5) Za prenos 8 jednakoverovatnih poruka kroz kanal sa šumom koristi se Hamingov zaštitni koder. (12)

a) Izabrati parametre odgovarajućeg Hamingovog koda tako da se omogući korekcija 1 greške

b) Odrediti matrice G i H traženog koda

c) Odrediti sve kodne reči i spektar koda

d) Ako je pri prenosu proizvoljno izabrane kodne reči došlo do greške na 3. bitu, pokazati kako se u prijemniku vrši korekcija greške

Rešenje:

a) 8 poruka $\Rightarrow$ 3b $\Rightarrow \boxed{k=3}$

$$2^m \geq 1+n = 1+m+k$$

$$2^m \geq m+3 \Rightarrow \boxed{m=3} \quad (2^3 \geq 6)$$

$\Rightarrow (6,3)$

$$\boxed{n = m+k = 6}$$

b)

	t_3	t_2	t_1	
1	0	0	$\boxed{1}$	t_1
2	0	$\boxed{1}$	0	t_2
3	0	1	1	i_1
4	$\boxed{1}$	0	0	t_3
5	1	0	1	i_2
6	1	1	0	i_3
~~7~~	~~1~~	~~1~~	~~1~~	~~t_4~~

info reč: $i_1 i_2 i_3$

kodna reč:

$x_1\ x_2\ x_3\ x_4\ x_5\ x_6$

$t_1\ t_2\ i_1\ t_3\ i_2\ i_3$

$t_1 = i_1 + i_2$

$t_2 = i_1 + i_3$

$t_3 = i_2 + i_3$

$s_1 = y_1 + y_3 + y_5$

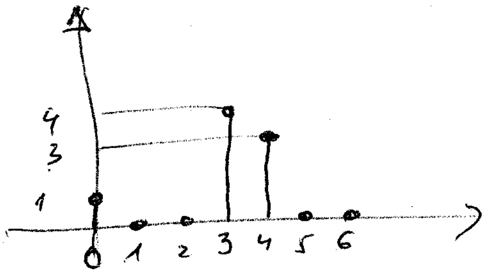
$s_2 = y_2 + y_3 + y_6$

$s_3 = y_4 + y_5 + y_6$

$$G = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \qquad H = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

c)

$$X = I\,G = \begin{bmatrix} 000 \\ 001 \\ 010 \\ 011 \\ 100 \\ 101 \\ 110 \\ 111 \end{bmatrix} \begin{bmatrix} 1&1&1&0&0&0 \\ 1&0&0&1&1&0 \\ 0&1&0&1&0&1 \end{bmatrix} = \begin{array}{cc} \begin{bmatrix} 0&0&0&0&0&0 \\ 0&1&0&1&0&1 \\ 1&0&0&1&1&0 \\ 1&1&0&0&1&1 \\ \hline 1&1&1&0&0&0 \\ 1&0&1&1&0&1 \\ 0&1&1&1&1&0 \\ 0&0&1&0&1&1 \end{bmatrix} & \begin{matrix} -0 \\ -3 \\ -3 \\ -4 \\ -3 \\ -4 \\ -4 \\ -3 \end{matrix} \end{array}$$



d)

$i = 100$

$x = 111000$

$e = 001000$

$y = x + e = 110000$

$S = yH^T = [110] \Rightarrow S_{321} = 011 = 3$

$\hat{x} = 11\boxed{1}0\boxed{00}$

$\hat{i} = 100$

4.5. a) Izvršiti dekodovanje sekvence [0000 0001 0001] ako se zna da je na predaji primenjen kod za korekciju i detekciju jedne greške.

b) Izvršiti dekodovanje sekvenci [0010 0100 0001] i [1011 1100 0011] ako se zna da je na predaji primenjen Hamingov (12, 7) kod.

