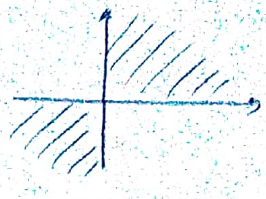
M2 АВГУСТ 2020

1. (10) $z = \sqrt{xy} + \ln\frac{x}{y}$ D: $xy \geq 0,\ \frac{x}{y} > 0,\ y \neq 0$



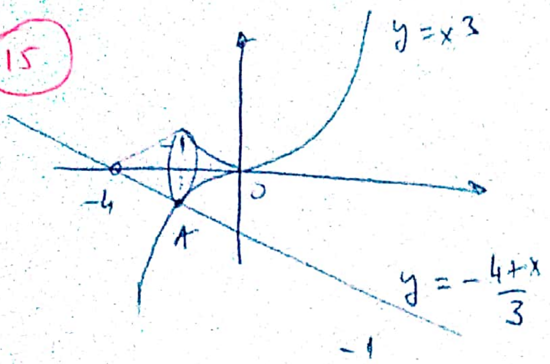
$z'_x = \frac{\sqrt{y}}{2\sqrt{x}} + \frac{1}{x}$ (1)

$z'_y = \frac{\sqrt{x}}{2\sqrt{y}} - \frac{1}{y}$ (1) (2)

$z''_{xx} = -\frac{\sqrt{y}}{4x\sqrt{x}} - \frac{1}{x^2}$ (2), $z''_{yy} = -\frac{\sqrt{x}}{4y\sqrt{y}} + \frac{1}{y^2}$ (2)

$y^2 \cdot z''_{yy} - x^2 \cdot z''_{xx} + y \cdot z'_y - x \cdot z'_x = 0$ ф-ја задовољава ... (2)

2. (15)



A: $3x^3 = -x - 4$

$3x^3 + x + 4 = 0$

$(x+1)(3x^2 - 3x + 3) = 0$ (8) за [illegible] и домен

$x = -1,\ y = -1$ $A(-1,-1)$

$$V = V_1 + V_2 = \pi \int_{-4}^{-1} \left(-\frac{4+x}{3}\right)^2 dx + \pi \int_{-1}^{0} (x^3)^2 dx = \pi \cdot 1 + \frac{\pi}{7} = \frac{8\pi}{7}$$

(3) (3) (1)

3. 1. (10)

$$\int \frac{3x^2+2x+3}{x^3+2x^2+3x+2}\,dx \overset{(2)}{=} \int \frac{3x^2+2x+3}{(x+1)(x^2+x+2)}\,dx$$

$$\overset{(3)}{=} \int \left(\frac{2}{x+1} + \frac{x-1}{x^2+2+x} \right) dx = 2\ln|x+1| \overset{(1)}{+} \frac{1}{2}\int \frac{2x+1-3}{x^2+x+2}\,dx$$

$$= 2\ln|x+1| + \frac{1}{2}\ln(x^2+x+2) - \frac{3}{2}\cdot\frac{2}{\sqrt{7}}\,\mathrm{arctg}\,\frac{2x+1}{\sqrt{7}} + C \quad (4)$$

3. 2.

$$\int \frac{\sqrt{x}}{(2+\sqrt[3]{x})^2}\,dx = \begin{bmatrix} x = t^6 \\ dx = 6t^5\,dt \end{bmatrix} = 6\int \frac{t^8}{(2+t^2)^2}\,dt \quad (5)$$

$$= 6\int \left(t^4 - 4t^2 + 12 - 16\,\frac{2t^2+3}{(2+t^2)^2} \right) dt$$

$$I = \int \frac{2t^2+3}{(2+t^2)^2}dt = 2\cdot\int\frac{t^2+2}{(t^2+2)^2}dt - \int\frac{dt}{(t^2+2)^2} =$$

$$= \sqrt{2}\,\text{arctg}\,\frac{t}{\sqrt{2}} - \frac{t}{(t^2+2)\cdot 4} - \frac{1}{4\sqrt{2}}\,\text{arctg}\,\frac{t}{\sqrt{2}} + C$$

$$\int\frac{\sqrt{x}}{(2+\sqrt[3]{x})^2}dt = 6\cdot\left(\frac{\sqrt[6]{x}^5}{5} - \frac{4}{3}\cdot\sqrt{x} + 12\sqrt[6]{x} + 16I\right)\Big|_{34\ t=\sqrt[6]{x}}$$ 10

3.3. 10 $$\int\frac{\sin(4\ln|\text{tg}\,x|)}{\cos^2 x}dx = \begin{bmatrix} z = \text{tg}\,x \\ dz = \frac{1}{\cos^2 x}dx\end{bmatrix} \overset{2}{=} \int \sin(4\ln z)\,dz$$

$$= \begin{bmatrix} t = \ln z \\ dz = e^t dt\end{bmatrix} \overset{1}{=} \int \sin(4t)\cdot e^t dt = \begin{bmatrix} u = \sin(4t) \\ du = 4\cos(4t) \\ v = e^t\end{bmatrix} =$$

$$= e^t\cdot\sin(4t) - 4\cdot\int\cos(4t)\cdot e^t dt =$$

$$= e^t\cdot\sin 4t - 4\left(e^t\cdot\cos(4t) + 4\underbrace{\int\sin(4t)\cdot e^t dt}_{I}\right)$$

