M1, јуни 2020

1. (15) $z = \begin{vmatrix} 2i+1 & 3i-1 & i+1 \\ i-1 & -i & i-1 \\ 1 & i & 1 \end{vmatrix} = \begin{vmatrix} i & 3i-1 & i+1 \\ 0 & -i & i-1 \\ 0 & i & 1 \end{vmatrix}$

$= \begin{vmatrix} i & 3i-1 & i+1 \\ 0 & -i & i-1 \\ 0 & 0 & i \end{vmatrix} = -i^3 = i$ (5)

$\sqrt[3]{8z+8} = 2\cdot\sqrt[3]{(1+i)}$

$1+i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ (5)

$z_k = 2\cdot\sqrt[3]{\sqrt{2}}\cdot\left(\cos\frac{\pi/4+2k\pi}{3} + i\sin\frac{\pi/4+2k\pi}{3}\right)$ (5) $k = 0, 1, 2$

2. (15)

$$\begin{array}{c} \begin{array}{cccc} z & y & x & t \end{array} \\ \left[\begin{array}{cccc|c} 1 & -2 & 2 & -1 & -1 \\ 2 & -6 & 5 & -7 & -8 \\ 3 & -4 & 5 & 2 & 3 \\ -1 & 4 & -3 & a & a+1 \end{array}\right] \end{array} \begin{array}{c} |-2 \;|-3\; | \end{array} \sim \left[\begin{array}{cccc|c} 1 & -2 & 2 & -1 & -1 \\ 0 & -2 & 1 & -5 & -6 \\ 0 & 2 & -1 & 5 & 6 \\ 0 & 2 & -1 & a-1 & a \end{array}\right]$$

$$\sim \begin{array}{c} \begin{array}{cccc} z & y & x & t \end{array} \\ \left[\begin{array}{cccc|c} 1 & -2 & 2 & -1 & -1 \\ 0 & -2 & 1 & -5 & -6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a-6 & a-6 \end{array}\right] \end{array}$$ (8)

$r(A_p) = r(A) = \begin{cases} 2, & a = 6 \\ 3, & a \neq 6 \end{cases}$ (2)

Системот увек има бесконечно многу решенија (1)

I $a \neq 6$ $(z, y, x, t) = (2-2\alpha, \alpha, 2\alpha-1, 1)$ $\alpha \in \mathbb{R}$ (2)

II $a = 6$ $(z, y, x, t) = (-2\beta-9\alpha+11, \beta, 2\beta+5\alpha-6, \alpha)$ $\alpha, \beta \in \mathbb{R}$ (2)

3. (10) $XA - 2I = A + 3X$

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$X(A-3I) = A + 2I$

$X = (A+2I)(A-3I)^{-1}$ (4)

за $\det(A-3I) \neq 0$

$(A-3I)^{-1} = \begin{bmatrix} 1 & 3 & 5/2 \\ 1 & 2 & 2 \\ 0 & 0 & -1/2 \end{bmatrix}$ (4)

$X = \begin{bmatrix} 6 & 15 & \frac{25}{2} \\ 5 & 11 & 10 \\ 0 & 0 & -3/2 \end{bmatrix}$ (2)

4. (23) a) $\lim\limits_{x\to 0} \dfrac{\sqrt{1+tg^2 x} - 1}{(e^{2x}-1)\cdot \ln(1+\arcsin x)} = \lim\limits_{x\to 0} \dfrac{\frac{(1+tg^2x)^{1/2}-1}{tg^2x}\cdot\frac{tg^2x}{x^2}\cdot x^2}{2x\cdot\frac{e^{2x}-1}{2x}\cdot\frac{\ln(1+\arcsin x)}{\arcsin x}\cdot\frac{\arcsin x}{x}\cdot x}$

$= \dfrac{\frac{1}{2}}{2} = \dfrac{1}{4}$ (10)

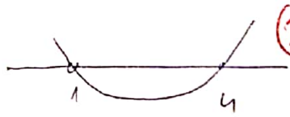
б) $\lim\limits_{x\to 0} \dfrac{x - \sin 2x}{x^3} \overset{\frac{0}{0}}{=} \lim\limits_{x\to 0}\dfrac{1-2\cos 2x}{3x^2} \overset{\frac{0}{0}}{=} \lim\limits_{x\to 0}\dfrac{4\sin 2x}{6x} = \dfrac{4}{3}$ (6)

в) $\lim\limits_{n\to\infty} \underbrace{\left(\dfrac{n+2}{n+5}\right)^n}_{(4)} + \underbrace{\dfrac{\cos(n!)}{\sqrt[3]{n}}}_{(3)}$ огр. $= \lim\limits_{n\to\infty}\left(1-\dfrac{3}{n+5}\right)^{-\frac{n+5}{3}\cdot\frac{-3}{n+5}\cdot n} + 0$

$= e^{-3}$ $\cos(n!)\cdot\dfrac{1}{\sqrt[3]{n}}$ огр. пута нула низ је нула низ

5. (15) $y = \sqrt{x^2-5x+4} - x$

1) $x^2-5x+4 = (x-1)(x-4) \geq 0$



(2) $x \in (-\infty, +1] \cup [4, +\infty)$

2) $\sqrt{x^2-5x+4} = x$, $x \geq 0$

$-5x+4 = 0$ $\quad x = \dfrac{4}{5}$ (2) нула ф-је

знак $\quad y \geq 0$ за $x \in \left(-\infty, \frac{4}{5}\right]$ (1)

