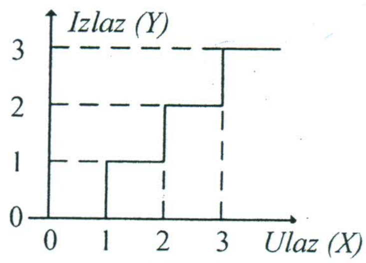
Statistička teorija telekomunikacija, 1. kolokvijum **30.11.2018.**

1. *(15 poena)*

Slučajna promenljiva X ima Gausou raspodelu srednje vrednosti 2 i varijanse 4. Slučajna promenljiva X transformiše se u slučajnu promenljivu Y prolaskom kroz kvantizer čija je funkcija prenosa prikazana na slici.

a) (*9p*) Odrediti i grafički predstaviti funkciju gustine verovatnoće slučajne promenljive Y na izlazu kvantizera.

b) (*6p*) Odrediti srednju vrednost i varijansu slučajne promenljive Y.



Slika 1

2. *(15 poena)*

Posmatra se naponski signal sinusoidalnog talasnog oblika sa slučajnom fazom, definisan izrazom: $X(t) = 5 - 5sin\,(\omega_0 t + \Theta)$, pri čemu je ω_0 konstanta. Faza Θ ovog slučajnog signala ima uniformnu raspodelu u intervalu $[-\pi, \pi]$.

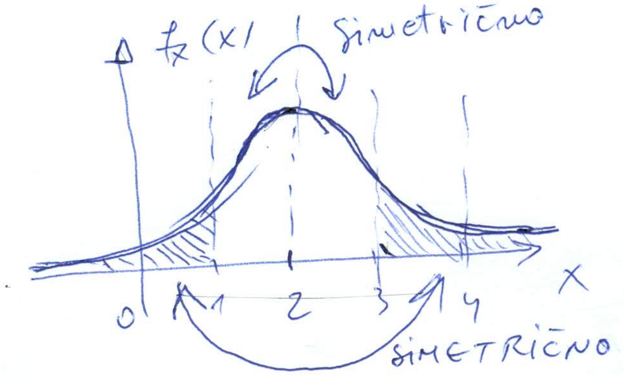
a) (*10p*) Odrediti i grafički prikazati autokorelacionu funkciju $R_X(\tau)$ slučajnog signala $X(t)$.

b) (*5p*) Odrediti srednju snagu jednosmerne komponente, naizmenične komponente i ukupnu srednju snagu slučajnog signala $X(t)$ (na otpornosti od 1Ω).

<u>NAPOMENA</u>: Dozvoljeno je korišćenje samo pribora za pisanje i neprogramabilnog džepnog kalkulatora. Kolokvijum traje 90 minuta. Nije dozvoljeno napuštanje kolokvijuma tokom prvih 60 minuta. Nije dozvoljeno iznošenje zadatka do kraja kolokvijuma.

1

a)



$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$\mu = 2$

$\sigma^2 = 4 \Rightarrow \sigma = \sqrt{4} = 2$

$P(Y=3) = P(Y=0)$ jer je zbog simetrije $P(Y=3) = P(X \geq 3) =$
$= P(Y=0) = P(X \leq 1)$

$$P(X \geq 3) = \int_3^{\infty} \frac{1}{\sqrt{\pi 4}} e^{-\frac{(x-2)^2}{2\cdot 4}} dx = \frac{1}{2\sqrt{2\pi}} \int_3^{\infty} e^{-\frac{(x-2)^2}{8}} dx$$

smena: $\frac{x-2}{\sqrt{8}} = u \Rightarrow$ granice: $x=3 \Rightarrow u = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}$

$x \to \infty \Rightarrow u \to \infty$

$dx = 2\sqrt{2}\, du$

$$P(X \geq 3) = \frac{2\sqrt{2}}{2\sqrt{2\pi}} \int_{\frac{1}{2\sqrt{2}}}^{\infty} e^{-u^2} du = \frac{1}{2} \cdot \frac{2}{\sqrt{\pi}} \int_{\frac{1}{2\sqrt{2}}}^{\infty} e^{-u^2} du = \frac{1}{2} \operatorname{erfc}\left(\frac{1}{2\sqrt{2}}\right)$$

$$P(X \geq 3) \simeq \frac{1}{2} \operatorname{erfc}(0{,}35) = \frac{1}{2}(1 - \operatorname{erf}(0{,}35)) = \frac{1}{2}(1 - 0{,}37938)$$

$P(X \geq 3) \approx 0{,}31 \Rightarrow \boxed{P(Y=0) = P(Y=3) = 0{,}31}$

Takođe zbog simetrije važi da je:

