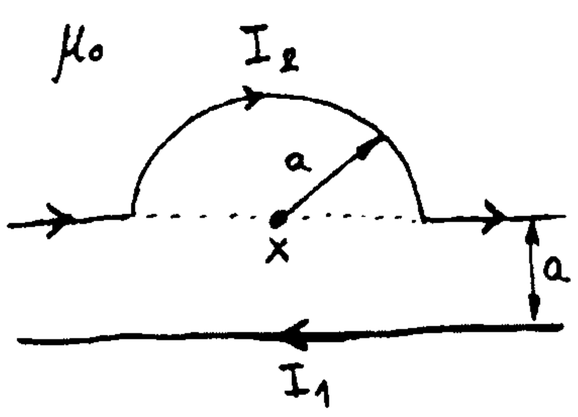
DRUGI KOLOKVIJUM IZ ELEKTROTEHNIKE
29.12.2014.

GRUPA 1

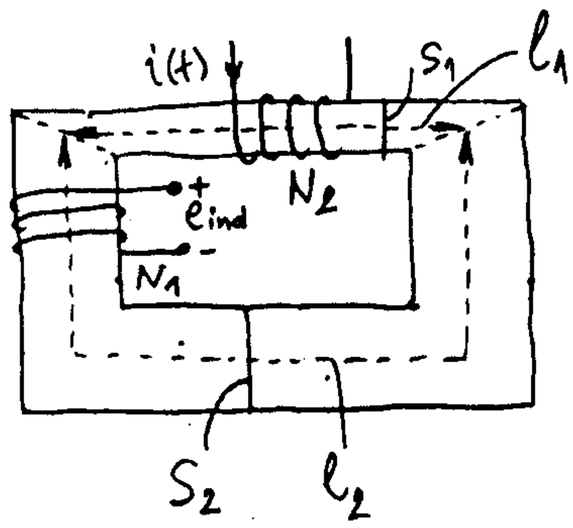
1. U istoj ravni u vazduhu nalaze se dva beskonačno dugačka provodnika, na međusobnom rastojanju a, kroz koje protiču struje intenziteta I_1 i I_2 (Slika 1). Na provodniku sa strujom I_2 se nalazi polukružni segment poluprečnika a. Odrediti vektor magnetne indukcije u tački X, koja se nalazi u centru polukružnog segmenta. **(5 poena)**

2. U kolu na Slici 2 prikazano je magnetno kolo sa dva namotaja. Namotaj sa N_1 navojaka je otvorenih krajeva, a kroz namotaj sa N_2 navojaka protiče struja intenziteta $i(t) = I_m \cos(\omega t)$. Jezgro je homogeno i sastoji se iz dva dela površina poprečnog preseka S_1 i S_2, čije dužine srednjih linija iznose l_1 i l_2. Relativna magnetna permeabilnost jezgra iznosi μ_r.

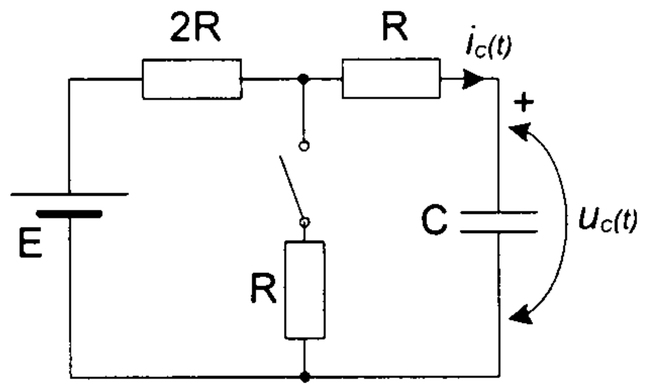
a) Odrediti izraz za fluks vektora magnetne indukcije u jezgru. **(3 poena)**
b) Odrediti izraz za induktivnost namotaja sa N_2 navojaka. **(2 poena)**
c) Odrediti izraz za elektromotornu silu indukovanu na krajevima namotaja sa N_1 navojaka. **(3 poena)**



