М2, 11. септембар 2020.

1 a) $\int \frac{dx}{7\cos x - 4\sin x + 8} \overset{t=tg\frac{x}{2}}{=\!=} \int \frac{\frac{2dt}{1+t^2}}{\frac{7-7t^2}{1+t^2} - 4\cdot\frac{2t}{1+t^2} + 8\cdot\frac{1+t^2}{1+t^2}} =$

$= \int \frac{2dt}{t^2 - 8t + 15} = \int \frac{2dt}{(t-3)(t-5)} = \frac{1}{2}\ln\left|\frac{tg\frac{x}{2} - 5}{tg\frac{x}{2} - 3}\right| + C$ 10

б) $\int \sqrt[3]{x^3 + 2x}\,dx = \int x\cdot(1 + 2x^{-2})^{1/3}\,dx$

$m=1, n=-2, p=1/3$

$\frac{m+1}{n} \in \mathbb{Z}$ $\qquad 1 + 2x^{-2} = t^3$

$x^{-2} = \frac{1}{2}(t^3 - 1)$

$x = \sqrt{2}\cdot(t^3-1)^{-1/2} \qquad dx = \sqrt{2}\cdot\left(-\frac{1}{2}\right)(t^3-1)^{-3/2}\cdot 3t^2\,dt$

$= \int \sqrt{2}\,(t^3-1)^{-1/2}\cdot t\cdot 3\sqrt{2}\cdot\left(-\frac{1}{2}\right)(t^3-1)^{-3/2}\cdot t^2\,dt$ 10

$= -3\int \frac{t^3}{(t^3-1)^2}\,dt = -3\int t\cdot\frac{t^2}{(t^3-1)^2}\,dt = \left[\begin{array}{l} u = t \\ v = \int \frac{-3t^2}{(t^3-1)^2}\,dt \\ \quad = \frac{1}{t^3-1} \end{array}\right]$

$= \frac{t}{(t^3-1)} - \int \left(\frac{A}{t-1} + \frac{Bt+C}{t^2+t+1}\right) dt = \ldots$

в) $\int_1^{+\infty} \frac{(\ln(x^2))^{5/2}}{x^3}\,dx = \left[\begin{array}{l} t = \ln x \\ x = e^t, dx = e^t dt \end{array}\right] = \int_0^{+\infty} \frac{(2t)^{5/2}}{e^{3t}}\cdot e^t\,dt =$

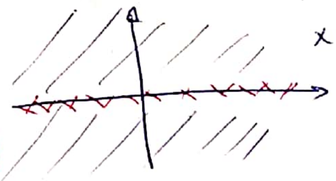
$= 2^{5/2}\int_0^{+\infty} t^{5/2}\cdot e^{-2t}\,dt \overset{z=2t}{=\!=} 2^{5/2}\cdot\frac{1}{2^{5/2}}\int_0^{+\infty} z^{5/2}\cdot e^{-z}\cdot\frac{1}{2}\,dz =$

$= \frac{1}{2}\cdot\Gamma\left(\frac{7}{2}\right) = \frac{1}{2}\cdot\frac{5\cdot 3\cdot 1}{2^3}\cdot\sqrt{\pi} = \frac{15\sqrt{\pi}}{16}$ 10

2. $V = \pi \int_0^{\sqrt{2}} \frac{9}{(2+x^2)^2}\,dx = \left\{ \begin{array}{l} \text{I начин парцијална} \\ \text{II начин смена} \\ x = tg\,t \cdot \sqrt{2} \end{array} \right\} = \frac{\pi(2+\pi)}{16\sqrt{2}}$ 10

3. 10

$z = arctg\left(\frac{x}{y}\right) + 4y + 2x \qquad D:\ y \neq 0,\ x \in \mathbb{R}$ 24



$\mathbb{R}^2$ без

x осе

$z'_x = \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot \frac{1}{y} + 2 = \frac{y}{x^2+y^2} + 2$ 1

$z'_y = \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot \frac{-x}{y^2} + 4 = \frac{-x}{y^2+x^2} + 4$ 1

$z''_{xx} = \frac{-2xy}{(x^2+y^2)^2}$ 2 $\qquad z''_{xy} = \frac{1\cdot(x^2+y^2) - y\cdot 2y}{(x^2+y^2)^2} = \frac{x^2-y^2}{(x^2+y^2)^2}$ 2

