1}{8}$

$\left.\begin{array}{l} r < 0 \\ rt - s^2 = \frac{1}{4} - \frac{1}{16} > 0 \end{array}\right\}$ 3 $\Rightarrow$ в точке S(4,4) ф-ция имеет локальн. макс.

$z(4,4) = 4 \cdot 2 - 16 - 4 + 24 + 3 = 15$

② $V = \pi \int_a^b g^2(x)\,dx$ ①

$$V = \pi \int_0^{\pi/2} \frac{1}{1+\sin x+\cos x}\,dx = \left(\begin{array}{l} \text{смена} \\ t = \operatorname{tg}\frac{x}{2} \\ dx = \frac{2dt}{1+t^2} \end{array} \quad \begin{array}{l} \sin x = \frac{2t}{1+t^2} \\ \cos x = \frac{1-t^2}{1+t^2} \end{array} \quad \begin{array}{c|c|c} x & 0 & \pi/2 \\ \hline t & 0 & 1 \end{array} \right)$$

⑤

$$= \pi \int_0^1 \frac{\frac{2}{1+t^2}}{1+\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}}\,dt = \pi \int_0^1 \frac{2}{1+t^2+2t+1-t^2}\,dt = \pi \int_0^1 \frac{2}{2+2t}\,dt$$

④

$$= \pi \int_0^1 \frac{dt}{1+t} = \pi \ln|1+t| \Big|_0^1 = \underline{\underline{\pi \ln 2}}$$

3 a) $\displaystyle\int \frac{x^2 - x - 1}{(x+1)(x^2+x+1)}\,dx$

$$\frac{x^2 - x - 1}{(x+1)(x^2+x+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+x+1}$$ 1

$$x^2 - x - 1 = A(x^2+x+1) + (Bx+C)(x+1) = x^2(A+B) + x(A+B+C) + A+C$$

$$\left.\begin{array}{l} A + B = 1 \Rightarrow B = 1 - A \\ A + B + C = -1 \\ A + C = -1 \Rightarrow C = -1 - A \end{array}\right\} \Rightarrow \cancel{A} + \cancel{1} - \cancel{A} - \cancel{1} - A = -1$$

$$-A = -1 \Rightarrow \boxed{A = 1}$$

$$\boxed{B = 0} \quad \boxed{C = -2}$$ 4

$$\Rightarrow \int \frac{x^2 - x - 1}{(x+1)(x^2+x+1)}\,dx = \int \left(\frac{1}{x+1} - \frac{2}{x^2+x+1}\right)dx$$

$$= \int \frac{dx}{x+1} - 2\int \frac{dx}{x^2+x+1} = \ln|x+1| - 2\int \frac{dx}{(x+\frac{1}{2})^2 + \frac{3}{4}}$$

$$= \ln|x+1| - 2 \cdot \frac{1}{\sqrt{\frac{3}{4}}} \cdot \operatorname{arctg} \frac{(x+\frac{1}{2})}{\sqrt{\frac{3}{4}}} + C$$ 7

$$= \ln|x+1| - \frac{4}{\sqrt{3}} \operatorname{arctg} \frac{2x+1}{\sqrt{3}} + C$$

③ б) $\int \cos x \sqrt{\sin^2 x + 1}\, dx = \begin{pmatrix} \sin x = t \\ \cos x\, dx = dt \end{pmatrix}$ ①

$= \int \sqrt{t^2+1}\, dt$

МОЖЕ НА ВИШЕ НАЧИНА ...

$I = \int \sqrt{1+t^2}\, dt = \int \frac{1+t^2}{\sqrt{1+t^2}}\, dt = \int \frac{dt}{\sqrt{1+t^2}} + \underbrace{\int \frac{t^2}{\sqrt{1+t^2}}\, dt}_{J} = (*)$

④

$J = \begin{pmatrix} u = t & dv = \frac{t\, dt}{\sqrt{1+t^2}} \\ du = dt & v = \sqrt{1+t^2} \end{pmatrix} = t\sqrt{1+t^2} - \int \sqrt{1+t^2}\, dt$