Slika 1



Slika 2



Slika 3

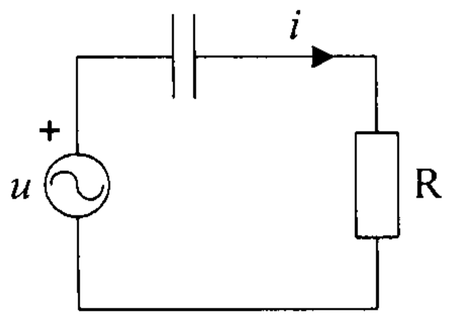
3. U kolu na Slici 3 poznato je E, R, i C. Prekidač je zatvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se otvara. Odrediti izraz za napon kondenzatora $u_c(t)$ i intenzitet struje $i_c(t)$ nakon otvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame. Odrediti minimalnu i maksimalnu vrednost energije električnog polja kondenzatora u toku prelaznog procesa. **(7 poena)**

4. Dva prijemnika vezana su paralelno i priključena na naizmenični napon efektivne vrednosti $U = 300\,\mathrm{V}$. Kompleksna impedansa prvog prijemnika iznosi $\overline{Z}_1 = 50 + j50\,\Omega$. Drugi prijemnik ima reaktivnu snagu $Q_2 = -600\,\mathrm{var}$ i prividnu snagu $S_2 = 750\,\mathrm{VA}$.

a) Odrediti aktivnu snagu i faktor snage drugog prijemnika. **(1 poen)**
b) Odretiti efektivne vrednosti struja I_1 i I_2 u prijemnicima. **(2 poena)**
c) Odrediti ukupnu aktivnu, ukupnu reaktivnu i ukupnu kompleksnu prividnu snagu celokupnog potrošača. **(3 poena)**

5. Na Slici 4 je prikazano kolo naizmenične struje koje se napaja naponom trenutne vrednosti: $u(t) = 20\sin(\omega t - \pi/2)\,\mathrm{V}$, gde je $\omega = 100\,\mathrm{rad/s}$. Poznate su sledeće vrednosti elemenata u kolu: $R = 1\,\Omega$, $C = 10\,\mathrm{mF}$.

a) Odrediti kompleksne izraze struje i napona na otporniku i kondenzatorū; **(3 p.)**
b) Predstaviti na fazorskom dijagramu struju i napone na svim elementima; **(1 p.)**
c) Odrediti aktivnu, reaktivnu i prividnu snagu celokupnog potrošača; **(2 p.)**
d) Odrediti trenutnu vrednost struje u kolu i napona na kondenzatoru, **(3 p.)**

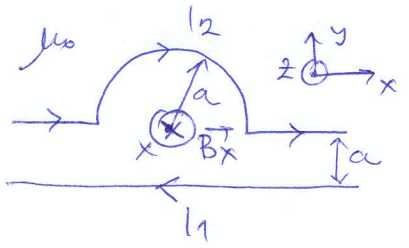


Slika 4

6. Na sistem trofaznog napona $3\times 5\,\mathrm{kV}$ priključen je trofazni potrošač povezan u trougao. Impedansa svake faze iznosi $\overline{Z}_f = 60 + j80\,\Omega$. Odrediti: efektivnu vrednost linijske struje, aktivnu, reaktivnu i prividnu snagu potrošača. **(5 poena)**

GRUPA 1

1.



$$\vec{B_x} = \vec{B_{x_1}} + \vec{B_{x_2}}$$

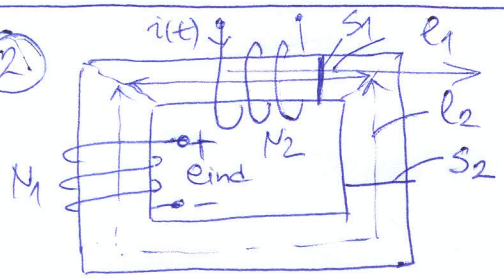
$$\vec{B_{x_2}} = \frac{1}{2}\,\frac{\mu_0 I_2}{2a}\,(-\vec{k}) = -\frac{\mu_0 I_2}{4a}\vec{k}$$

$$\vec{B_{x_1}} = \frac{\mu_0 I_1}{2\pi a}\,(-\vec{k}) = -\frac{\mu_0 I_1}{2\pi a}\vec{k}$$

$$\boxed{\vec{B_x} = -\frac{\mu_0}{4a}\left(I_2 + \frac{2I_1}{\pi}\right)\vec{k}}$$

5p

2.



