Osobine autokorelacije

1) Autokorelacija u nuli ($\tau=0$) jednaka je sr. kvadratnoj vrednosti signala (ukupna snaga).

$$R_X(\tau=0) = E[X(t)\,X(t+\tau)]\Big|_{\tau=0} = E[X^2(t)] = P_{SR}.$$

$$R_X(\tau=0) = \int_{-\infty}^{\infty} S_X(jf)\, e^{j2\pi f\tau}\, df\Big|_{\tau=0} = \int_{-\infty}^{\infty} S_X(jf)\,df = P_{SR}$$

$$\boxed{P_{SR} = R_X(0) = \int_{-\infty}^{\infty} S_X(jf)\,df}$$

2) Autokorelacija je parna funkcija

$$R_X(-\tau) = R_X(\tau)$$

3) $\lim_{\tau\to\infty} R_X(\tau) = 0$, ako $\overline{X(t)} \neq 0$ i $X(t)$ nema periodičnih komponenata

4) Ako je $X(t)$ periodičan $R_X(\tau)$ je periodična

I) Sinusoida sa slučajnom fazom

$$X(t) = A\cos(\omega_0 t + \theta), \qquad f_\theta = \begin{cases} \frac{1}{2\pi}, & \theta \in (0, 2\pi) \\ 0, & \text{inače} \end{cases}$$

$$R_X(\tau) = E[X(t)X(t+\tau)] = \frac{A^2}{2\pi}\int_0^{2\pi} \cos(\omega_0 t+\theta)\cos[\omega_0(t+\tau)+\theta]\,d\theta$$

$$R_X(\tau) = \frac{A^2}{2}\cos(\omega_0\tau)$$

$$S_X(f) = \mathcal{F}\{R_X(\tau)\} = \int_{-\infty}^{\infty} \frac{A^2}{2}\underbrace{\cos(2\pi f_0\tau)}_{\frac{1}{2}(e^{j2\pi f_0\tau}+e^{-j2\pi f_0\tau})} e^{-j2\pi f\tau}\,d\tau = \frac{A^2}{4}\left(\delta(f-f_0)+\delta(f+f_0)\right)$$

$$= \frac{A^2}{4}\int_{-\infty}^{\infty} e^{-j2\pi(f-f_0)\tau}\,d\tau + \frac{A^2}{4}\int_{-\infty}^{\infty} e^{-j2\pi(f+f_0)\tau}\,d\tau$$

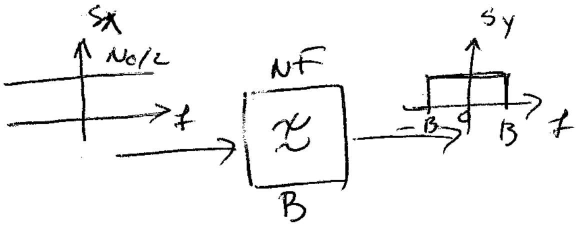
II) Beli šum

$$S_X(f) = N_0/2$$

$$R_X(\tau) = \mathcal{F}^{-1}\left\{\frac{N_0}{2}\right\} = \frac{N_0}{2}\underbrace{\left(\int_{-\infty}^{\infty} e^{j2\pi f\tau}\,df\right)}_{\delta(\tau)} = \frac{N_0}{2}\,\delta(\tau)$$

$$(\text{ili}) = \frac{N_0}{2}\mathcal{F}^{-1}\left\{e^{-j2\pi f\cdot 0}\right\} = \frac{N_0}{2}\,\delta(\tau-0) = \frac{N_0}{2}\,\delta(\tau)$$

III)



B – PROPUSNI OPSEG

$$S_y(f) = \begin{cases} N_0/2, & |f| \le B \\ 0, & \text{inače} \end{cases}$$

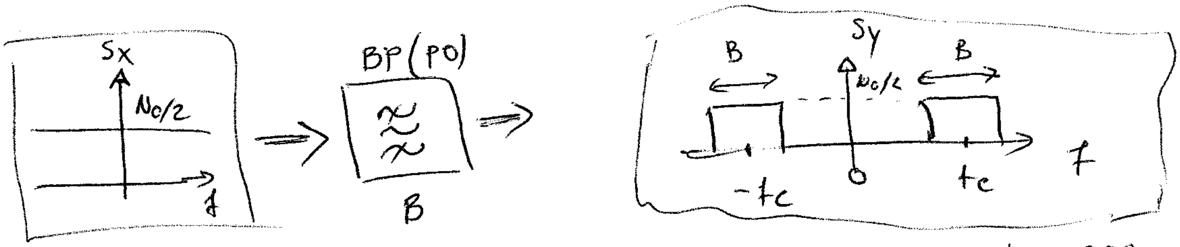
$$R_y(f) = \mathcal{F}^{-1}\{S_y(f)\} = \int_{-\infty}^{\infty} S_y(f) e^{+j2\pi f\tau} df = \frac{N_0}{2}\int_{-B}^{B} e^{j2\pi f\tau} df$$

