1. $$\begin{vmatrix} z-4i & 1+z & 2i \\ -2-2i & z-i & 1+i \\ -2z-2i & 0 & z+i \end{vmatrix} = \begin{vmatrix} z & 1+z & 2i \\ 0 & z-i & 1+i \\ 0 & 0 & z+i \end{vmatrix}$$

$/ +2$

$$= z\,(z^2+1) = z^3 + z \quad 4$$

$$z^3 + z = z + 1 - i\sqrt{3}$$

$$z^3 = 1 - i\sqrt{3} = 2\cdot\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right) \quad 3$$

$$z_k = \sqrt[3]{2}\cdot\left(\cos\frac{-\frac{\pi}{3}+2k\pi}{3} + i\sin\frac{-\frac{\pi}{3}+2k\pi}{3}\right) \quad 3 \quad k=0,1,2$$

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2. $2x+y+z=1$

$(m+6)x+2y+3z=1$

$x-y+2z=m-1$

$2x+(m-1)y+4z=m$

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ m+6 & 2 & 3 & 1 \\ 1 & -1 & 2 & m-1 \\ 2 & m-1 & 4 & m \end{array}\right] \sim \begin{array}{c} \begin{array}{cccc} z & \quad & x & \quad \end{array} \\ \left[\begin{array}{ccc|c} \boxed{1} & 1 & 2 & 1 \\ 3 & 2 & m+6 & 1 \\ 2 & -1 & 1 & m-1 \\ 4 & m-1 & 2 & m \end{array}\right] \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & -1 & m & -2 \\ 0 & -3 & -3 & m-3 \\ 0 & m-5 & -6 & m-4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & -1 & m & -2 \\ 0 & 0 & -3(1+m) & m+3 \\ 0 & 0 & (m+1)(m-6) & 6-m \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & -1 & m & -2 \\ 0 & 0 & -3(1+m) & m+3 \\ 0 & 0 & 0 & (m-6)\frac{m}{3} \end{array}\right]$$ 12

I $m \neq 0$ $m \neq 6$

$r(A) < r(A_p)$ нема р. 2

II $m=0$ $r(A)=r(A_p)=3$ 2

$(x,y,z)=(-1,2,1)$

III $m=6$ $(x,y,z)=\left(-\frac{3}{7}, -\frac{4}{7}, \frac{17}{7}\right)$ 2

12

3. Дате су матрице: $A=\begin{bmatrix} 1 & a & 2 \\ -1 & 0 & 1 \end{bmatrix}$, $a \in R$, $B=\begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$

и $C=\begin{bmatrix} 1 & 12 \\ -3 & 9 \\ 1 & 0 \end{bmatrix}$

а) Испитати за које вр. $a \in R$ матрица AC има инверзну

б) За $a=0$ решити матричну једн. $ACX=B$

а) $AC=\begin{bmatrix} 3-3a & 12+9a \\ 0 & -12 \end{bmatrix}$ $\det(AC)=-36+36a \neq 0$

за $a \neq 1$

б) $X=(AC)^{-1}\cdot B = -\frac{1}{36}\begin{bmatrix} -12 & -12 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} = -\frac{1}{36}\begin{bmatrix} -24 & -36 \\ 9 & 3 \end{bmatrix}$

20 8

4. a) $\lim\limits_{x\to 0}\left(e^{2x}+x\right)^{\frac{1}{x}}=A$

$$\ln A=\lim_{x\to 0}\frac{\ln(e^{2x}+x)}{x}=\lim_{x\to 0}\frac{\frac{1}{e^{2x}+1}\cdot(2e^{2x}+1)}{1}=3$$

$A=e^{3}$

7 б) $\lim\limits_{x\to 0}\dfrac{\arcsin x-x}{x^{3}}\overset{\frac{0}{0}}{\underset{L}{=}}\dots=\dfrac{1}{6}$

