Математика 3, 5.2.2021.

1. 1) $a_n = \sin\left(\frac{1}{n}\right)\operatorname{tg}\left(\frac{1}{n}\right) \sim \frac{1}{n}\cdot\frac{1}{n} = \frac{1}{n^2}$ п.к. $\sum a_n$ конв. 3

2) $a_n = \dfrac{(n!)^3\cdot n^n}{((2n)!)^2}$

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^3\cdot(n+1)^n\cdot(n+1)}{n^n\cdot(2n+2)^2\cdot(2n+1)^2} = \left(\frac{n+1}{n}\right)^n\cdot\frac{1}{4}\cdot\frac{(n+1)^2}{(2n+1)^2}$$ 6

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n\cdot\frac{n^2+2n+1}{4n^2+4n+1}\cdot\frac{1}{4} = \frac{e}{16} < 1 \overset{\text{Д.К.}}{\Longrightarrow} \sum a_n \text{ конв}$$

3) $a_n = \dfrac{(2n-1)!!}{2^{n+1}\cdot n\cdot(n+1)!}$

$a_{n+1} = \dfrac{(2n+1)\cdot(2n-1)!!}{2\cdot 2^{n+1}\cdot(n+1)\cdot(n+2)\cdot(n+1)!}$

$$\frac{a_{n+1}}{a_n} = \frac{(2n+1)\cdot n}{(2n+2)(n+2)} \to 1,\ n\to\infty$$ 2 + 4 па Д.К. не помаже и

примењујемо Раабов крит. 6

$$\lim_{n\to\infty} n\cdot\left(1-\frac{a_{n+1}}{a_n}\right) = \lim_{n\to\infty} n\cdot\left(\frac{5n+4}{(2n+2)(n+2)}\right) = \frac{5}{2} > 1 \Rightarrow \sum a_n \text{ конв.}$$

2. $$\frac{|a_{n+1}|}{|a_n|} = \left[\frac{\frac{(2n+1)!!}{8\cdot 8^n(n+1)\cdot n!}}{\frac{(2n-1)!!}{8^n\cdot n!}}\right]^p\cdot\left(\frac{2n+5}{2n+3}\right)^q\cdot\frac{2n+1}{2n+3} \to \frac{1}{4^p},\quad n\to\infty$$ 6

$p<0$ $\sum a_n$ див. 1

$p>0$ $\sum a_n$ апс. конв. 1

$\boxed{p=0}$: $|a_n| \sim \dfrac{(2n+3)^q}{2n+1} \sim \dfrac{1}{2^{1-q}}\cdot\dfrac{1}{n^{1-q}}$ 2

- за $1-q>1$, тј. $q<0$ $\sum a_n$ А.К. 1
- за $1-q\le 0$, тј. $q\ge 1$ $\sum a_n$ див 1
- за $0\le q<1$ испитујемо условну конв: 8

1. $|a_n| \to 0,\ n\to\infty$ Лајб. $\Rightarrow \sum$ у.к.
2. $f(x) = \dfrac{(2x+3)^q}{2x+1}$ $f'(x) = \dots < 0$ $f \searrow$

3. $f(x) = x^2 \ln\left(\frac{2+x}{2-x}\right) = x^2 \cdot g(x)$ (15)

$$g'(x) = \frac{1}{2}\left(\frac{1}{1+\frac{x}{2}} + \frac{1}{1-\frac{x}{2}}\right) = \frac{1}{2}\sum_{n=0}^{\infty}\binom{-1}{n}\cdot\left(\left(-\frac{x}{2}\right)^n + \left(\frac{x}{2}\right)^n\right)$$

$$= \frac{1}{2}\sum_{n=0}^{\infty}(-1)^n\cdot\left(\frac{(-1)^n+1}{2^n}\right)\cdot x^n = \frac{1}{2}\sum_{n=0}^{\infty}\left(1+(-1)^n\right)\cdot\left(\frac{x}{2}\right)^n$$

$$= \frac{1}{2}\sum_{k=0}^{\infty}\left(\frac{x}{2}\right)^{2k} \quad (5) \qquad x \in (-2, 2) \ (2)$$

$$g(x) = \frac{1}{2}\sum_{k=0}^{\infty}\frac{x^{2k+1}}{(2k+1)\cdot 2^{2k}} + C \quad (1)$$

$$g(0) = 0 = C \quad (1)$$

$$f(x) = x^2\cdot\frac{1}{2}\sum_{k=0}^{\infty}\frac{x^{2k+1}}{(2k+1)\cdot 2^{2k}} = x^3\,\frac{1}{2}\cdot\sum_{k=0}^{\infty}\frac{1}{2k+1}\cdot\left(\frac{x}{2}\right)^{2k} \quad (1)$$

za S: $\frac{x^2}{4} = \frac{1}{3} \Rightarrow x = \pm\frac{2\sqrt{3}}{3}$

$$f\left(\frac{2\sqrt{3}}{3}\right) = \frac{8\cdot 3\cdot\sqrt{3}}{3^3}\cdot\frac{1}{2}\cdot S \Rightarrow S = \frac{9}{4\sqrt{3}}\, f\left(\frac{2\sqrt{3}}{3}\right) \quad (5)$$