$$R_y(f) = \frac{N_0}{2} \frac{e^{j2\pi B\tau} - e^{-j2\pi B\tau}}{j2\pi\tau} = \frac{N_0}{2} \frac{\sin(2\pi B\tau)}{\pi\tau}$$

$$R_y(f) = BN_0 \frac{\sin(2\pi B\tau)}{2\pi B\tau} = BN_0 \operatorname{sinc}(2B\tau)$$

$$P_Y = \int_{-\infty}^{\infty} S_y(f) df = \int_{-B}^{B} \frac{N_0}{2} df = N_0 B = R_y(0)$$

IV)



B – propusni opseg

f_c – centralna učestanost

$$R_y(\tau) = \mathcal{F}^{-1}\{S_y(f)\} = \int_{-f_0-B/2}^{-f_0+B/2} \frac{N_0}{2} \underbrace{e^{j2\pi f\tau}} df + \int_{f_0-B/2}^{f_0+B/2} \frac{N_0}{2} \underbrace{e^{j2\pi f\tau}} df$$

$$\cos(2\pi f\tau) + j\sin(2\pi f\tau)$$

PARNA F-JA

NEPARNA F-JA

$$R_y(\tau) = 2\frac{N_0}{2} \int_{f_0-B/2}^{f_0+B/2} \cos(2\pi f\tau) df$$

$$R_y(\tau) = N_0 \int_{-B/2}^{B/2} \cos[2\pi(u + f_0)\tau] du \qquad (u = f - f_0 \text{, SMENA})$$

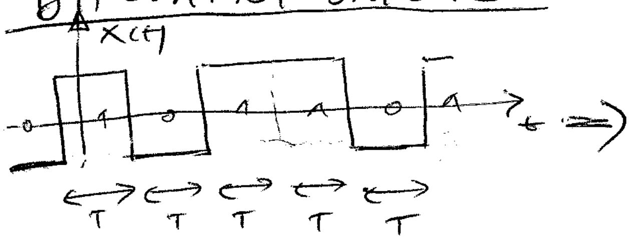
$$R_y(\tau) = N_0 \int_{-B/2}^{B/2} \cos(2\pi u\tau)\cos(2\pi f_0\tau) du - \underbrace{\int_{-B/2}^{B/2} \sin(2\pi u\tau)\sin(2\pi f_0\tau) du}_{\emptyset \text{ (NEPARNO)}}$$

$$R_y(\tau) = N_0 \cos(2\pi f_0\tau) \int_{-B/2}^{B/2} \cos(2\pi u\tau) du$$

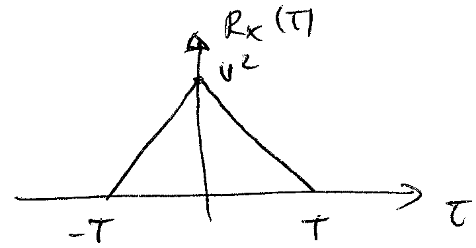
$$R_y(\tau) = 2N_0 \cos(2\pi f_0\tau) \frac{1}{2\pi\tau} \sin(\pi B\tau) = BN_0 \cos(2\pi f_0\tau) \operatorname{sinc}(B\tau)$$

$$P_Y = \int_{-\infty}^{\infty} S_y(f) df = 2\int_{f_0-B/2}^{f_0+B/2} N_0/2 \, df = N_0 B = R_y(0)$$

V) BIPOLARNI SIGNAL



$$\Rightarrow R_X(\tau) = \begin{cases} U^2\left(1-\frac{|\tau|}{T}\right), & |\tau| \le T \\ 0, & |\tau| > T \end{cases}$$



$$S_X(f) = \mathcal{F}\{R_X(\tau)\} = U^2 \mathcal{F}\left\{\Lambda\left(\frac{t}{T}\right)\right\}$$

$$S_X(f) = U^2 T \operatorname{sinc}^2(fT) \leftarrow \text{TABLICA}$$

IZVOĐENJE

$$S_X(f) = \mathcal{F}\{R_X(\tau)\} = \int_{-\infty}^{\infty} R_X(\tau)\, \underbrace{e^{-j2\pi f\tau}}_{\uparrow}\, d\tau = 2\int_0^T U^2\left(1-\frac{\tau}{T}\right)\cos(2\pi f\tau)\, d\tau$$

