1\. 1) $\left|\frac{a_{n+1}}{a_n}\right| = \frac{(n+1)!}{(2n+3)^p \cdot ((n+1)!)^q} \cdot \frac{(2n+1)^p \cdot (n!)^q}{n!}$

$= \left(\frac{2n+1}{2n+3}\right)^p \cdot \frac{1}{(n+1)^{q-1}}$ (3)

$\lim\limits_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \begin{cases} \infty, & q<1 \\ 0, & q>1 \\ 1, & q=1 \end{cases}$ (1)

Σ А.К. за $q>1$ (1)

Σ див. за $q<1$ (1)

$q=1$ $\quad |a_n| = \frac{1}{(2n+1)^p} \sim \frac{1}{2^p n^p}$ (1)

$p>1,\ q=1$ $\quad \Sigma$ А.К. (1)

$p\le 0,\ q=1$ $\quad \Sigma$ див (1)

$0<p\le 1$ $\quad$ 1. $|a_n|\to 0,\ n\to\infty$; 2. $|a_{n+1}|<|a_n|$ } Лајбниц (5) Σ усл. конв. за $0<p\le 1$, $q=1$ (1)

2) $a_n = \frac{n^a}{\ln n}$ $\quad a>0$ $\quad \lim\limits_{n\to\infty} a_n \overset{L}{=} \lim\limits_{n\to\infty} \frac{a n^{a-1}}{\frac{1}{n}} = \lim\limits_{n\to\infty} a n^a \ne 0$ (5)

$\Rightarrow \Sigma a_n$ див.

3) $S(x) = \sum\limits_{n=1}^{\infty} \frac{x^n}{n(n+1)}$

$S'(x) = \sum\limits_{n=1}^{\infty} \frac{n x^{n-1}}{n(n+1)} = \frac{1}{x^2}\sum\limits_{n=1}^{\infty} \frac{x^{n+1}}{n+1}$

$\sum\limits_{n=1}^{\infty} \frac{x^{n+1}}{n+1} = \sum\limits_{k=2}^{\infty} \frac{x^k}{k} = \sum\limits_{k=1}^{\infty} \frac{x^k}{k} - x = -\ln(1-x) - x$

(10)

$S'(x) = -\frac{\ln(1-x)+x}{x^2} \Rightarrow S(x) = -\int \frac{\ln(1-x)+x}{x^2}\,dx =$

$= \frac{\ln(1-x)}{x} - \ln(1-x) + C$, $\quad C = 1$

$S(0) = \lim\limits_{x\to 0}(\ ) = -1 + C$

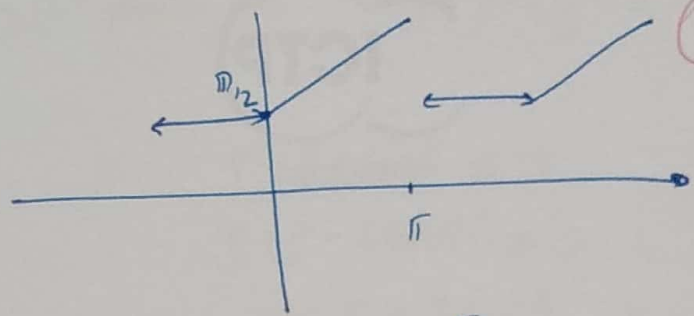
$x = \frac{2}{3}$

$S = \frac{\ln\left(\frac{1}{3}\right)}{\frac{2}{3}} - \ln\left(\frac{1}{3}\right) + 1$

$= \ln 3 + 1 - \frac{3}{2}\ln 3 = 1 - \frac{1}{2}\ln 3$

II начин $\quad S(x) = \sum\limits_{n=1}^{\infty} \frac{n+1-n}{n(n+1)} x^n = \sum\limits_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+1}\right)x^n = \ldots$

2. $f(x)$



1

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\left(\int_{-\pi}^{0} \frac{\pi}{2}\,dx + \int_{0}^{\pi}\left(x+\frac{\pi}{2}\right)dx\right) = \frac{3\pi}{2}$$ 1