$y < 0$ за $x \in \left(\frac{4}{5}, 1\right] \cup [4, +\infty)$ (1)

3) $y' = \dfrac{2x-5}{2\sqrt{x^2-5x+4}} - 1 = \dfrac{x-\frac{5}{2}}{\sqrt{x^2-5x+4}} - 1$ (2)

$y'' = \dfrac{\sqrt{x^2-5x+4} - (x-\frac{5}{2})\cdot(2x-5)\cdot 1/2\sqrt{x^2-5x+4}}{x^2-5x+4} = \dfrac{x^2-5x+4 - (x-\frac{5}{2})^2}{(x^2-5x+4)^{3/2}}$

$y'' = \dfrac{4-\frac{25}{4}}{(x^2-5x+4)^{3/2}} = \dfrac{-9}{4(x^2-5x+4)^{3/2}}$ (3)

$y(0) = 2$ $\quad y'(0) = \dfrac{-5/2}{2} - 1 = -\dfrac{9}{4}$ $\quad y''(0) = \dfrac{-9}{4\cdot 8}$

$M_2 = y(0) + y'(0)\cdot x + y''(0)\cdot\dfrac{x^2}{2} = 2 - \dfrac{9}{4}x - \dfrac{9}{64}x^2$ (4)

6. 22 $y=(x+1)\ln^2(x+1)$

1) $x\in(-1,+\infty)$ 1

2) $y\geq 0$ за $x\in D_f$ 1 $\quad y=0$ за $x+1=1,\ x=0$

3) Није ни парна ни неп. јер домен није симетричан у односу на коорд. почетак 1

4) $\lim\limits_{x\to+\infty} y(x)=+\infty$ нема Х.А. 1 $\quad \lim\limits_{x\to+\infty}\frac{f(x)}{x}=+\infty$ 1 нема К.А.

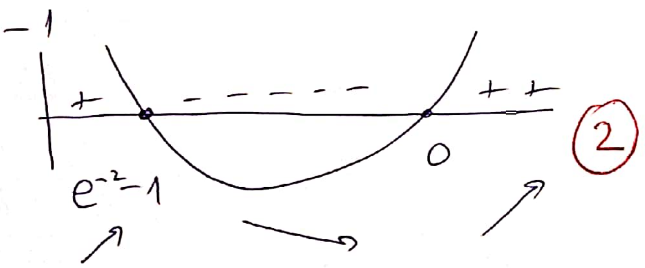
$$\lim_{x\to-1+0} y(x) \overset{0\cdot\infty}{=} \lim_{x\to-1+0}\frac{\ln^2(x+1)}{\frac{1}{x+1}} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to-1+0}\frac{\frac{2\ln(x+1)}{x+1}}{-\frac{1}{(x+1)^2}} = -2\lim_{x\to-1+0}\frac{\ln(x+1)}{\frac{1}{x+1}}$$

$$\overset{LP}{=} -2\lim_{x\to-1+0}\frac{\frac{1}{x+1}}{-\frac{1}{(x+1)^2}} = 2\cdot\lim_{x\to-1+0}(x+1)=0$$ 5 нема В.А.

5) $y'=\ln^2(x+1)+2\ln(x+1)=\ln(x+1)\left(\ln(x+1)+2\right)$ 2

$\ln(x+1)=0$ за $x+1=1,\ x=0$

$\ln(x+1)=-2$ за $x+1=e^{-2},\ x=e^{-2}-1$



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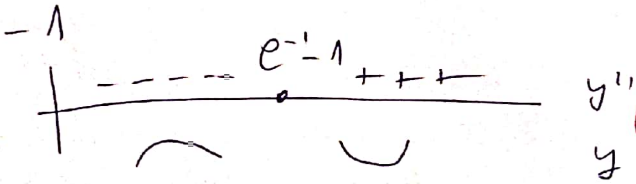
$y_{max}\left(e^{-2}-1,\ \frac{4}{e^2}\right)$ 1

$y_{min}(0,0)$

6) $y''=\dfrac{2(\ln(x+1)+1)}{x+1}$ 3



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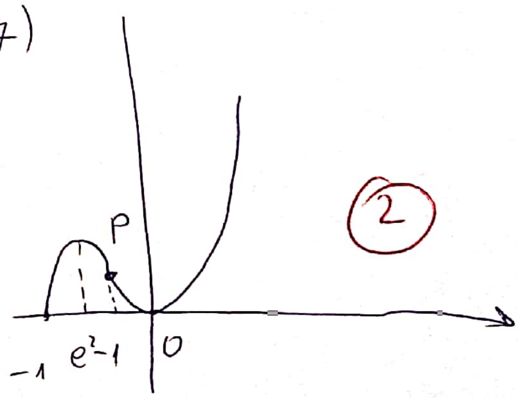
$P\left(e^{-1}-1,\ \frac{1}{e}\right)$ 1

7)



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