$i(t) = I_m \cos \omega t$

a) $\oint \vec{H}(t)\,d\vec{l} = \Sigma I$

$$H_1(t)\,l_1 + H_2(t)\,l_2 = N_2\,i(t)$$

$$\frac{B_1(t)\,l_1}{\mu_0\mu_r} + \frac{B_2(t)\,l_2}{\mu_0\mu_r} = N_2\,i(t)$$

$$\frac{\Phi(t)\,l_1}{S_1\mu_0\mu_r} + \frac{\Phi(t)\,l_2}{S_2\mu_0\mu_r} = N_2\,i(t)$$

$$\boxed{\Phi(t) = \frac{\mu_0\mu_r N_2\,i(t)}{l_1/S_1 + l_2/S_2}}$$

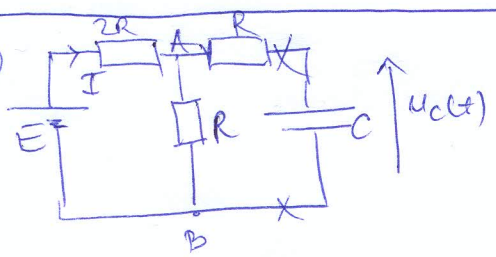
a) 3p
b) 2p
c) 3p

b) $$\boxed{L_2 = \frac{N_2\,\Phi(t)}{i(t)} = \frac{N_2^2\,\mu_0\mu_r}{l_1/S_1 + l_2/S_2}}$$

c) $$e_{ind} = -N_1\frac{d\Phi(t)}{dt} = -\frac{N_1 N_2\,\mu_0\mu_r}{l_1/S_1 + l_2/S_2}\,\frac{di(t)}{dt} = -\frac{N_1 N_2\,\mu_0\mu_r}{l_1/S_1 + l_2/S_2}\,I_m(-\sin\omega t)\,\omega$$

$$\boxed{e_{ind} = \frac{N_1 N_2\,\mu_0\mu_r\,I_m\omega}{l_1/S_1 + l_2/S_2}\sin\omega t}$$

3.



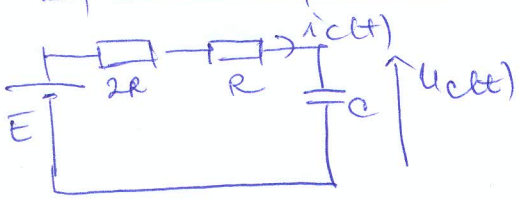
STAȚIONAR STARE:

$$I = \frac{E}{3R}$$

$$U_{C_0} = U_{AB} = R\cdot I = \frac{E}{3}$$

$$Q_{C_0} = U_{C_0}\cdot C = \frac{EC}{3}$$

PRELAZIM PRIN ÎC:



$$E - 3R\,i_c(t) - u_c(t) = 0$$

$$i_c(t) = C\frac{du_c}{dt}$$

$$E - 3RC\frac{du_c}{dt} - u_c = 0$$

$$\frac{du_c}{dt} + \frac{u_c}{3RC} = \frac{E}{3RC}$$

$$\frac{du_c}{dt} + \frac{u_c}{3RC} = \frac{E}{3RC} \qquad \tau = 3RC \qquad K = \frac{E}{3RC}$$

$$u_c(t) = Ae^{-t/\tau} + B$$

$$B = K \cdot \tau = 3RC \cdot \frac{E}{3RC} = E$$

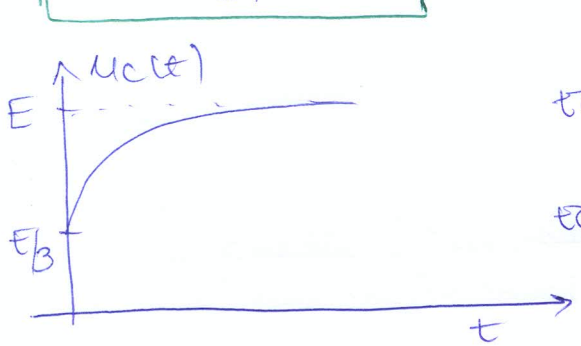
$$A + B = U_{c0} \Rightarrow A = U_{c0} - B = \frac{E}{3} - E = -\frac{2E}{3}$$