$z''_{yy} = \frac{2xy}{(x^2+y^2)^2}$ 2

$$\frac{1}{x^2+y^2} \cdot \left(y \cdot \frac{y}{x^2+y^2} + x \cdot \frac{-x}{x^2+y^2} \right) + \frac{x^2-y^2}{(x^2+y^2)^2} = 0$$

$0 = 0$

4. 17

a) $P\,dx + Q\,dy = 0$

$P = y^2 + e^x / \sqrt{3-2e^x-2e^{2x}} + \cos^2 x \qquad Q = 2xy$

$P'_y = 2y = Q'_x$ 3

$$\int P\,dx = y^2x + \int \frac{e^x}{\sqrt{3-2e^x-2e^{2x}}}\,dx + \int \cos^2 x\,dx =$$

$e^x = t$

$$= y^2x + \int \frac{dt}{\sqrt{2\left(\frac{7}{4} - (t+1/2)^2\right)}} + \int \frac{1+\cos 2x}{2}\,dx =$$

8 4

$$= y^2x + \frac{1}{\sqrt{2}} \arcsin \frac{e^x + 1/2}{\frac{\sqrt{7}}{2}} + \frac{1}{2}x + \frac{1}{4}\sin 2x + C$$

$$\frac{\partial}{\partial y} \int P dx = 2xy \quad (1)$$

ОПШТЕ РЕШ: $y^2x + \frac{1}{\sqrt{2}} \arcsin \frac{2e^x + 1}{\sqrt{7}} + \frac{1}{2}x + \frac{1}{4}\sin 2x = C$ (1)

5) (18) $$y' = \frac{1}{\ln^2\left(\frac{y}{x}\right) + \arccos\left(\frac{y}{x}\right)}$$

$z = \frac{y}{x}$

$y' = z' \cdot x + z$

$$z'x + \cancel{z} = \frac{1}{\ln^2 z + \arccos z} + \cancel{z} \quad (3)$$

$$\int (\ln^2 z + \arccos z)\, dz = \ln|x| + C$$

$$I_1 = \int \ln^2 z\, dz = \left[\begin{matrix} u = \ln^2 z, & du = 2\ln z \cdot \frac{1}{z}\, dz \\ v = z \end{matrix}\right]$$

$$= z \ln^2 z - \int 2\ln z\, dz = z\ln^2 z - 2(z\ln z - z) + C_1 \quad (7)$$

$$I_2 = \int \arccos z\, dz = \left[\begin{matrix} u = \arccos z, & du = \frac{-1}{\sqrt{1-z^2}}\, dz \\ v = z \end{matrix}\right]$$

$$= z\arccos z + \int \frac{z}{\sqrt{1-z^2}}\, dz = z\arccos z - 2\sqrt{1-z^2} + C_2 \quad (7)$$

ОПШТЕ РЕШ.

$$\frac{y}{x}\ln^2\frac{y}{x} - 2\frac{y}{x}\ln\frac{y}{x} + 2\frac{y}{x} + \frac{y}{x}\arccos\frac{y}{x} - 2\sqrt{1-\left(\frac{y}{x}\right)^2} =$$

$$= \ln|x| + C \quad (1)$$

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5. 15

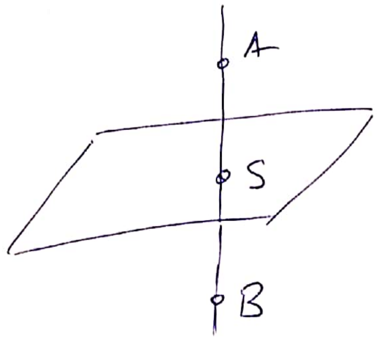
a) $A(1,-2,1)$

$\alpha: x-3y+2z+5=0$

5

$A \in p,\ p \perp \alpha \Rightarrow \vec{v}_p = (1,-3,2)$ $\quad p: \dfrac{x-1}{1} = \dfrac{y+2}{-3} = \dfrac{z-1}{2}$

б)



$A_t \in p \quad A_t(1+t,\ -2-3t,\ 1+2t)$

10

$S = p \cap \alpha$

$1+t-3(-2-3t)+2(1+2t)+5=0$

$14t+14=0 \Rightarrow t=-1$

$S(0,1,-1)$

$x_S = \dfrac{x_A + x_B}{2}$ ... $\Rightarrow B(-1,4,-3)$