(13)

1. $z(x,y) = (x+y^2)\cdot\sqrt{e^{3x}}$ $\qquad D = \mathbb{R}^2$ (1)

$z'_x = \sqrt{e^{3x}}\left(\dfrac{2+3(x+y^2)}{2}\right)$ (1) $\qquad$ стац. точка

$M\left(-\frac{2}{3}, 0\right)$ (3)

$z'_y = 2y\cdot\sqrt{e^{3x}}$ (1)

$z''_{xx} = \sqrt{e^{3x}}\cdot\frac{3}{2}\left(\dfrac{4+3(x+y^2)}{2}\right)$ (1)

$M: \Delta = \begin{vmatrix} \sqrt{e^{-2}}\cdot\frac{3}{2} & 0 \\ 0 & 2\sqrt{e^{-2}} \end{vmatrix} > 0$ (3)

$z''_{xy} = 3y\cdot\sqrt{e^{3x}}$ (1)

$z''_{yy} = 2\cdot\sqrt{e^{3x}}$ (1)

$z''_{xx}(M) > 0$

locmin (1)

2. 1) $\displaystyle\int \frac{e^{3x}dx}{(3+4e^{2x})^2} \underset{=}{\overset{t=e^x}{=}} \int \frac{t^2dt}{(3+4t^2)^2} =$ (10)

$\displaystyle = \int \frac{t^2+\frac{3}{4}-\frac{3}{4}}{4^2\cdot\left(t^2+\frac{3}{4}\right)^2}dt = \frac{1}{4^2}\int\frac{dt}{t^2+\frac{3}{4}} - \frac{3}{4^3}\int\frac{dt}{\left(t^2+\frac{3}{4}\right)^2}$

$\displaystyle = \dots = \frac{\sqrt{3}}{48}\,\mathrm{arctg}\,\frac{2e^x}{\sqrt{3}} - \frac{1}{48}\cdot\frac{6e^x}{4e^{2x}+3} + C$

II начин $\displaystyle\int\frac{t^2dt}{(3+4t^2)^2} = \int t\,\frac{t\,dt}{(3+4t^2)^2} \overset{\text{парц.}}{=} \left[\begin{aligned} u &= t \\ v &= \int\frac{t\,dt}{(3+4t^2)^2} = \dots \end{aligned}\right]$

2) $\displaystyle\int\frac{dx}{x-\sqrt{x^2-1}} = \left[\begin{aligned} &\sqrt{x^2-1} = x+t \\ &x = \frac{-1-t^2}{2t},\ dx = \frac{-2t^2+2}{4t^2}dt \end{aligned}\right]$ (8)

$\displaystyle = \frac{1}{2}\int\frac{t^2-1}{t^2}dt = \frac{1}{2}\left(t+\frac{1}{t}\right) + C \qquad t = \dots$

3) $\displaystyle\int\frac{3\,\mathrm{tg}^2x+2\,\mathrm{tg}x+5}{\mathrm{tg}^3x+2\,\mathrm{tg}^2x+4\,\mathrm{tg}x+3}\cdot\frac{dx}{\cos^2x} \overset{t=\mathrm{tg}x}{=} \int\left(\frac{2}{t+1}+\frac{t-1}{t^2+t+3}\right)$ (2) (2) (8) до крај (12)

$\displaystyle = \frac{1}{2}\ln|t^2+t+3| + 2\ln|t+1| - \frac{3}{\sqrt{11}}\,\mathrm{arctg}\,\frac{2t+1}{\sqrt{11}} + C$

3. $l = \int_0^{\pi/3} \sqrt{1+(y')^2}\, dx = \int_0^{\pi/3} \sqrt{1+\frac{\sin^2 x}{\cos^2 x}}\, dx$

$= \int_0^{\pi/3} \frac{dx}{\cos x} \overset{t=\operatorname{tg}\frac{x}{2}}{\longrightarrow} \ln\left|\frac{1+\operatorname{tg}\frac{\pi}{6}}{1-\operatorname{tg}\frac{\pi}{6}}\right|$

$\overset{t=\sin x}{\longrightarrow} \frac{1}{2}\ln\left|\frac{1+\sin x}{1-\sin x}\right|\Bigg|_0^{\pi/3}$

12

4. $\frac{d\lambda}{\lambda} = -\frac{(x^2+y^2+2x-2x)}{0-(x^2+y^2)}\, dx = dx \Rightarrow \lambda = e^x$ 10

$\underbrace{e^x\left(2xy + x^2y + \frac{y^3}{3}\right)}_{P_1}dx + \underbrace{e^x\left(x^2+y^2\right)}_{Q_1}dy = 0 \qquad \frac{\partial P_1}{\partial y} = \frac{\partial Q_1}{\partial x}$

$\int Q_1 dy = e^x x^2 y + e^x \frac{y^3}{3} \Rightarrow e^x y\left(x^2 + \frac{y^2}{3}\right) = C$ 10

$\int\left(P_1 - \frac{\partial}{\partial x}\int Q_1 dy\right)dx = \dots = 0$ О.Р.

5. $y = y'x + y' + y'^2 \qquad y' = p, \quad dy = p\,dx$

$y = px + p + p^2 \quad / d$ 7

$dy = p\,dx + x\,dp + dp + 2p\,dp$ општо реш.

$0 = (x+1+2p)\,dp \Rightarrow$ $dp = 0 \Rightarrow p = C \Rightarrow y = Cx + C + C^2$

или $\begin{cases} x = -1-2p \\ y = -p^2 \end{cases}$ сингр. реш.

6. $\alpha: x - y + z + 8 = 0$

$\vec{p} = (1,-2,-3) \qquad P(0,2,3)$ 3 x 6 ~~4~~

б) $n: \frac{x}{1} = \frac{y-2}{-1} = \frac{z-3}{1} = t \quad \dots \quad S(3,5,0)$

$P_1(-6,8,3)$

$s: \frac{x+6}{1} = \frac{y-8}{-2} = \frac{z-3}{-3}$ г) $d = 3\sqrt{3}$