5 в) $$\lim_{n\to\infty}\left(\sqrt[3]{n^{3}+2n^{2}}-n\right)\cdot\frac{\sqrt[3]{(n^{3}+2n^{2})^{2}}+n\sqrt[3]{n^{3}+2n^{2}}+n^{2}}{\text{''}}=$$

$$=\lim_{n\to\infty}\frac{n^{3}+2n^{2}-n^{3}}{n^{2}\cdot\left(\sqrt[3]{(1+\frac{2}{n})^{2}}+\sqrt[3]{1+\frac{2}{n}}+1\right)}=\frac{2}{3}$$

6. 20 $f(x)=\ln\sqrt{\dfrac{2x}{x-1}}$

1) $x\in(-\infty,0)\cup(1,+\infty)$ 1

2) $(-1,0)$ 1 $\quad f(x)>0 \quad x\in(-\infty,-1)\cup(1,+\infty)$

$f(x)<0 \quad x\in(-1,0)$ 1

3) домен нит симетр ... 1

4) $\lim\limits_{x\to 0^-}f(x)=-\infty$ 1 $\quad \lim\limits_{x\to 1^+}f(x)=+\infty$ 1 $\quad x=0,\ x=1$ В.А. 1

$\lim\limits_{x\to\pm\infty}f(x)=\ln\sqrt{2}$ 2 Х.А., нема К.А. 1

5) $f'(x)=\dfrac{-1}{2x(x-1)}$ 3

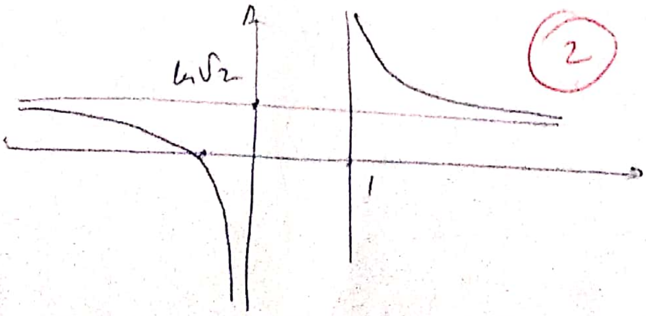
	$-\infty$	0	1	
$f'(x)$	$-$			$-$
f	$\searrow$			$\searrow$

1

6) $f''(x)=\dfrac{1}{2}\cdot\dfrac{2x-1}{x^{2}(x-1)^{2}}$ 3

	$-\infty$	0	1	
f''	$-$			$+$
f	$\cap$			$\cup$

1



2

5. $f(x)=(2x+1)\cdot e^{-\frac{1}{x}}$

20

а) домен, нули, знак и асимптоти

б) T_2 околу $x_0=1$

а) $x\neq 0$ 1

$2x+1$: $- - - \; -\frac{1}{2} \; + + +$, 0 1

$-\frac{1}{2}$ 1

$\lim\limits_{x\to\pm\infty} f(x) = \pm\infty$ 2 нема х.а.

$\left.\begin{array}{l}\lim\limits_{x\to 0^+} f(x) = 1\cdot 0 = 0 \quad 1\\ \lim\limits_{x\to 0^-} f(x) = +\infty \quad 1\end{array}\right\}$ $x=0$ 1 в.а. од лева стр.

$k=2$ 1

$n=\lim\limits_{x\to+\infty}\left(2x\cdot e^{-\frac{1}{x}}+e^{-\frac{1}{x}}-2x\right)=\lim\limits_{x\to+\infty}\left((-2)\cdot\frac{e^{-\frac{1}{x}}-1}{-\frac{1}{x}}+e^{-\frac{1}{x}}\right)$ 3

$=-2+1=-1$

$y=2x-1$ к.а. 1

б) $f'(x)=\frac{2x^2+2x+1}{x^2}e^{-\frac{1}{x}}$

$f''(x)=\frac{1}{x^4}\cdot e^{-\frac{1}{x}}$

$f'(1)=\frac{5}{e}$ $\quad f''(1)=\frac{1}{e}$ $\quad f(1)=\frac{3}{e}$

$T_2=\frac{3}{e}+\frac{5}{e}(x-1)+\frac{1}{2e}(x-1)^2$