1)

$$\left|\frac{a_{n+1}}{a_n}\right| = \left(\frac{4\cdot 7\cdots(3n+1)(3n+4)}{(2n+2)!!}\right)^p \cdot \ln\left(\frac{n^2+3n+5}{n^2+n+3}\right) \cdot \left(\frac{(2n)!!}{4\cdot 7\cdots(3n+1)}\right)^p \cdot \frac{1}{\ln\left(\frac{n^2+3n+1}{n^2-n+2}\right)} = \left(\frac{3n+4}{2n+2}\right)^p \cdot \frac{\ln\left(1+\frac{4n+2}{n^2+n+3}\right)}{\ln\left(1+\frac{4n-1}{n^2-n+2}\right)} \overset{n\to\infty}{\sim} \left(\frac{3}{2}\right)^p \cdot \frac{\frac{4n+2}{n^2+n+3}}{\frac{4n-1}{n^2-n+2}}$$

$$\xrightarrow[n\to\infty]{} \left(\frac{3}{2}\right)^p \cdot$$

6

3

За $p>0$ $\lim\limits_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|>1 \Rightarrow$ ред ~~А.К.~~ по Даламберу јер $a_n \not\to 0$

3

За $p<0$ $\lim\limits_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|<1 \Rightarrow$ ред А.К. по Даламберу.

1

За $p=0$ $\lim\left|\frac{a_{n+1}}{a_n}\right|=1 \Rightarrow$ неодлучно по Даламберу.

За $p=0$ ред постаје

$$\sum_{n=1}^{\infty}(-1)^n \ln\left(1+\frac{4n-1}{n^2-n+2}\right)$$

$$\lim_{n\to\infty}\ln\left(1+\frac{4n-1}{n^2-n+2}\right)=\ln 1 = 0$$

$$f(x)=\ln\left(1+\frac{4x-1}{x^2-x+2}\right)$$

$\Rightarrow$ ред У.К. по Лајбницу јер $\ln\left(1+\frac{4n-1}{n^2-n+2}\right)\sim\frac{4}{n}$, $\sum\frac{4}{n}$ див.

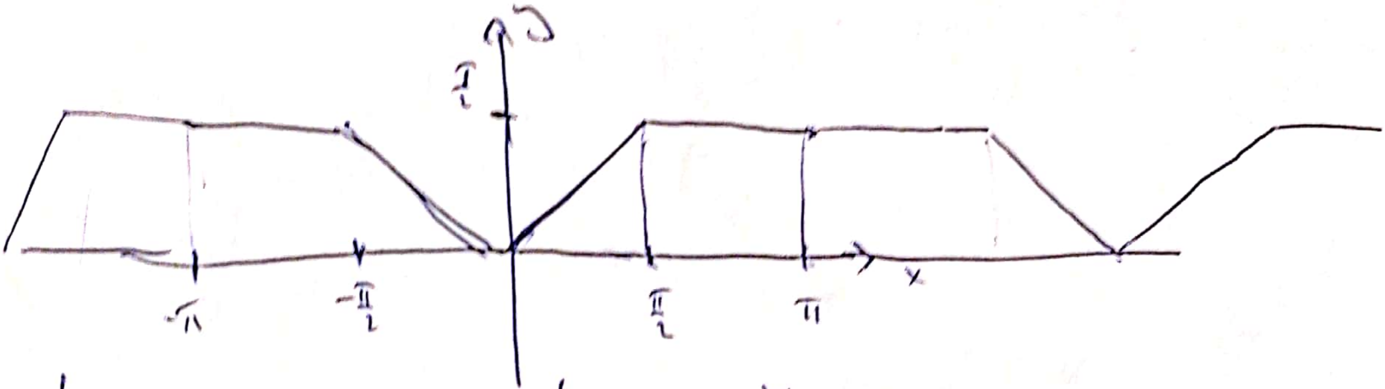
$$f'(x)=\frac{\cancel{x^2-x+2}}{x^2+3x+1}\cdot\frac{4(x^2-x+2)-(4x-1)(2x-1)}{(x^2-x+2)^{\cancel{2}}}$$

7

$$=\frac{-4x^2+2x+7}{(x^2+3x+1)(x^2-x+1)} = \frac{-\cancel{x^2}\left(4+\frac{2}{x}+\frac{7}{x^2}\right)}{x^{\cancel{4}}\left(1+\frac{3}{x}+\frac{1}{x^2}\right)\left(1-\frac{1}{x}+\frac{1}{x^2}\right)} = \frac{-\left(4+\frac{2}{x}+\frac{7}{x^2}\right)}{\left(1+\frac{1}{x}\ldots\right)}$$

$\boxed{\text{за } x\geq 6}$ <0

2) $f(x) = \begin{cases} |x|, & x \in [-\frac{\pi}{2}, \frac{\pi}{2}] \\ \frac{\pi}{2}, & x \in [-\pi, -\frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi] \end{cases}$ $\quad f(x+2\pi) = f(x)$



