1

(A) $\int \frac{e^x\,dx}{\sqrt[4]{e^{4x}+e^{2x}}}$ $\left[\begin{array}{l} e^x=t \\ e^x dx=dt \end{array}\right] = \int \frac{dt}{\sqrt[4]{t^4+t^2}} = \int (t^4+t^2)^{-1/4}dt$

$= \int t^{-1}(1+t^{-2})^{-1/4}dt$ $\left[\begin{array}{lll} m=-1,\ n=-2 & 1+t^{-2}=z^4 & t=\frac{1}{\sqrt{z^4-1}} \\ p=-1/4, \ \frac{m+1}{n}=0\in\mathbb{Z} & t^{-2}=z^4-1 & dt=\frac{4z^3dz}{(z^4-1)^{3/2}} \end{array}\right]$

$= \int (z^4-1)^{1/2}\cdot(z^4)^{-1/4}\cdot\frac{4z^3dz}{(z^4-1)^{3/2}} = 4\int\frac{z^2dz}{z^4-1} = \ldots$ 10

(Б) $\int \frac{\cos x\,dx}{\sin x+\sqrt{\sin^2 x+2\sin x+2}}$ $\left[\begin{array}{l} \sin x=t \\ \cos x\,dx=dt \end{array}\right] = \int\frac{dt}{t+\sqrt{t^2+2t+2}}$

$\left[\begin{array}{ll} \sqrt{t^2+2t+2}=z-t & t=\frac{z^2-2}{2(1+z)} \\ \cancel{t^2}+2t+2=z^2-2zt+\cancel{t^2} & \\ 2t(1+z)=z^2-2 & dt=\frac{z^2+z+2}{(1+z)^2} \end{array}\right] = \int\frac{\frac{z^2+z+2}{(1+2z)^2}dz}{z} = \int\frac{z^2+z+2}{z(1+2z)^2}dz$

10 $= \ldots$

(B) $\int_0^{+\infty}\frac{\sqrt[4]{x}}{(1+x)^2}dx = \int_0^{+\infty}\frac{x^{1/4}}{(1+x)^2}$ $\left[\begin{array}{ll} a-1=1/4, & a=5/4 \\ a+b=2, & b=3/4 \end{array}\ \int_0^{+\infty}\frac{x^{a-1}}{(1+x)^{a+b}}=B(a,b)\right]$

$= B\left(5/4, 3/4\right) = \ldots$ 10

[13]

2. За $-\frac{\sqrt{3}}{2} \le x \le -1/2$, $f(x) = \frac{\arcsin x}{x^2} < 0$, зато ③

$$P = -\int_{-\sqrt{3}/2}^{-1/2} \frac{\arcsin x}{x^2}\,dx \quad \left[\begin{matrix} \arcsin x = u & v = \int \frac{dx}{x^2} \\ \frac{1}{\sqrt{1-x^2}}dx = du \ \text{④} & v = -\frac{1}{x} \end{matrix}\right]$$

$$= -\left[-\frac{\arcsin x}{x}\Big|_{-\sqrt{3}/2}^{-1/2} + \int_{-\sqrt{3}/2}^{-1/2} \frac{dx}{x\sqrt{1-x^2}} \ \text{⑥} \right] = \ldots$$

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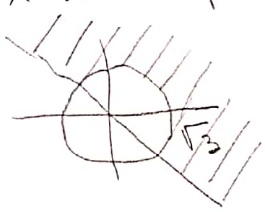
3. $z(x,y) = \ln (x+y)(x^2+y^2-3)$

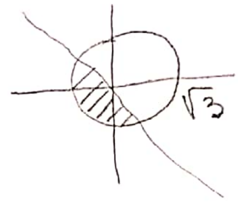
Δ ОИЕН: $(x+y)(x^2+y^2-3) > 0$

$(x+y>0$ и $x^2+y^2-3>0)$ или $(x+y<0$ и $x^2+y^2-3<0)$

$y>-x$ и $x^2+y^2>3$ или $y<-x$ и $x^2+y^2<3$

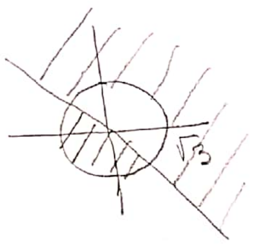


или



⑦

Δ ОИЕН:



$$\frac{\partial z}{\partial x} = \frac{1}{(x+y)(x^2+y^2-3)} \cdot \left(x^2+y^2-3+(x+y)\cdot 2x\right) = \frac{3x^2+y^2+2xy-3}{(x+y)(x^2+y^2-3)} \ \text{①}$$

$$\frac{\partial z}{\partial y} = \frac{1}{(x+y)(x^2+y^2-3)} \cdot \left(x^2+y^2-3+(x+y)\cdot 2y\right) = \frac{x^2+3y^2+2xy-3}{(x+y)(x^2+y^2-3)} \ \text{①}$$

$$\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 2\,\frac{x^2-y^2}{(x+y)(x^2+y^2-3)} = \frac{2(x-y)}{x^2+y^2-3}\,. \ \text{①}$$