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0} \frac{\pi}{2}\,\cancel{dx}\cdot \cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi}\left(x+\frac{\pi}{2}\right)\cos nx\,dx =$$

$$= \frac{1}{2}\cdot\frac{1}{n}\sin(nx)\Big|_{-\pi}^{0} + 0 + \frac{1}{\pi}\int_{0}^{\pi} x\cos nx\,dx =$$

$u = x \quad v = \frac{1}{n}\sin(nx)$

$$= \frac{1}{\pi}\cdot 0 + \frac{1}{\pi}\cdot\frac{1}{n^2}\cos nx\Big|_{0}^{\pi} = \frac{(-1)^n - 1}{n^2\pi}$$ 5

$$b_n = \frac{1}{\pi}\cdot\frac{\pi}{2}\int_{-\pi}^{0}\sin nx\,dx + \frac{1}{\pi}\cdot\int_{0}^{\pi} x\sin nx\,dx + \frac{1}{\pi}\int_{0}^{\pi}\frac{\pi}{2}\cdot\sin nx\,dx$$

0

$$= \frac{1}{\pi}\cdot\frac{-x}{n}\cos(nx)\Big|_{0}^{\pi} + \frac{1}{n\pi}\int_{0}^{\pi}\cos(nx)\,dx = \frac{-(-1)^n}{n}$$ 5

$$\Phi(x) = \frac{\pi}{4} + \frac{9\pi^2}{16} + \sum_{n=1}^{\infty}\left(\frac{(-1)^n - 1}{n^2\pi}\cdot\cos(nx) + \frac{(-1)^{n-1}}{n}\cdot\sin(nx)\right]$$ 1

$$\Phi(0) = \frac{\pi}{2} = f(0)$$ 1

$$\Phi(\pi) = \frac{3\pi/2 + \pi/2}{2} = \pi$$ 1 ЗА $x = \pi + 2k\pi$ РАЗВОЈ НЕ ВАЖИ

$$\frac{\pi}{2} = \Phi(0) = \frac{\pi}{4} + \frac{9\pi^2}{16} + \sum_{n=1}^{\infty}\frac{(-1)^n - 1}{n^2\pi} = \frac{\pi}{4} + \frac{9\pi^2}{16} + \sum_{n=1}^{\infty}\frac{(-1)^{2n+1} - 1}{(2n-1)^2\pi}$$

$$\frac{2}{\pi}S_1 = \frac{9\pi^2}{16} - \frac{\pi}{4} \Rightarrow S_1 = \frac{9\pi^3 - 4\pi^2}{32}$$ 5 $$= \frac{\pi}{4} + \frac{9\pi^2}{16} - \frac{2}{\pi}S_1$$

3. $\iint\limits_D \sqrt{(x^2+y^2+2y+1)^3}\,dx\,dy = I \qquad D: x^2+y^2 \le 1$

$x = \rho\cos\theta$

$y = \rho\sin\theta - 1$ (2) $\qquad D': \ \rho^2 - 2\rho\sin\theta \le 0$

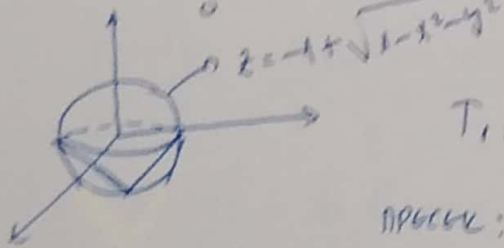
$J = \rho \qquad\qquad \rho(\rho - 2\sin\theta) \le 0$ (5)

$0 \le \rho \le 2\sin\theta$ и $0 \le \theta \le \pi$

$$I = \int_0^{\pi}\int_0^{2\sin\theta} \sqrt{(\rho^2)^3}\,\rho\,d\rho\,d\theta = \int_0^{\pi}\int_0^{2\sin\theta} \rho^4\,d\rho\,d\theta = \int_0^{\pi} \frac{2^5\cdot\sin^5\theta}{5}\,d\theta =$$

$$= -\frac{32}{5}\int_0^{\pi}(1-\cos^2\theta)^2\cdot d(\cos\theta) \overset{t=\cos\theta}{=\!=} \frac{32}{5}\int_{-1}^{1}(1-t^2)^2\,dt =$$ (8)

$$= 2\cdot\frac{32}{5}\int_0^1(1-2t^2+t^4)\,dt = \frac{64}{5}\left(t - \frac{2}{3}t^3 + \frac{t^5}{5}\right)\Big|_0^1 = \frac{64}{5}\cdot\frac{8}{15}$$

4. 