ф-ја је парна $\Rightarrow b_n = 0 \quad \forall n \geq 1$

ф-ја је непрекидна $\Rightarrow S(x) = f(x) \quad \forall x \in \mathbb{R}$. 4

$$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x)\,dx = \frac{2}{\pi} \cdot \left(\frac{(\frac{\pi}{2})^2}{2} + \left(\frac{\pi}{2}\right)^2 \right) = \frac{2}{\pi} \cdot \frac{3}{2} \cdot \frac{\pi^2}{4} = \frac{3\pi}{4}$$

3

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx\,dx = \frac{2}{\pi} \int_0^{\pi/2} x \cos nx\,dx + \frac{2}{\pi} \int_{\frac{\pi}{2}}^{\pi} \frac{\pi}{2} \cos nx\,dx$$

$$= \frac{2}{\pi} \left(x \cdot \frac{1}{n} \sin nx \Big|_0^{\pi/2} + \frac{1}{n^2} \cos nx \Big|_0^{\frac{\pi}{2}} \right) + \frac{1}{n} \sin nx \Big|_{\frac{\pi}{2}}^{\pi}$$

$$= \frac{2}{\pi} \left(\cancel{\frac{\pi}{2} \cdot \frac{1}{n} \sin \frac{n\pi}{2}} + \frac{\cos \frac{n\pi}{2} - 1}{n^2} \right) \cancel{- \frac{1}{n} \sin \frac{n\pi}{2}}$$

$$= \frac{2}{\pi} \frac{\cos \frac{n\pi}{2} - 1}{n^2} = \begin{cases} \frac{2}{\pi} \left(\frac{(-1)^k - 1}{4k^2} \right), & n = 2k, \quad k = 1, 2, \ldots \\ -\frac{2}{\pi n^2}, & n = 2k - 1 \end{cases}$$

8

$$\varphi(x) = \frac{3\pi}{8} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos\frac{n\pi}{2} - 1}{n^2} \cos nx$$

$$= \frac{3\pi}{8} + \frac{2}{\pi} \left(\sum_{n=1}^{\infty} \frac{(-1)^n - 1}{4n^2} \cos 2nx - \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos(2n-1)x \right)$$

$$\varphi(0) = \frac{3\pi}{8} + \frac{2}{\pi} \left(\sum_{n=1}^{\infty} \frac{(-1)^n - 1}{4n^2} - \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \right)$$

$$\varphi(\pi) = \frac{3\pi}{8} + \frac{2}{\pi} \left(\sum_{n=1}^{\infty} \frac{(-1)^n - 1}{4n^2} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \right)$$

$$\varphi(0) + \varphi(\pi) = \frac{6\pi}{8} + \frac{2}{\pi} \cdot \frac{2}{4} \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n^2} = \frac{3\pi}{4} + \frac{1}{\pi} S_1$$

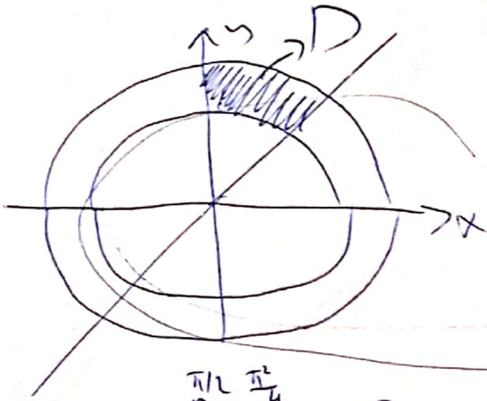
$$\frac{\pi}{2} = \frac{3\pi}{4} + \frac{1}{\pi} S_1 \Rightarrow \boxed{S_1 = -\frac{\pi^2}{4}}$$

6 + 6

$$\varphi(\pi) - \varphi(0) = \frac{4}{\pi} \cdot \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{4}{\pi} S_2$$

$$\frac{\pi}{2} = \frac{4}{\pi} S_2 \Rightarrow \boxed{S_2 = \frac{\pi^2}{8}}$$

3 a) $I=\iint\limits_D \frac{\sin\sqrt[4]{x^2+y^2}}{\sqrt{x^2+y^2}}\,dx\,dy$, $D=\left\{(x,y)\in\mathbb{R}^2 \;\middle|\; \frac{\pi^4}{256}\le x^2+y^2\le\frac{\pi^4}{16},\ x\ge 0,\ y\ge 0,\ y\ge x\right\}$



СМЕНА
$x=\rho\cos\varphi$
$y=\rho\sin\varphi$
$J=\rho$

$\varphi\in\left[\frac{\pi}{4},\frac{\pi}{2}\right]$
$\rho\in\left[\frac{\pi^2}{16},\frac{\pi^2}{4}\right]$

8

$$I=\int_{\pi/4}^{\pi/2}\int_{\pi^2/16}^{\pi^2/4}\sin\sqrt{\rho}\,d\rho\,d\varphi=\frac{\pi}{4}\int_{\pi^2/16}^{\pi^2/4}\sin\sqrt{\rho}\,d\rho=\left(\begin{matrix}\sqrt{\rho}=t\\ \rho=t^2\\ d\rho=2t\,dt\end{matrix}\right)$$