4. (A) $y\sqrt{y^2+1}\,dx + \dfrac{y+y^2x+x}{\sqrt{y^2+1}}\,dy = 0$

$$P(x,y) = y\sqrt{y^2+1}, \quad \frac{\partial P}{\partial y} = \sqrt{y^2+1} + \frac{y^2}{\sqrt{y^2+1}}$$

$$Q(x,y) = \frac{y+y^2x+x}{\sqrt{y^2+1}}, \quad \frac{\partial Q}{\partial x} = \sqrt{y^2+1}$$

$$\frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}, \quad \frac{\partial z}{\partial x} = y, \quad \frac{\partial z}{\partial y} = x$$

$$\frac{dz}{z} = \frac{\dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y}}{\dfrac{\partial z}{\partial y}P - \dfrac{\partial z}{\partial x}Q} = \frac{\cancel{\sqrt{y^2+1}} - \cancel{\sqrt{y^2+1}} - \frac{y^2}{\sqrt{y^2+1}}}{xy\sqrt{y^2+1} - \dfrac{y^2+y^3x+yx}{\sqrt{y^2+1}}}\,dv = dv$$

$$\int \frac{dz}{z} = \int dv, \quad \ln|z| = v, \quad \ln z = xy, \quad z = e^{xy}.$$ 9

$$e^{xy}y\sqrt{y^2+1}\,dx + \frac{y+y^2x+x}{\sqrt{y^2+1}}e^{xy}\,dy = 0$$

$$P_1(x,y) = e^{xy}y\sqrt{y^2+1}, \quad Q_1(x,y) = \frac{y+y^2x+x}{\sqrt{y^2+1}}e^{xy}$$ 8

$$\int P_1(x,y)\,dx = \int e^{xy}y\sqrt{y^2+1}\,dx = e^{xy}\sqrt{y^2+1}$$

$$\int\left(Q_1(x,y) - \frac{\partial}{\partial y}\int P_1(x,y)\,dx\right)dy = \int\left(\frac{(y+y^2x+x)e^{xy}}{\sqrt{y^2+1}} - xe^{xy}\sqrt{y^2+1} + \frac{y}{\sqrt{y^2+1}}e^{xy}\right)dy$$

$= 0$, ΔΑΚΛΕ $u(x,y) = e^{xy}\sqrt{y^2+1}$.

Ο.Ρ. $e^{xy}\sqrt{y^2+1} = c$.

4 б) $y' - y \cdot tg x - \dfrac{1}{\cos(x) \cdot (\sin(x) + \cos(x) + 2)^2} = 0$

$y = e^{-\int p(x)dx} \left(C + \int q(x) e^{\int p(x) dx} dx \right)$

$\int p(x)dx = -\int tg x \, dx = -\int \dfrac{\sin x}{\cos x} dx \overset{t = \cos x}{=\!=\!=} \int \dfrac{dt}{t} = \ln|\cos x|$ 6

$\int q(x) \cdot e^{\int p(x)} dx = \int \dfrac{1}{(\sin x + \cos x + 2)^2} dx = \left[t = tg \frac{x}{2} \right] =$ 12

$= \int \dfrac{\frac{2dt}{1+t^2}}{\left(\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} + 2 \right)^2} = \int \dfrac{2 \cdot (1+t^2) dt}{(3 + 2t + t^2)^2} = \ldots$

5. $p: \begin{cases} x = 2t + 3 \\ y = 3t + 5 \\ z = -2t - 2 \end{cases}, \ t \in \mathbb{R}$

A)

$S = p \cap \alpha$: $6(2t+3) + 3(3t+5) + 2 \cdot (-2t-2) + 12 = 0$

$12t + 9t - 4t + 18 + 15 - 4 + 12 = 0$

$17t = -17 \Rightarrow t = -1 \Rightarrow S(1, 2, 0)$ 3

$\cos \varphi = \dfrac{\vec{p} \cdot \vec{n_\alpha}}{|\vec{p}| \cdot |\vec{n_\alpha}|} = \dfrac{(2,3,-2) \cdot (6,3,2)}{\sqrt{4+9+4} \cdot \sqrt{36+9+4}} = \dfrac{17}{7 \cdot \sqrt{17}} = \dfrac{\sqrt{17}}{7}$ 2

б) $\vec{q} = \vec{n_\alpha} \times \vec{p} = \begin{vmatrix} i & j & k \\ 6 & 3 & 2 \\ 2 & 3 & -2 \end{vmatrix} = (-12, 16, 12) = 4 \cdot (-3, 4, 3)$

$\vec{n_\beta} = \vec{q} \times \vec{p} = \begin{vmatrix} i & j & k \\ -3 & 4 & 3 \\ 2 & 3 & -2 \end{vmatrix} = (-17, 0, -17) = -17(1, 0, 1)$

$\beta: 1 \cdot (x-3) + 0 \cdot (y-5) + 1 \cdot (z+2) = 0$ 7

$x + z - 1 = 0$

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