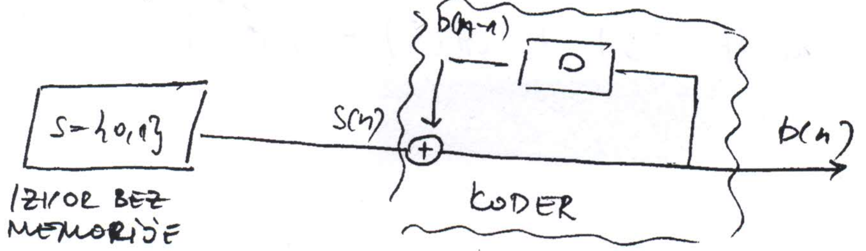
7. Binarni izvor bez memorije sa verovatnoćama simbola $P(\text{"1"}) = p$ i $P(\text{"0"}) = q = 1-p$ dovodi se na ulaz kodera kao na slici:



Odrediti entropiju binarnog niza na izlazu kodera sa slike.

Rešenje:

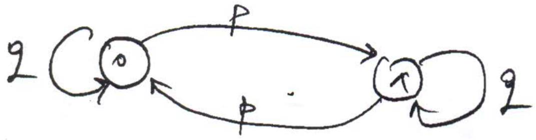
$$b(n) = s(n) \oplus b(n-1)$$

$$\left.\begin{array}{l} s(n) = \{0\,0\,1\,0\,0\,1\,1\,0\,0\,0\,0\,1\,0\,0 \\ b(n) = \boxed{0}\,0\,0\,1\,1\,1\,0\,1\,1\,1\,1\,1\,0\,0\,0 \end{array}\right\} \Rightarrow \text{diferencijalni koder}$$

$b(-1)$ = početno stanje

Očigledno izvor je sa memorijom I reda:

$$P(b(n)=0 \mid b(n-1)=0) = P(0/0) = P(s(n)=0) = q$$
$$P(b(n)=1 \mid b(n-1)=0) = P(1/0) = P(s(n)=1) = p$$
$$P(b(n)=0 \mid b(n-1)=1) = P(0/1) = P(s(n)=1) = p$$
$$P(b(n)=1 \mid b(n-1)=1) = P(1/1) = P(s(n)=0) = q$$



$$P(0) = P(b(n)=0) = P(b(n)=0)\cdot q + P(b(n)=1)\cdot p = q\,P(0) + p\,P(1)$$

$$\left.\begin{array}{ll} (1) & p \cdot P(1) = (1-q)\cdot P(0) \\ (2) & P(0)+P(1) = 1 \end{array}\right\} \Rightarrow P(1) = P(0) = 0{,}5 \quad \Rightarrow H(B) = \sum_{i=1}^{2} P(b_i)\, ld \frac{1}{P(b_i)} = 1$$

$$H(^mB) = \sum_{B^{m+1}} P(b_{i1}, b_{i2}, \dots b_{im}, b_j)\, ld \frac{1}{P(b_j \mid b_{i1}, b_{i2}, \dots b_{im})}$$

$$H(^1B) = \sum_{i=1}^{2}\sum_{j=1}^{2} P(b_j|b_i)\, P(b_i)\, ld \frac{1}{P(b_j|b_i)}$$

$$H(^1B) = P(0/0)\,P(0)\, ld \frac{1}{P(0/0)} + P(1/0)\,P(0)\, ld \frac{1}{P(1/0)} + P(0/1)\,P(1)\, ld \frac{1}{P(0/1)}$$
$$+ P(1/1)\,P(1)\, ld \frac{1}{P(1/1)}$$

$$H(^1B) = q/2\; ld \frac{1}{q} + p/2\; ld \frac{1}{p} + p/2\; ld \frac{1}{p} + q/2\; ld \frac{1}{q}$$