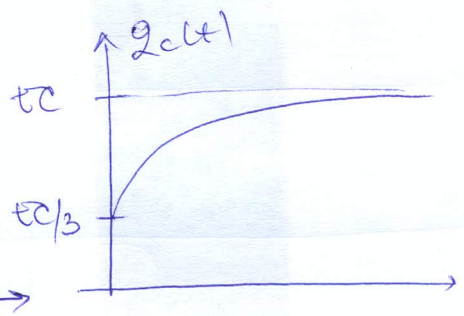
$$u_c(t) = -\frac{2E}{3}e^{-t/\tau} + E = E\left(1 - \frac{2}{3}e^{-t/\tau}\right)$$

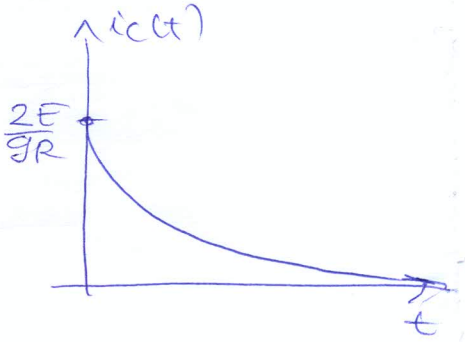
$$\boxed{u_c(t) = E\left(1 - \frac{2}{3}e^{-t/\tau}\right)} \qquad \boxed{q_c(t) = u_c(t) \cdot C = CE\left(1 - \frac{2}{3}e^{-t/\tau}\right)}$$

$$i_c(t) = C\frac{du_c(t)}{dt} = CE\left(-\frac{2}{3} \cdot \frac{-1}{\tau}e^{-t/\tau}\right) = \frac{2CE}{3\tau}e^{-t/\tau} = \frac{2CE}{9RC}e^{-t/\tau}$$

$$\boxed{i_c(t) = \frac{2E}{9R}e^{-t/\tau}}$$







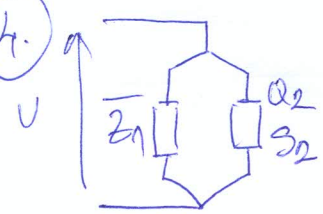
$$W_c(t) = \frac{1}{2}Cu_c^2(t)$$

$$W_{cMIN} = \frac{1}{2}Cu_{cMIN}^2(t) = \frac{1}{2}C \cdot \frac{E^2}{9} = \boxed{\frac{CE^2}{18}}$$

$$W_{cMAX} = \frac{1}{2}Cu_{cMAX}^2(t) = \boxed{\frac{1}{2}CE^2}$$

7p

4.



a) $P_2 = \sqrt{S_2^2 - Q_2^2} = \sqrt{750^2 - (-600)^2} = \boxed{450\,W}$

$$\cos\varphi_2 = \frac{P_2}{S_2} = \frac{450}{750} = \boxed{0{,}6}$$

b) $Z_1 = \sqrt{50^2 + 50^2} = 50\sqrt{2}\,\Omega$

$$I_1 = \frac{U}{Z_1} = \frac{300}{50\sqrt{2}} = \boxed{3\sqrt{2}\,A}$$

$$I_2 = \frac{S_2}{U} = \frac{750}{300} = \boxed{2{,}5\,A}$$

c) $P_1 = R_1 I_1^2 = 50 \cdot 9 \cdot 2 = 900\,W$

$P = P_1 + P_2 = \boxed{1350\,W}$

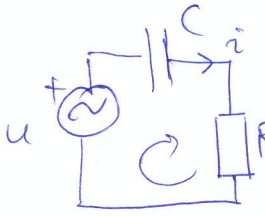
$Q_1 = X_1 I_1^2 = 50 \cdot 9 \cdot 2 = 900\,VAr$

$Q = Q_1 + Q_2 = \boxed{300\,VAr}$

$\boxed{\underline{S} = P + jQ = (1350 + j300)\,VA}$

a) 1p
b) 2p
c) 3p

5.



