1. $z = e^{y-x}(y^2-2x^2)$ $D=\mathbb{R}^2$ 1

$\boxed{15}$

$z'_x = e^{y-x}(2x^2-y^2-4x)$ 2

$z'_y = e^{y-x}(y^2+2y-2x^2)$ 2

Стационарне тачке:

$$\begin{cases} z_x=0\\ z_y=0\end{cases} \Leftrightarrow \begin{cases}2x^2-y^2-4x=0\\ y^2+2y-2x^2=0\end{cases}\Leftrightarrow$$

$$\Leftrightarrow \begin{cases}2x^2-y^2-4x=0\\ 2y-4x=0\end{cases} \Leftrightarrow \begin{cases}y=2x\\ 2x^2+4x=0\end{cases}\Leftrightarrow \begin{matrix}M_1=(0,0)\\ M_2=(-2,-4)\end{matrix}$$ 3

$z''_{xx}=e^{y-x}(y^2-2x^2+8x-4)$ $z''_{xy}=e^{y-x}(2x^2-y^2-2y-4x)=z''_{yx}$

$z''_{yy}=e^{y-x}(y^2+4y-2x^2+2)$ 3

$M_1=(0,0)$ $\Delta = e^{-4}\begin{vmatrix}-4&0\\0&2\end{vmatrix}<0 \Rightarrow$ z нема е. в. у M_1 2

$M_2=(-2,-4)$ $\Delta=e^{-4}\begin{vmatrix}-12&8\\8&\underbrace{-6}_{<0}\end{vmatrix}>0 \Rightarrow$ z у M_2 има лок. макс.

$z(M_2)=\frac{8}{e^2}$ 2

2. А) $\boxed{10}$

$$\int_0^{+\infty}\frac{7\cdot 2^x\,dx}{(3\cdot2^x+7)^{3/2}} = \left[\begin{matrix}\text{смена:}\\ 3\cdot2^x+7=t\\ 3\cdot\ln2\;2^x dx=dt\end{matrix}\right] = \frac{7}{3\ln 2}\int_{10}^{+\infty} t^{-3/2}\,dt =$$

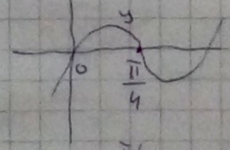
$$=\frac{-14}{3\ln2}\,\frac{1}{\sqrt t}\Big|_{10}^{+\infty} = \frac{14\sqrt{10}}{30\ln2}=\frac{7\sqrt{10}}{15\ln2}$$ 5

Б)

$$\int_0^1 x^4(1-x^2)^{5/2}\,dx = \left[\begin{matrix}\text{смена:}\\ x^2=t\\ dx=\frac12 t^{-\frac12}dt\end{matrix}\right] = \frac12\int_0^1 t^{3/2}(1-t)^{5/2}\,dt =$$

$$=\frac12 B\left(\frac52,\frac72\right)=\frac{3\pi}{2^9}$$ 5

3. $\boxed{20}$ $y=\frac12\sin4x$



$P=\int_0^{\pi/4}\frac12\sin4x\,dx = \frac{-1}{8}\cos4x\Big|_0^{\pi/4}=\frac14$ 5

$V=\pi\int_0^{\pi/4}\frac14\sin^2 4x = \frac{\pi}{8}\int_0^{\pi/4}1-\cos8x\,dx=\frac{\pi}{8}\left(x+\frac18\sin8x\right)\Big|_0^{\pi/4}=\frac{\pi^2}{32}$ 15

4) $y' - \frac{y}{x} = \left(1+\frac{y}{x}\right)\sqrt{\frac{\frac{y}{x}-1}{\frac{y}{x}+1}}$

$\boxed{20}$

Смена:

$\boxed{\begin{array}{l} y = zx, \quad z = z(x) \\ y' = z'x + z \end{array}}$

$z'x = (1+z)^2\sqrt{\frac{z-1}{z+1}}$ 2 $\quad z+1 \neq 0$

1. $z-1 \neq 0$ => смемо да поделимо са $(1+z)^2\sqrt{\frac{z-1}{z+1}}$

$\frac{1}{(1+z)^2}\sqrt{\frac{z+1}{z-1}}\,dz = \frac{dx}{x}$ 3 $\Rightarrow \ln(cx) = \int\sqrt{\frac{z+1}{z-1}}\,\frac{dz}{(1+z)^2}$