$P(Y=1) = P(1 \leq X \leq 2) = P(2 \leq x \leq 3) = P(Y=2)$

$\sum P(Y=y_i) = 1 \Rightarrow P(Y=0) + P(Y=1) + P(Y=2) + P(Y=3) = 1$

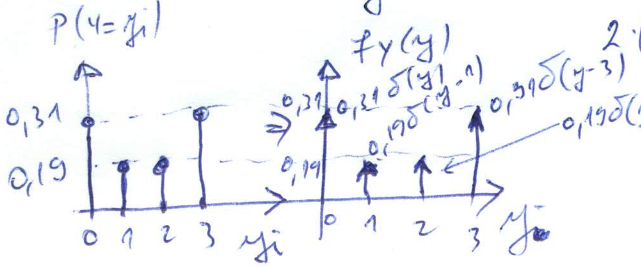
$2 \cdot (P(Y=0) + P(Y=1)) = 1$

$P(Y=0) + P(Y=1) = 1/2$

$P(Y=1) = 1/2 - P(Y=0)$

$\boxed{P(Y=1) = 0{,}19}$

$\boxed{P(Y=2) = 0{,}19}$



b)

$$\overline{Y} = \sum_{i=1}^{4} y_i P(Y=y_i) = 0 \cdot P(Y=0) + 1 \cdot P(Y=1) + 2 \cdot P(Y=2) + 3 \cdot P(Y=3)$$
$$= 0{,}19 + 2 \cdot 0{,}19 + 3 \cdot 0{,}31 = 1{,}5$$

$$\overline{Y^2} = \sum_{i=1}^{4} y_i^2 P(Y=y_i) = 0^2 \cdot P(Y=0) + 1^2 \cdot P(Y=1) + 2^2 \cdot P(Y=2) + 3^2 \cdot P(Y=3)$$
$$= 0{,}19 + 4 \cdot 0{,}19 + 9 \cdot 0{,}31 = 3{,}74$$

$\boxed{Var(Y) = \overline{Y^2} - (\overline{Y})^2 = 1{,}49}$

② $x(t) = 5 - 5\sin(\omega_0 t + \theta)$ $\qquad f_\theta(\theta) = \begin{cases} \frac{1}{2\pi}, & -\pi \le \theta \le \pi \\ 0, & \text{inače} \end{cases}$

a)

$$R_x(\tau) = \overline{x(t)\cdot x(t+\tau)} = \int_{-\pi}^{\pi} [5 - 5\sin(\omega_0 t + \theta)][5 - 5\sin(\omega_0(t+\tau)+\theta)] \underbrace{\frac{1}{2\pi}}_{f_\theta(\theta)} d\theta$$

$$R_x(\tau) = \int_{-\pi}^{\pi} 25\frac{1}{2\pi}d\theta + \underbrace{\int_{-\pi}^{\pi}\frac{(-25)}{2\pi}\sin(\omega_0 t+\theta)\,d\theta}_{0} + \underbrace{\int_{-\pi}^{\pi}\frac{(-25)}{2\pi}\sin(\omega_0(t+\tau)+\theta)\,d\theta}_{0}$$

$$+ \int_{-\pi}^{\pi}\frac{25}{2\pi}\sin\underbrace{(\omega_0 t+\theta)}_{\alpha}\sin\underbrace{(\omega_0(t+\tau)+\theta)}_{\beta}\,d\theta$$

$$R_x(\tau) = 25\cdot\frac{1}{2\pi}\underbrace{\theta\Big|_{-\pi}^{\pi}}_{2\pi} + 0 + 0 + \frac{25}{2\cdot 2\pi}\int_{-\pi}^{\pi}[\cos(\omega_0\tau) - \cos(\omega_0(2t+\tau)+2\theta)]\,d\theta$$

$$R_x(\tau) = 25 + \frac{25}{4\pi}\int_{-\pi}^{\pi}\cos(\omega_0\tau)\,d\theta - \underbrace{\frac{25}{4\pi}\int_{-\pi}^{\pi}\cos(\omega_0(2t+\tau)+2\theta)\,d\theta}_{0}$$

$$R_x(\tau) = 25 + \frac{25}{4\pi}\cos(\omega_0\tau)\underbrace{\int_{-\pi}^{\pi}d\theta}_{2\pi}$$

$$\boxed{R_x(\tau) = 25 + \frac{25}{2}\cos(\omega_0\tau)}$$

b) $P_{DC} = 25\,W \qquad P_{UK} = R_x(\tau=0) = 25 + \frac{25}{2} = 37{,}5\,W$

$P_{AC} = \frac{25}{2} = 12{,}5\,W \qquad (P_{UK} = P_{AC} + P_{DC})$

$\omega_0\tau = 2\pi$

$\tau = \frac{2\pi}{\omega_0}$