$$H(^1B) = p\, ld \frac{1}{p} + q\, ld \frac{1}{q} = H(S)$$

IZVOR SA MEMORIJOM PRVOG REDA KOJI EMITUJE SIMBOLE S_1, S_2, S_3 I S_4 OPISAN JE TRANZICIONOM MATRICOM

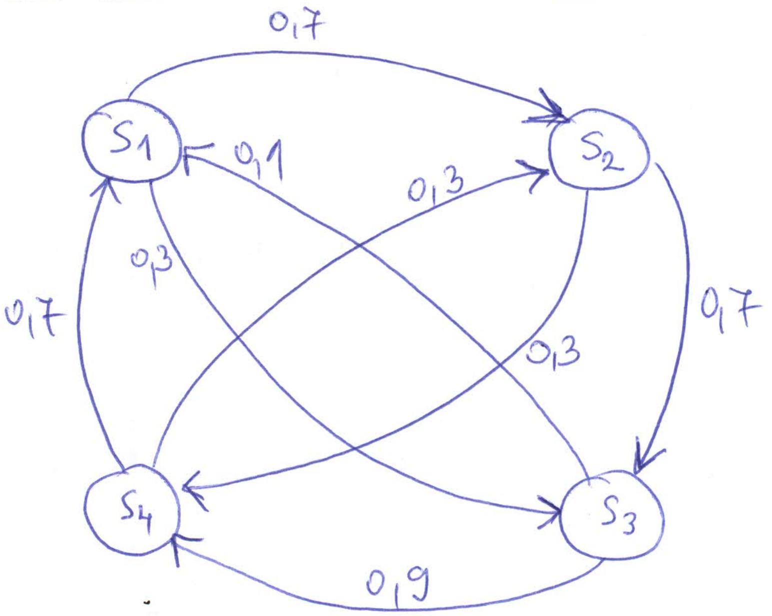
$$\Pi = \underbrace{\begin{bmatrix} 0 & 0{,}7 & 0{,}3 & 0 \\ 0 & 0 & 0{,}7 & 0{,}3 \\ 0{,}1 & 0 & 0 & 0{,}9 \\ 0{,}7 & 0{,}3 & 0 & 0 \end{bmatrix}}_{S_j} \Bigg\} S_i \qquad \Pi = P(S_j|S_i)$$

a) NACRTATI DIJAGRAM STANJA KOJI ODGOVARA OVOM IZVORU. ODREDITI VEROVATNOĆE PRIDRUŽENOG IZVORA (VEROVATNOĆE STANJA) I ENTROPIJU PRIDRUŽENOG IZVORA.

b) ODREDITI ENTROPIJU IZVORA SA MEMORIJOM.

REŠENJE:

a)



$$\left.\begin{aligned} P(S_1) &= 0{,}1 \cdot P(S_3) + 0{,}7\,P(S_4) \\ P(S_2) &= 0{,}7\,P(S_1) + 0{,}3\,P(S_4) \\ P(S_3) &= 0{,}3\,P(S_1) + 0{,}7\,P(S_2) \\ P(S_4) &= 0{,}3\,P(S_2) + 0{,}9\,P(S_3) \end{aligned}\right\}$$

JEDNU OD JEDNAČINA IZBACITI JER JE LINEARNO ZAVISNA, A UMESTO NJE KORISTITI:

$$\sum_{i=1}^{4} P(S_i) = 1$$

REŠENJE OVOG SISTEMA JE:

$$P(S_1) = 0{,}23$$
$$P(S_2) = 0{,}25$$
$$P(S_3) = 0{,}24$$
$$P(S_4) = 0{,}29$$

$$H(\bar{S}) = \sum_{i=1}^{4} P(S_i)\, ld \frac{1}{P(S_i)} = 1{,}994 \ Sh/simbol$$

b)

$$H(s) = \sum_{i=1}^{4} \sum_{j=1}^{4} P(s_i) P(s_j | s_i) \, ld \frac{1}{P(s_j|s_i)}$$

$$\begin{aligned} H(s) = P(s_1) & \left(0{,}7 \, ld \frac{1}{0{,}7} + 0{,}3 \, ld \frac{1}{0{,}3}\right) \\ + P(s_2) & \left(0{,}7 \, ld \frac{1}{0{,}7} + 0{,}3 \, ld \frac{1}{0{,}3}\right) \\ + P(s_3) & \left(0{,}1 \, ld \frac{1}{0{,}1} + 0{,}9 \, ld \frac{1}{0{,}9}\right) \\ + P(s_4) & \left(0{,}7 \, ld \frac{1}{0{,}7} + 0{,}3 \, ld \frac{1}{0{,}3}\right) \end{aligned}$$

$H(s) = 0{,}7825$ Sh/simbol

$\Delta H(s) = H(\bar{s}) - H(s) = 1{,}2112$ Sh/simbol

$\boxed{H(\bar{s}) \geq H(s)}$!

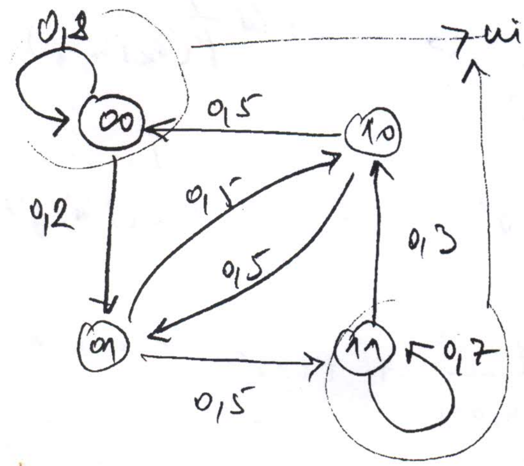
⑤ Nacrtati dijagram stanja Markovljevskog izvora II reda sa binarnom listom simbola $s=\{0,1\}$ i sa uslovnim verovatnoćama pojavljivanja simbola:

$$P(0/00)=0{,}8 \qquad P(1/11)=0{,}7$$
$$P(1/00)=0{,}2 \qquad P(0/11)=0{,}3$$
$$P(0/01)=P(1/01)=P(0/10)=P(1/10)=0{,}5$$

ENTROPIJA II REDA PRORAJ!!

Odrediti verovatnoće stanja pridruženog izvora i verovatnoće izlaznih simbola.