7

$$17I = e^t(\sin(4t) - 4\cos(4t)) + C$$

$$I = \frac{1}{17}e^{\ln(\text{tg}\,x)}\left(\sin(4\ln(\text{tg}\,x)) - 4\cos(4\ln(\text{tg}\,x))\right) + C$$

4.1. 10 $$y' = \frac{7x+2y+3}{6x-y+8}$$

$$x = X+\alpha, \quad y = Y+\beta \quad \cdots \quad \begin{matrix}\alpha = -1 \\ \beta = 2\end{matrix}$$ 2

$$Y' = \frac{7X+2Y}{6X-Y} \qquad z = \frac{Y}{X}$$

$$z'\cdot X + z = \frac{7+2z}{6-z} \Rightarrow z'\cdot X = \frac{7+2z}{6-z} - z = \frac{z^2-4z+7}{6-z}$$ 3

$$\int\frac{(6-z)dz}{z^2-4z+7} = -\frac{1}{2}\int\frac{2z-4-8}{z^2-4z+7}dz =$$

$$= \frac{1}{2}\ln(z^2-4z+7) + \frac{4}{\sqrt{3}}\,\text{arctg}\,\frac{z-2}{\sqrt{3}}$$

$$\frac{1}{2}\ln\left(\left(\frac{y-2}{x+1}\right)^2-4\cdot\frac{y-2}{x+1}+7\right)+\frac{4}{\sqrt{3}}\operatorname{arctg}\frac{\frac{y-2}{x+1}-2}{\sqrt{3}}=\ln|x+1|+C$$ 5

4.2. 18 $$xy^2+2(x^2+1)\cdot y\cdot y'=\sqrt{x^2+1}\cdot(\sin^5x\cdot\cos^2x+1)\cdot\frac{1}{\sin^3x}$$

БЕРНУЛЛИ 1.Д. СМЕНА $z=y^2$

$$z'+\underbrace{\frac{x}{x^2+1}}_{p(x)}\cdot z=\underbrace{\frac{\sin^5x\cdot\cos^2x+1}{\sin^3x}\cdot\frac{1}{\sqrt{x^2+1}}}_{q(x)}$$ 3

$$\int p(x)dx=\ln\sqrt{x^2+1}$$ 2

$$\int q(x)\,e^{\int p(x)dx}dx=\int\left(\sin^2x\cdot\cos^2x+\frac{1}{\sin^3x}\right)dx=I_1+I_2$$

$$I_1=\frac{1}{4}\int\sin^2(2x)dx=\frac{1}{4}\int\frac{1-\cos4x}{2}dx=\frac{1}{8}\left(x-\frac{1}{4}\sin4x\right)$$ 6

$$I_2=\int\frac{dx}{\sin^3x}=\left[t=\operatorname{tg}\frac{x}{2}\right]=\int\frac{(t^2+1)^2}{4t^3}dt=$$

$$=\frac{1}{8}\operatorname{tg}^2\frac{x}{2}+\frac{1}{2}\ln\left|\operatorname{tg}\frac{x}{2}\right|-\frac{1}{8\operatorname{tg}^2\frac{x}{2}}$$ 6

$$z=y^2=\frac{1}{\sqrt{x^2+1}}(C+I_1+I_2)$$ 1

5. 12 $$p:\ \frac{x-\frac{2}{3}}{2}=\frac{y-\frac{2}{3}}{-1}=\frac{z}{1}=t$$ 2 $$P\left(\frac{2}{3},\frac{2}{3},0\right)\quad \vec{PA}=\left(\frac{1}{3},\frac{4}{3},8\right)$$

$$d=\frac{|\vec{p}\times\vec{PA}|}{|\vec{p}|}=\frac{1}{3}\cdot\frac{\sqrt{28^2+47^2+(-9)^2}}{\sqrt{4+1+1}}=\frac{1}{3}\cdot\sqrt{\frac{1537}{3}}$$ 2

$$P_t\in p\Rightarrow P_t\left(\frac{2}{3}+2t,\ -t+\frac{2}{3},\ t\right)$$

$$0=\vec{p}\cdot\vec{AP_t}=6t-8+\frac{2}{3}\Rightarrow t=\frac{11}{9}$$ 8

$$S\left(\frac{28}{9},-\frac{5}{9},\frac{11}{9}\right)\quad x_S=\frac{x_A+x_B}{2}\ \ldots\Rightarrow B\left(\frac{37}{9},-\frac{28}{9},-\frac{50}{9}\right)$$