---

a) $n = 12$
$k = ?$

$(11,7) \xrightarrow{\text{ili}} \text{(proširenje } z_4) \rightarrow (12,7) \text{ kod} \rightarrow e_c = 1, \ e_d = 2$

$(15,11) \rightarrow \text{skraćenje za 3 bita} \rightarrow \boxed{(12,8) \text{ kod} \rightarrow e_d = e_c = 1}$ (sve zavisi)

skraćeni (15,11) kod

$z_4 z_3 z_2 z_1$	
000$\boxed{1}$	$x_1 = z_1$
00$\boxed{1}$0	$x_2 = z_2$
0011	$x_3 = i_1$
0$\boxed{1}$00	$x_4 = z_3$
0101	$x_5 = i_2$
0110	$x_6 = i_3$
0111	$x_7 = i_4$
$\boxed{1}$000	$x_8 = z_4$
1001	$x_9 = i_5$
1010	$x_{10} = i_6$
1011	$x_{11} = i_7$
1100	$x_{12} = i_8$

$z_1 = \ldots$
$z_2 = \ldots$
$z_3 = \ldots$
$z_4 = \ldots$
} nije potrebno

$S_1 = y_1 \oplus y_3 \oplus y_5 \oplus y_7 \oplus y_9 \oplus y_{11} = 0$

$S_2 = y_2 \oplus y_3 \oplus y_6 \oplus y_7 \oplus y_{10} \oplus y_{11} = 0$

$S_3 = y_4 \oplus y_5 \oplus y_6 \oplus y_7 \oplus y_{12} = 1$

$S_4 = y_8 \oplus y_9 \oplus y_{10} \oplus y_{11} \oplus y_{12} = 0$

$y = [000\boxed{0}\ 0001\ 0001]$ (y_8, y_{12})

$S_{4321} = 0100_2 = 4$ — greška

$\hat{x} = [0001\ 0001\ 0001]$

$\hat{i} = [0000\ 0001]$

b)

(11,7) || (15,7) SKRAĆEN ZA 3 bita

$z_4 z_3 z_2 z_1$	
$000\boxed{1}$	$x_1 = z_1$
$00\,1\,0$	$x_2 = z_2$
0011	$x_3 = i_1$
$0\boxed{1}00$	$x_4 = z_3$
0101	$x_5 = i_2$
0110	$x_6 = i_3$
0111	$x_7 = i_4$
$\boxed{1}000$	$x_8 = z_4$
1001	$x_9 = i_5$
1010	$x_{10} = i_6$
1011	$x_{11} = i_7$
	$x_{12} = z_5 = \sum_{i=1}^{11} x_i$

mod 11 bita (11,7)
$$\begin{cases} S_1 = y_1 \oplus y_3 \oplus y_5 \oplus y_7 \oplus y_9 \oplus y_{11} \\ S_2 = \text{— — —} \\ S_3 = \text{— — —} \\ S_4 = \text{— — —} \end{cases}$$

dodatni $\left\{ S_5 = \sum_{i=1}^{12} y_i \right.$

H za (11,7)

$$\begin{array}{c} \\ H = \end{array} \left[\begin{array}{ccccccccccc|c} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ \hline 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right] \begin{array}{l} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{array}$$

$y = [0010\ 0100\ 0001]$

$s = yH^T = \underset{s_1 s_2 s_3 s_4 s_5}{[1\,0\,1\,0\,1]}$ $\quad \left.\begin{matrix} s_5 = 1 \\ s_{4321} = 0101_2 = 5 \end{matrix}\right\}$ jedna greška na poziciji 5

$\hat{x} = [0010\,\boxed{1}100\ 0001]$

$\hat{i} = [111\,0000]$

$y = [1011\ 1100\ 0011]$

$s = yH^T = [\underset{s_1}{0}111\underset{s_5}{1}] \Rightarrow \left.\begin{matrix} s_5 = 1 \\ s_{4321} = 1110_2 = 14 \end{matrix}\right\}$ jedna greška na poziciji 14 ???