$= t\sqrt{1+t^2} - I + C$

$(*) = \ln|t + \sqrt{1+t^2}| + t\sqrt{1+t^2} - I + C$

$\Rightarrow 2I = \ln|t+\sqrt{1+t^2}| + t\sqrt{1+t^2} + C$

⑦

$I = \frac{1}{2}\left(\ln|t+\sqrt{1+t^2}| + t\sqrt{1+t^2}\right) + C$

$\Rightarrow \int \cos x \sqrt{\sin^2 x + 1}\, dx = \frac{1}{2}\left(\ln|\sin x + \sqrt{\sin^2 x + 1}| + \sin x \sqrt{1+\sin^2 x}\right) + C$

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3 h) $\int_0^{+\infty} \sqrt{x}\, e^{-\sqrt{x}}\, dx = \left(\begin{array}{ll} \sqrt{x} = t & dx = 2t\,dt \\ x = t^2 & \begin{array}{c|c|c} x & 0 & +\infty \\ \hline t & 0 & +\infty \end{array} \end{array} \right)$ 2

$= 2\int_0^{+\infty} t^{1/2} e^{-t}\, t\, dt = 2\int_0^{+\infty} t^{3/2} e^{-t}\, dt = 2\,\Gamma\left(\frac{5}{2}\right) = 2 \cdot \frac{3}{2} \cdot \frac{1}{2}\,\Gamma\left(\frac{1}{2}\right)$

$= \underline{\underline{\frac{3}{2}\sqrt{\pi}}}$. 6

(4) a) $y' + y \operatorname{ctg} x = e^{x}$

$p(x) = \operatorname{ctg} x$

$g(x) = e^{x}$

$$y = e^{-\int p(x)dx}\left(c + \int e^{\int p(x)dx} \cdot g(x)\right) \quad (2)$$

$$\int p(x)dx = \int \operatorname{ctg} x\, dx = \int \frac{\cos x}{\sin x} dx = \ln \sin x \quad (2)$$

$$\int e^{\int p(x)dx} \cdot g(x)\, dx = \int e^{x} \sin x \cdot dx = \dots = \frac{e^{x}(\sin x - \cos x)}{2} \quad (7)$$

$$y = \frac{1}{\sin x}\left(c + \frac{e^{x}(\sin x - \cos x)}{2}\right) \quad (1)$$

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4) $xy' + y = \sqrt{x^2+y^2} \quad /:x \neq 0, \quad y(1) = 0$

$$y' + \frac{y}{x} = \sqrt{1 + \left(\frac{y}{x}\right)^2}$$

(смена $\frac{y}{x} = z$ $\quad y' = z + z'x$) 2

$$z + z'x + z = \sqrt{1+z^2}$$

$$z'x = \sqrt{1+z^2} - 2z$$

$$x\frac{dz}{dx} = \sqrt{1+z^2} - 2z$$

$$\frac{dz}{\sqrt{1+z^2}-2z} = \frac{dx}{x} \Big/ \int$$

$$\underbrace{\int \frac{dz}{\sqrt{1+z^2}-2z}}_{I} = \ln|x| + C \quad 1$$

$$I = \int \frac{dz}{\sqrt{z^2+1} - 2z} =$$

(Ојлерова смена $\sqrt{1+z^2} = z + t$) 2 $\quad ^2$

$$\cancel{1} + \cancel{z^2} = \cancel{z^2} + 2zt + t^2$$

$$\boxed{z = \frac{1-t^2}{2t}} \qquad \boxed{dz = \frac{-1-t^2}{2t^2}dt}$$

$$\boxed{\sqrt{1+z^2} = z + t = \frac{1-t^2}{2t} + t = \frac{1+t^2}{2t}}$$

$$= \int \frac{\frac{-1-t^2}{2t^2}dt}{\frac{1+t^2}{2t} - 2\frac{1-t^2}{2t}} = -\int \frac{\frac{1+t^2}{\cancel{2}t\cancel{^2}}}{\frac{3t^2-1}{\cancel{2t}}} dt$$

$$= -\int \frac{1+t^2}{(3t^2-1)t}dt = -\frac{1}{3}\int \frac{1+t^2}{(t^2-\frac{1}{3})t}dt = -\frac{1}{3}\int \frac{1+t^2\,dt}{(t-\frac{1}{\sqrt{3}})(t+\frac{1}{\sqrt{3}})t} = (*)$$