Rešenje:



→ nisu apsorbujuća stanja!

$$P(00)=P(0/00)P(00)+P(0/10)\cdot P(10)$$

$$P(00)=0{,}8\cdot P(00)+0{,}5\cdot P(10) \qquad (1)$$
$$P(01)=0{,}2\cdot P(00)+0{,}5P(10) \qquad (2)$$
$$P(10)=0{,}5P(01)+0{,}3P(11) \qquad (3)$$
$$\left[P(11)=0{,}7P(11)+0{,}5P(01)\right] \qquad X$$
$$P(00)+P(01)+P(10)+P(11)=1 \qquad (4)$$

(suma prethodne četiri jednačine)

(1) ⇒ $0{,}2P(00)=0{,}5P(10) \Rightarrow P(00)=2{,}5P(10)$

(2) ⇒ $P(01)=0{,}5P(10)+0{,}5P(10)=P(10) \Rightarrow P(01)=P(10)$

(3) ⇒ $0{,}5P(10)=0{,}3P(11) \rightarrow P(11)=\frac{5}{3}P(10)$

(4) → $2{,}5P(10)+P(10)+P(10)+\frac{5}{3}P(10)=1 \rightarrow \boxed{P(10)=\frac{6}{37}}$

$$P(00)=\frac{5}{2}\cdot P(10)=\frac{15}{37}$$
$$P(01)=P(10)=\frac{6}{37}$$
$$P(11)=\frac{5}{3}P(10)=\frac{5}{3}\cdot\frac{6}{37}=\frac{10}{37}$$

$$P(0)=P(0/00)P(00)+P(0/01)P(01)+P(0/10)P(10)+P(0/11)P(11)$$
$$P(0)=0{,}8\cdot\frac{15}{37}+0{,}5\cdot\frac{6}{37}+0{,}5\cdot\frac{6}{37}+0{,}3\cdot\frac{10}{37}=\frac{21}{37}$$
$$P(1)=1-P(0)=\frac{16}{37}$$

$\left[\text{ili } P(0)=P(00)+\frac{1}{2}\left[P(01)+P(10)\right]\right]$

ONO SU STANJA, A NE SIMBOLI IZVORA!

Pridruženi izvor ($\bar{S}$)

		P_i
stanje ~~s_1~~	00	15/37
stanje ~~s_2~~	01	6/37
stanje ~~s_3~~	10	6/37
stanje ~~s_4~~	11	10/37

PAZI!

$$\cancel{H(\bar{S}) = \sum_{i=1} P_i \operatorname{ld} \frac{1}{P_i} = 1{,}8894 \ \frac{Sh}{simb}}$$

$s_1 - 0$

$s_2 - 1$

$$\Rightarrow \bar{S} = \left\{ \begin{array}{l} P(0) = \frac{21}{37} \\ P(1) = 16/37 \end{array} \right\} \Rightarrow H(\bar{S}) = 0{,}9751 \ \frac{Sh}{simb}$$

$$H(^mS) = \sum_{S^{m+1}} P(s_{i_1}, s_{i_2}, \ldots s_{i_m}, s_j) \operatorname{ld} \frac{1}{P(s_j | s_{i_1}, s_{i_2}, \ldots, s_{i_m})}$$

$$H(^mS) = H(^2S) = \sum_{i=0}^{1} \sum_{j=0}^{1} \sum_{k=0}^{1} P(s_i, s_j, s_k) \operatorname{ld} \frac{1}{P(s_k | s_i, s_j)}$$

$$H(^2S) = \sum_{i=0}^{1} \sum_{j=0}^{1} \sum_{k=0}^{1} P(s_k | s_i, s_j) \, P(s_i, s_j) \operatorname{ld} \frac{1}{P(s_k | s_i, s_j)}$$

$$H(^2S) = P(0|00) P(00) \operatorname{ld} \frac{1}{P(0|00)} + P(1|00) P(00) \operatorname{ld} \frac{1}{P(1|00)} +$$

$$+ P(0|01) P(01) \operatorname{ld} \frac{1}{P(0|01)} + P(1|01) P(01) \operatorname{ld} \frac{1}{P(1|01)}$$

$$+ P(0|10) P(10) \operatorname{ld} \frac{1}{P(0|10)} + P(1|10) P(10) \operatorname{ld} \frac{1}{P(1|10)}$$

$$+ P(0|11) P(11) \operatorname{ld} \frac{1}{P(0|11)} + P(1|11) P(11) \operatorname{ld} \frac{1}{P(1|11)}$$

$$H(^2S) = 0{,}8552 \ \frac{Sh}{simb.}$$

$$\Delta H = H(\bar{S}) - H(S) \cong 0{,}12 \ \frac{Sh}{simb}$$

Ako bi se kodovalo Hafmenom (n-to proširenje) moglo bi da se priđe entropiji $H(\bar{S})$ ali nikad $H(S)$

$$H(\bar{S}) \Rightarrow \lim_{n \to \infty} \frac{L_n}{n} \cong H(\bar{S})$$

tj. ostala bi ΔH neiskorišćena !!!.