$$\underbrace{\cos(2\pi f\tau)}_{\text{PARNO}} + j\underbrace{\sin(2\pi f\tau)}_{\text{NEPARNO}}$$

$$S_X(f) = 2U^2\left\{\int_0^T \cos(2\pi f\tau)\, d\tau - \frac{1}{T}\int_0^T \tau\cos(2\pi f\tau)\, d\tau\right\}$$

$$\int_0^T \cos(2\pi f\tau)\, d\tau = \frac{1}{2\pi f}\sin(2\pi f\tau)\Big|_0^T = \frac{\sin(2\pi fT)}{2\pi f}$$

$$\int_0^T \sin(2\pi f\tau)\, d\tau = \frac{-1}{2\pi f}\cos(2\pi f\tau)\Big|_0^T = \frac{1-\cos(2\pi f\tau)}{2\pi f}$$

$$u = \tau, \quad v = \int \cos(2\pi f\tau)\, d\tau = \frac{\sin(2\pi f\tau)}{2\pi f}$$

$$\int_0^T \tau\cos(2\pi f\tau)\, d\tau = \tau\frac{\sin(2\pi f\tau)}{2\pi f}\Big|_0^T - \int_0^T \frac{\sin(2\pi f\tau)}{2\pi f}\, d\tau$$

$$= T\frac{\sin(2\pi fT)}{2\pi f} - \frac{1-\cos(2\pi fT)}{(2\pi f)^2}$$

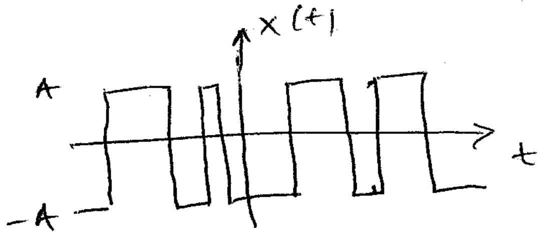
$$S_X(f) = 2U^2\left\{\cancel{\frac{\sin(2\pi fT)}{2\pi f}} - \cancel{\frac{\sin(2\pi fT)}{2\pi f}} + \frac{1-\cos(2\pi fT)}{T(2\pi f)^2}\right\}$$

$$S_X(f) = \frac{2U^2T}{4}\, 2\cdot\frac{\sin^2(\pi fT)}{(\pi fT)^2}$$

$$S_X(f) = U^2 T \operatorname{sinc}^2(fT)$$

$$P_X = \int_{-\infty}^{\infty} S_X(f) = U^2 \underbrace{\int_{-\infty}^{\infty} \operatorname{sinc}^2(fT)\, d(fT)}_{=1} = U^2 = R_X(0)$$

VI TELEGRAFSKI SIGNAL



$$\boxed{R_x(\tau) = A^2 e^{-2c|\tau|}}$$

c – SREDNJI BROJ PRESEKA t-OSE U JEDINICI VREMENA

$$S_x(f) = A^2 \mathcal{F}\{e^{-2c|\tau|}\} = A^2 \mathcal{F}\{e^{-\frac{|\tau|}{\underbrace{1/2c}_{\alpha}}}\} = A^2 \frac{2\cdot\underbrace{\frac{1}{2c}}_{\alpha}}{1+(2\pi f\underbrace{\frac{1}{2c}}_{\alpha})^2}$$

$$S_x(f) = \frac{A^2/c}{1+\left(\frac{\pi f}{c}\right)^2} = \frac{A^2 c}{c^2+(\pi f)^2} = \frac{4A^2 c}{(2c)^2+(2\pi f)^2}$$

ili

$$S_x(f) = \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f\tau} d\tau = A^2 \int_{-\infty}^{\infty} e^{-2c|\tau|} e^{-j2\pi f\tau} d\tau$$

$$S_x(f) = A^2 \int_{-\infty}^{0} e^{-j2\pi f\tau+2c\tau} d\tau + A^2 \int_{0}^{\infty} e^{-j2\pi f-2c\tau} d\tau$$

$$S_x(f) = A^2 \frac{1}{2c-j2\pi f}(1-0) + A^2\cdot\frac{1}{-2c-j2\pi f}(0-1)$$

$$S_x(f) = A^2\left(\frac{1}{2c-j2\pi f}+\frac{1}{2c+j2\pi f}\right)$$

$$S_x(f) = A^2 \frac{2c+\cancel{j2\pi f}+2c-\cancel{j2\pi f}}{(2c)^2+(2\pi f)^2} = \frac{4A^2 c}{(2c)^2+(2\pi f)^2}$$