T_1:  $\qquad T_2$: остатак

пресек: $\quad S_2 \to (z+2)^2 = x^2+y^2$

заменом у S_1: $\quad (z+2)^2 + (z+1)^2 = 1$

$2z^2 + 6z + 4 = 0 \Rightarrow z_1 = -1,\ z_2 = -2$ (3)

$D: \ x^2+y^2 = (-1+2)^2 = 1$

$$V(T_1) = \iint\limits_D \left(\sqrt{1-x^2-y^2} - 1 - \sqrt{x^2+y^2} + 2\right)dx\,dy = \left[\begin{array}{ll} x = \rho\cos\theta, & 0 \le \rho \le 1 \\ y = \rho\sin\theta, & -\pi \le \theta \le \pi \\ J = \rho, & \end{array}\right]$$

$$= \int_{-\pi}^{\pi}\int_0^1 \left(\sqrt{1-\rho^2} + 1 - \rho\right)\rho\,d\rho\,d\theta = 2\pi\Big(\underbrace{\int_0^1 \sqrt{1-\rho^2}\cdot\rho\,d\rho}_{t=\sqrt{1-\rho^2}} + \int_0^1(\rho-\rho^2)\,d\rho\Big)$$ (10)

$$= 2\pi\cdot\left(\frac{1}{3} + \frac{1}{2} - \frac{1}{3}\right) = \pi$$

II начин $\quad V(T_1) = \underset{H=1,\ r=1}{V(\text{купе})} + \underset{r=1}{V(\text{полулопте})} = \frac{1}{3}\cdot 1^2\pi\cdot 1 + \frac{1}{2}\cdot\frac{4}{3}1^3\pi = \pi$

$V(T_2) = V(\text{лопте}) - V(T_1) = \frac{4}{3}\pi - \pi = \frac{\pi}{3}$ (2)

5. a) $x^2y'' - xy' + y = x$ ОЗЛОРОВА Е. Л.

$t = \ln x$, $x = e^t$, $y' = \frac{1}{x}\dot{y}$, $y'' = \frac{1}{x^2}(\ddot{y} - \dot{y})$

$\ddot{y} - 2\dot{y} + y = e^t$ 5

$y_h = C_1 e^t + C_2 t \cdot e^t$ 3

$y_p = A \cdot t^2 \cdot e^t$, $A = \frac{1}{2}$ 3

$y = C_1 e^t + C_2 t e^t + \frac{1}{2} t^2 e^t = C_1 x + C_2 x \ln x + \frac{1}{2} x \ln^2 x$ 2

б) $(1+y^2) y \cdot y'' = (3y^2+1) \cdot y'^2$

смена $y' = z$, $y'' = z' \cdot z$

$(1+y^2) \cdot y \cdot \cancel{z} \cdot z' = (3y^2+1) \cdot z^{\cancel{2}}$ $/ : z$, за $z \neq 0$

$y' = 0$
није С.Р.

$$\int \frac{dz}{z} = \int \frac{(3y^2+1)\,dy}{y^3+y}$$

$\ln|z| = \ln C_1 \cdot |y^3+y|$

$$\frac{dy}{dx} = C_1 \cdot (y^3+y)$$

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$$\int \frac{dy}{y(y^2+1)} = C_1 x + C_2$$

$\ln|y| - \frac{1}{2}\ln(y^2+1) = C_1 x + C_2$

$y / \sqrt{y^2+1} = e^{C_1 x} \cdot e^{C_2}$