$u(t) = 20\sin(\omega t - \pi/2)$ [V] , $\omega = 100$ rad/s

a) $\bar{U} = \frac{20}{\sqrt{2}} e^{-j\pi/2} = -j10\sqrt{2}$ V

$\bar{Z}_C = -j\frac{1}{\omega C} = -j\frac{1}{100 \cdot 10 \cdot 10^{-3}} = -j\Omega$

$\bar{Z}_R = R = 1\Omega$

$\bar{U} - \bar{Z}_C\bar{I} - \bar{Z}_R\bar{I} = 0 \Rightarrow \bar{I} = \frac{\bar{U}}{\bar{Z}_C + \bar{Z}_R} = \frac{-j10\sqrt{2}}{1-j} \cdot \frac{1+j}{1+j}$

$\bar{I} = \frac{+10\sqrt{2}(-j-j^2)}{1-j^2} = \frac{10\sqrt{2}(1-j)}{2} = 5\sqrt{2}(1-j)$ A

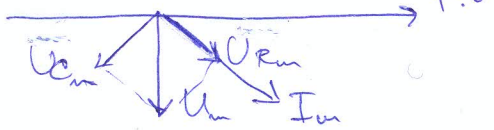
$\bar{I}_R = \bar{I}_C = \bar{I} = 5\sqrt{2}(1-j)$ A

$\bar{U}_R = \bar{Z}_R \cdot \bar{I} = 5\sqrt{2}(1-j)$ V

$\bar{U}_C = \bar{Z}_C \cdot \bar{I} = -j5\sqrt{2}(1-j) = 5\sqrt{2}(-j+j^2) = 5\sqrt{2}(-1-j)$

$\bar{U}_C = 5\sqrt{2}(-1-j)$ V

b)



c) $\bar{S} = \bar{U} \cdot \bar{I}^*$

$\bar{S} = -j10\sqrt{2} \cdot 5\sqrt{2}(1+j)$

$\bar{S} = 100(-j-j^2) = 100(1-j)$

$P = 100$ W | $Q = -100$ VAr

$S = 100\sqrt{2}$ VA

d) $i(t) = 10\sqrt{2}\sin(\omega t - \pi/4)$ [A] $\omega = 100$ rad/s

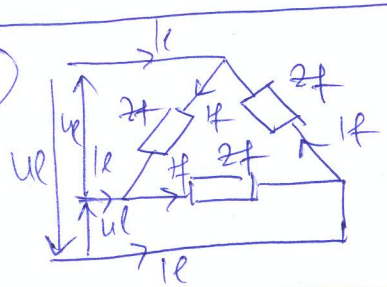
$I = 5\sqrt{2} \cdot \sqrt{2} = 10$A $\Rightarrow$ $I_m = 10\sqrt{2}$ $\quad \psi = -\pi/4$

$U_C = 5\sqrt{2} \cdot \sqrt{2} = 10$V $\Rightarrow U_{Cm} = 10\sqrt{2}$ $\quad \theta_C = -3\pi/4$

$u_C(t) = 10\sqrt{2}\sin(\omega t - 3\pi/4)$ [V] $\quad \omega = 100$ rad/s

a) 3p
b) 1p
c) 2p
d) 3p

6.



3×5kV $\Rightarrow$ $U_l = 5$kV $\quad \Delta \Rightarrow U_f = U_l = 5$kV

$\bar{Z}_f = (60+j80)\Omega \Rightarrow Z_f = \sqrt{60^2+80^2} = 100\Omega$

$I_f = \frac{U_f}{Z_f} = \frac{5000\text{V}}{100\Omega} = 50$A

$I_l = I_f \cdot \sqrt{3} = 50\sqrt{3} = 86{,}6\text{A} = I_l$

$\cos\varphi = \frac{R}{Z_f} = \frac{60}{100} = 0{,}6$

$\sin\varphi = \frac{X}{Z_f} = \frac{80}{100} = 0{,}8$

$P = 3U_f I_f \cos\varphi = 450$ kW

$Q = 3U_f I_f \sin\varphi = 600$ kVAr

$S = 3U_f I_f = 750$ kVA

5p.