Смена:

$\sqrt{\frac{z+1}{z-1}} = t$

$z = \frac{t^2+1}{t^2-1}$

$dz = \frac{-4t\,dt}{(t^2-1)^2}$

$\ln(cx) = \int -\frac{dt}{t^2} = \cancel{\sqrt{\frac{z+1}{z-1}}} = \frac{1}{t} = \sqrt{\frac{z-1}{z+1}}$

$\boxed{\ln(cx) = \sqrt{\frac{y-x}{y+x}}}$ ОР или

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$\boxed{y = x\,\frac{1+\ln^2(cx)}{1-\ln^2(cx)}}$ ОР

2. $z-1=0 \Rightarrow y = x$ јесте решење али није СР 2

$\left(\lim_{c\to+\infty}\right)$

5) $P'_y = -\frac{1+\sqrt{\ }}{y^2}$ $\quad Q'_x = \frac{2x}{y}$ $\quad \cancel{\frac{d\mu}{\mu}} = \frac{dz}{z} = -\frac{P'_y - Q'_x}{P}\,dy$

20

$\ln z = \int\frac{dy}{y} \Rightarrow z = y$ 10

$P_1 = 2xy + \sqrt{1+x^2} + 1 \qquad Q_1 = 1+x^2$

$u = \varphi(x) + \int Q\,dy = \varphi(x) + y + x^2y$

$u'_x = P_{1x} \Rightarrow \varphi'(x) = \sqrt{1+x^2}+1 \Rightarrow \varphi = x + \int\sqrt{1+x^2}\,dx$

$\int\sqrt{1+x^2}\,dx = \int\frac{1+x^2}{\sqrt{\ }}\,dx = \ln(x+\sqrt{\ }) + \int x\,d\left(\sqrt{1+x^2}\right) =$

$= \ln(x+\sqrt{\ }) + x\sqrt{\ } + \int\sqrt{1+x^2}\,dx$

$\boxed{y + x^2y + x + \frac{1}{2}\ln\left(x+\sqrt{1+x^2}\right) + \frac{x}{2}\sqrt{1+x^2} = C}$ 10

О.Р.

Нема С.Р.

6) $p: \begin{cases} x+2y+z=-24 \\ -x-y+z=6 \end{cases} \Leftrightarrow \begin{cases} y+2t=-18 \\ x=-y+z-6 \end{cases} \Leftrightarrow \begin{cases} z=t \\ y=-18-2t \\ x=18+2t+t-6 \end{cases}$

$\boxed{15}$

$p: \begin{cases} x=12+3t \\ y=-18-2t \\ z=t \end{cases}$ $\quad V_p=(3,-2,1)$

$P=(12,-18,0)$

A) $g \parallel p \Rightarrow g: \dfrac{x-1}{3}=\dfrac{y-1}{-2}=\dfrac{z-1}{1}$

$Q \in g \nearrow$ 5

Б) $\left.\begin{array}{l} P, Q \in \alpha \Rightarrow \overrightarrow{PQ} \perp n_\alpha \\ p \subseteq \alpha \Rightarrow v_p \perp n_\alpha \end{array}\right\} \Rightarrow n_\alpha = \lambda \cdot (\overrightarrow{PQ} \times V_p) = \lambda \begin{vmatrix} i & j & k \\ -11 & 19 & 1 \\ 3 & -2 & 1 \end{vmatrix} =$

$= \lambda(21, 14, -35) \underset{\lambda=\frac{1}{7}}{=\!=\!=} (3, 2, -5)$

$Q \in \alpha$

$\alpha: 3(x-1)+2(y-1)-5(z-1)$

$\alpha: 3x+2y-5z=0$ 5

B) $d(Q,p) = \dfrac{|\overrightarrow{PQ} \times V_p|}{|V_p|} = \dfrac{7|(3,2,-5)|}{|(3,-2,1)|} = \sqrt{7}\cdot\sqrt{19}$ 5