Ovo nije moguće jer je $n = 12 < 14$ pa je zapravo bila trostruka greška!

4) U telekomunikacionom sistemu mogu se otkrivati impulsne smetnje trajanja $\tau = 1{,}5\,ms$ sa periodom $T_p = 22{,}5\,ms$. Binarni protok na izlazu sistema je $V_b = 2000\,b/s$. 9

a) Odrediti ~~stepen~~ minimalni stepen interlivinga

b) Izvršiti izbor i konstruisati odgovarajućeg Hemingovog koda koji omogućava korekciju jedne greške.

c) Izvršiti kodovanje proizvoljne sekvence informacionih bita. Pokazati kako se postupkom interlivinga i deinterlivinga mogu eliminisati paketi grešaka a)

$$T_b = \frac{1}{V_b} = \frac{1}{2000} = 0{,}5\,ms$$

$$T_p = 22{,}5\,ms \Rightarrow \frac{22{,}5}{0{,}5} = 45 \text{ bita}$$

$$I = 1{,}5\,ms \Rightarrow \frac{1{,}5}{0{,}5} = 3 \text{ bita} \Rightarrow$$ minimalni stepen interlivinga $\boxed{l = 3}$

b)

$$\frac{45}{3} = 15 \Rightarrow n \le 15 \Rightarrow \boxed{n = 15}$$

$$2^{n-k} \ge 1 + n \quad (1 \text{ GREŠKA})$$

$$2^m \ge 16 \Rightarrow \boxed{m = 4} \Rightarrow \boxed{k = n - m = 11}$$

$\Rightarrow$ kod $(15, 11)$

	$z_4 z_3 z_2 z_1$	
1	0 0 0 $\boxed{1}$	z_1
2	0 0 $\boxed{1}$ 0	z_2
3	0 0 1 1	i_1
4	0 $\boxed{1}$ 0 0	z_3
5	0 1 0 1	i_2
6	0 1 1 0	i_3
7	0 1 1 1	i_4
8	$\boxed{1}$ 0 0 0	z_4
9	1 0 0 1	i_5
10	1 0 1 0	i_6
11	1 0 1 1	i_7
12	1 1 0 0	i_8
13	1 1 0 1	i_9
14	1 1 1 0	i_{10}
15	1 1 1 1	i_{11}

$$z_1 = i_1 + i_2 + i_4 + i_5 + i_7 + i_9 + i_{11}$$

$$z_2 = i_1 + i_3 + i_4 + i_6 + i_7 + i_{10} + i_{11}$$

$$z_3 = i_2 + i_3 + i_4 + i_8 + i_9 + i_{10} + i_{11}$$

$$z_4 = i_5 + i_6 + i_7 + i_8 + i_9 + i_{10} + i_{11}$$

11

c) $\overbrace{0000\ 0000\ 000}^{11}$ | $\overbrace{1111\ 1111\ 111}^{11}$ | $\overbrace{0000\ 0000\ 001}^{11}$ | . . . . .

$z_1=z_2=z_3=z_4=0$ | $z_1=z_2=z_3=z_4=1$ | $z_1=z_2=1$, $z_3=z_4=1$

kod: $\underbrace{0000\ 0000\ 0000\ 000}_{15}$ | $\underbrace{1111\ \boxed{1111}\ 1111\ 111}_{15}$ | $\underbrace{1101\ 0001\ 0000\ 001}_{15}$ |

interliver

→ 01$\boxed{1}$|$\boxed{01}$1|010|011|010|010|010| . . . . |011

greške paket 3 bite

prijem 0$\boxed{10}$|$\boxed{1}$01| . . . . |011

deinterliver

y: 0 $\boxed{1}$ 00 0000 0000 000 ⇒ $S_4S_3S_2S_1$ = 0010 = 2

1 $\boxed{0}$ 11 1111 1111 111 ⇒ S_{4321} = 0010 = 2

$\boxed{0}$ 101 0001 0000 001 ⇒ S_{4321} = 0001 = 1

⇒ 0000 0000 0000 000

⇒ 1111 1111 1111 111

⇒ 1101 0001 0000 001

($z_1\ z_2$, z_3, z_4)