7

$$=\frac{\pi}{2}\int_{\pi/4}^{\pi/2}\cancel{t}\,t\sin t\,dt=\frac{\pi}{2}\left(-t\cos t\Big|_{\pi/4}^{\pi/2}+\sin t\Big|_{\pi/4}^{\pi/2}\right)$$

$$=\boxed{\frac{\pi}{2}\left(\frac{\pi}{4}\frac{\sqrt{2}}{2}+1-\frac{\sqrt{2}}{2}\right)}$$

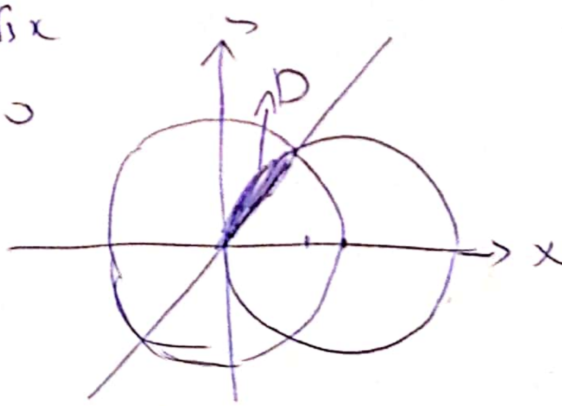
б) $S_1: z=2-\sqrt{x^2+y^2}$

$S_2: \cancel{z}(x-2)^2+y^2=4$

$S_3: y=\sqrt{3}x$

$S_4: z=0$

$$V(T)=\iint\limits_D\left(2-\sqrt{x^2+y^2}\right)dx\,dy$$



$x=\rho\cos\varphi$
$y=\rho\sin\varphi$
$J=\rho$
$\varphi\in\left[\frac{\pi}{3},\frac{\pi}{2}\right)$
$0\le\rho\le 4\cos\varphi$

8

$$V=\int_{\pi/3}^{\pi/2}\int_0^{4\cos\varphi}(2-\rho)\rho\,d\rho\,d\varphi=\int_{\pi/3}^{\pi/2}\left(\rho^2-\frac{\rho^3}{3}\Big|_0^{4\cos\varphi}\right)d\varphi=\ldots=\boxed{-\frac{128}{9}+6\sqrt{3}+\frac{4}{3}\pi}$$

7

4 $y''' - 2y'' + 2y' = 6\sin^2 x$

$\lambda^3 - 2\lambda^2 + 2\lambda = 0$

$\lambda(\lambda^2 - 2\lambda + 2) = 0$

$\lambda = 0 \vee (\lambda-1)^2 + 1 = 0$

$\lambda_1 = 0 \vee \lambda_{2,3} = 1 \pm i$

$$\boxed{y_h = C_1 + C_2 e^x \cos x + C_3 e^x \sin x, \quad C_1, C_2, C_3 \in \mathbb{R}}$$

10

$f(x) = 6\,\dfrac{1-\cos 2x}{2} = 3 - 3\cos 2x$

$f_1(x) = 3$

$f_2(x) = -3\cos 2x$

O.R.

$y = C_1 + C_2 e^x \cos x + C_3 e^x \sin x + \frac{3}{2}x - \frac{3}{10}\cos 2x + \frac{3}{20}\sin 2x$

y_{p1}

$\alpha = 0, \beta = 0$

$z = 0$ je rješenje kar. jedn. krat. 1

$\Rightarrow y_{p_1} = Ax$.

$y_{p1}' = A$

$y_{p1}'' = y_{p1}''' = 0$

$\} \; 2A = 3 \Rightarrow A = \frac{3}{2}$

5

$$\Rightarrow \boxed{y_{p1} = \frac{3}{2}x}$$

$y_{p2}' = -2A\sin 2x + 2B\cos 2x$

$y_{p2}'' = -4A\cos 2x - 4B\sin 2x$

$y_{p2}''' = 8A\sin 2x - 8B\cos 2x$

y_{p2}

$\alpha = 0, \beta = 2$

$z = 0 \pm 2i$ nije rj. kar. jed.

$y_{p2} = A\cos 2x + B\sin 2x$ 10

$8A\sin 2x - 8B\cos 2x - 2(-4A\cos 2x - 4B\sin 2x)$
$+ 2(-2A\sin 2x + 2B\cos 2x) = -3\cos 2x$

$\sin 2x(4A + 8B) + \cos 2x(8A - 4B) = -3\cos 2x$

$4A + 8B = 0$
$8A - 4B = -3$ $\Big\} \Rightarrow$ $20A = -6$, $\boxed{A = -\frac{3}{10}}$ $\boxed{B = \frac{3}{20}}$

$$\Rightarrow \boxed{y_{p2} = -\frac{3}{10}\cos 2x + \frac{3}{20}\sin 2x}$$