4. $\iint_D \frac{dx\,dy}{\sqrt{4-(x+1)^2-y^2}} = I \qquad D:\ x^2+y^2 \le 1$ (15)

pomerene polarne koord. $\quad x+1 = \rho\cos\theta$, $\ y = \rho\sin\theta$, $\ J = \rho$

$D: (\rho\cos\theta - 1)^2 + (\rho\sin\theta)^2 \le 1$

$\rho^2 - 2\rho\cos\theta \le 0 \Rightarrow 0 \le \rho \le 2\cos\theta$ (7)

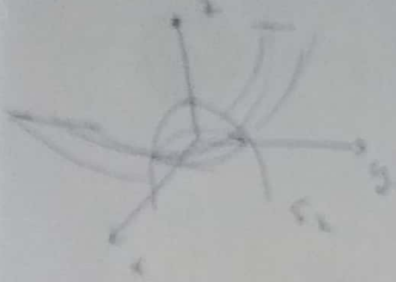
$\cos\theta \ge 0, \quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$$I = \iint_D \frac{\rho\,d\rho\,d\theta}{\sqrt{4-\rho^2}} = \int_{-\pi/2}^{\pi/2} d\theta \int_0^{2\cos\theta}\frac{\rho\,d\rho}{\sqrt{4-\rho^2}} = \int_{-\pi/2}^{\pi/2}\underbrace{\left(\sqrt{4}-\sqrt{4-4\cos^2\theta}\right)}_{2-2|\sin\theta|}d\theta \quad (8)$$

$t = \sqrt{4-\rho^2}$, $dt = -\frac{\rho\,d\rho}{\sqrt{4-\rho^2}}$

5. 15

$V = \iint_D (1 - x^2 - 4y^2 - 3x^2)\,dx\,dy$ 3



$D: \; 3x^2 = 1 - x^2 - 4y^2$

$x^2 + y^2 \le \left(\frac{1}{2}\right)^2$ 2

$x = \rho\cos\theta, \; y = \rho\sin\theta, \; J = \rho$

$0 \le \rho \le \frac{1}{2}$

$-\pi \le \theta \le \pi$

10

$V = \int_{-\pi}^{\pi}\int_0^{1/2} (1 - 4\rho^2)\rho\,d\rho\,d\theta = 2\pi \cdot \int_0^{1/2} (\rho - 4\rho^3)\,d\rho =$

$= 2\pi \cdot \left(\frac{\rho^2}{2} - \rho^4\right)\Big|_0^{1/2} = 2\pi \cdot \left(\frac{1}{8} - \frac{1}{16}\right) = \frac{\pi}{8}$

6. c1 6 $y''x\ln x + y'(1 + \ln x) = 0$ не содржи y

смена $z = y'$

$z'\,x\ln x + z(1 + \ln x) = 0$

$\frac{dz}{z} = -\frac{(1 + \ln x)}{x\ln x}\,dx$ за $z \ne 0$ $z = 0 \Rightarrow y' = 0$

$y = C$ [illegible]

$\ln|z| = -\int \frac{1 + \ln x}{\ln x}\frac{dx}{x} \overset{t = \ln x}{=\!=} -\int\left(\frac{1}{t} + 1\right)dt = -\ln|t| - t + \ln C_1$

$\ln|z| = \ln\frac{C_1}{|t|} - t^{\ln x} \Rightarrow z = \frac{C_1}{\ln x} \cdot \frac{1}{x}$ 6

$y = C_1 \int \frac{dx}{x\ln x} = C_1 \cdot \ln|\ln x| + C_2$ o.r.

c 2. $y_0 = y_h + y_p$

$y_h \quad k^3 - 3k^2 + 2k = 0$

$k(k^2 - 3k + 2) = 0 \quad k_1 = 0, \; k_2 = 1, \; k_3 = 2$

$y_h = C_1 e^{0x} + C_2 e^x + C_3 e^{2x}$ 6

$y_p = (Ax + B) \cdot x \cdot e^x = -\frac{x^2}{2} e^x$ 7

$y_0 = C_1 + C_2 e^x + C_3 e^{2x} - \frac{x^2}{2} e^x$ 1