⇓

0000 0000 000
1111 1111 111
0000 0000 001

$S_1 = y_1+y_3+y_5+y_7+y_9+y_{11}+y_{13}+y_{15}$

$S_2 = y_2+y_3+y_6+y_7+y_{10}+y_{11}+y_{14}+y_{15}$

$S_3 = y_4+y_5+y_6+y_7+y_{12}+y_{13}+y_{14}+y_{15}$

$S_4 = y_8+y_9+y_{10}+y_{11}+y_{12}+y_{13}+y_{14}+y_{15}$

$$2^{n-k} \geq \sum_{i=1}^{ec}\binom{n}{i} = 1+\binom{n}{1}\binom{n}{2} = 1+n+\frac{n(n-1)}{2\cdot 1}$$

$$2^{15-k} \geq 1+15+\frac{15\cdot \cancel{14}^{7}}{\cancel{2}} = 121$$

$$2^7 = 128 \geq 121 \Rightarrow \boxed{n=7}\ \boxed{k=8}\ \boxed{n=15}$$

za ~~KOREK~~ korekciju 2 greške

(15, 8)

V)

$\tau = 2{,}5\ ms$

$T_p = 13{,}5\ ms$

$v_b = 2000\ b/s$

$T_b = 0{,}5\ ms$

a) $\dfrac{T_p}{T_b} = \dfrac{22{,}5}{0{,}5} = 27$

$\dfrac{\tau}{T_b} = \dfrac{2{,}5}{0{,}5} = 3 \Rightarrow l \geq 3$

$\boxed{l = 3}$

b) $\dfrac{45}{5} = \boxed{9 \text{ bita} = n}$

$n = 9 \quad k = ? \qquad 2^m \geq 1 + n = 10$

$\boxed{m = 4} \Rightarrow k = 9 - 4 = 5 \Rightarrow (9,5)$ kod

$(15,11) \xrightarrow[\text{6 bita}]{\text{skrati za}} (9,5)$

$H^T =$ ili obrnuto

$z_4 z_3 z_2 z_1$	
000$\boxed{1}$	$x_1 = z_1$
00$\boxed{1}$0	$x_2 = z_2$
0011	$x_3 = i_1$
0$\boxed{1}$00	$x_4 = z_3$
0101	$x_5 = i_2$
0110	$x_6 = i_3$
0111	$x_7 = i_4$
$\boxed{1}$000	$x_8 = z_4$
1001	$x_9 = i_5$

$z_1 = i_1 \oplus i_2 \oplus i_4 \oplus i_5 = 1$

$z_2 = i_1 \oplus i_3 \oplus i_4 = 0$

$z_3 = i_2 \oplus i_3 \oplus i_4 = 0$

$z_4 = i_5 = 1$

$i = [00001]$

$x = [100\ 000\ 011]$

c) $\underbrace{[00001]}_{i}\underbrace{[00001]}_{i}\underbrace{[00001]}_{i}$

↓ kod

[100 000 011][100 000 011][100 000 011] ← memorija $(n \times l)$ bita

↓ interliver

GREŠKA

[111 000 000|000 $\boxed{000}$|000|000 111 111]

↓ KANAL SA IMPULSNOM GREŠKOM dužine 3 bita

[111 000 000|000 $\boxed{111}$|000|000 111 111]

↓ deinterliver

[100 0$\boxed{1}$0 011][100 0$\boxed{1}$0 011][100 0$\boxed{1}$0 011] ← greška se rasporedila u 3 kodne reči

$y = [100\ 010\ 011] \Rightarrow S = yH^T = [1010]$

$S_{4321} = 0101_2 = 5$

$\hat{x} = [100\ 000\ 011]$

$\hat{i} = [00001]$