6

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(4) 5) ВАСТАВАМЕ

$$\int \frac{1+t^2}{\left(t-\frac{1}{\sqrt3}\right)\left(t+\frac{1}{\sqrt3}\right)t} = \int\left(\frac{A}{t-\frac{1}{\sqrt3}} + \frac{B}{t+\frac{1}{\sqrt3}} + \frac{C}{t}\right)dt$$

$$\begin{cases} A = 2 \\ B = 2 \\ C = -3 \end{cases}$$

$$\int = 2\ln\left|t-\frac{1}{\sqrt3}\right| + 2\ln\left|t+\frac{1}{\sqrt3}\right| - 3\ln|t|$$

$$= 2\ln\left|\frac{t-\frac{1}{\sqrt3}}{t+\frac{1}{\sqrt3}}\right| - 3\ln t = \ln\left(\frac{t-\frac{1}{\sqrt3}}{t+\frac{1}{\sqrt3}}\right)^2 + \ln t^{-3}$$

$$= \ln\frac{\left(t-\frac{1}{\sqrt3}\right)^2}{t^3\left(t+\frac{1}{\sqrt3}\right)^2} =$$

$$= \ln\frac{\left(\sqrt{z^2+1}-z-\frac{1}{\sqrt3}\right)^2}{\left(\sqrt{z^2+1}-z\right)^3\left(\sqrt{z^2+1}-z+\frac{1}{\sqrt3}\right)^2}$$

2

$$= \boxed{\ln\frac{\left(\sqrt{\left(\frac{y}{x}\right)^2+1}-\frac{y}{x}-\frac{1}{\sqrt3}\right)^2}{\left(\sqrt{\left(\frac{y}{x}\right)^2+1}-\frac{y}{x}\right)^3\left(\sqrt{\left(\frac{y}{x}\right)^2+1}-\frac{y}{x}+\frac{1}{\sqrt3}\right)^2}} = F(x,y)$$

O.P. $-\frac{1}{3}F(x,y) = \ln|x| + C$ ЗА $x=1 \Rightarrow y=0$

$$-\frac{1}{3}F(1,0) = \ln 1 + C$$

$$-\frac{1}{3}\ln\frac{\left(\sqrt1-0-\frac{1}{\sqrt3}\right)^2}{\left(\sqrt1-0\right)^3\left(\sqrt{0+1}-0+\frac{1}{\sqrt3}\right)^2} = 0 + C$$

1

$\boxed{C = \dots}$

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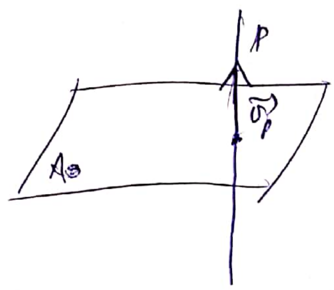
5 a) $A(1,2,2)$ $\quad p: \frac{x-2}{0} = \frac{y-5}{1} = \frac{z-1}{1}$

$\alpha \ni A$

2

$\alpha \perp p \Rightarrow \vec{v}_p = \vec{n}_\alpha \Rightarrow \vec{n}_\alpha = (0,1,1)$

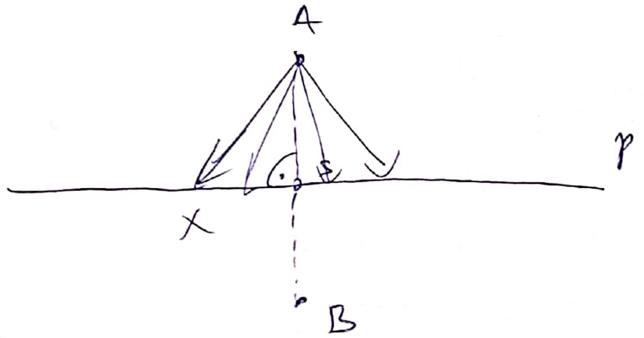


$\alpha: 0(x-1) + 1(y-2) + 1(z-2) = 0$

$\boxed{y + z - 4 = 0}$

2

б)



$p: x = 2$
$\quad y = 5 + t$
$\quad z = 1 + t$

$X \in p \Rightarrow X(2, 5+t, 1+t)$

$\vec{AX} = (1, 3+t, t-1)$

2

$\vec{AX} \circ \vec{v}_p = 0$

$\boxed{S(2,4,0)}$

$0 + 1(3+t) + 1(t-1) = 0$

$3 + t + t - 1 = 0$

$2t = -2$

$t = -1$

6

$S = \frac{A+B}{2} \Rightarrow B = 2S - A$

$\boxed{B = (4,8,0) - (1,2,2) = (3,6,-2)}$

2