DRUGI KOLOKVIJUM IZ ELEKTROTEHNIKE
29.12.2014.

GRUPA 2

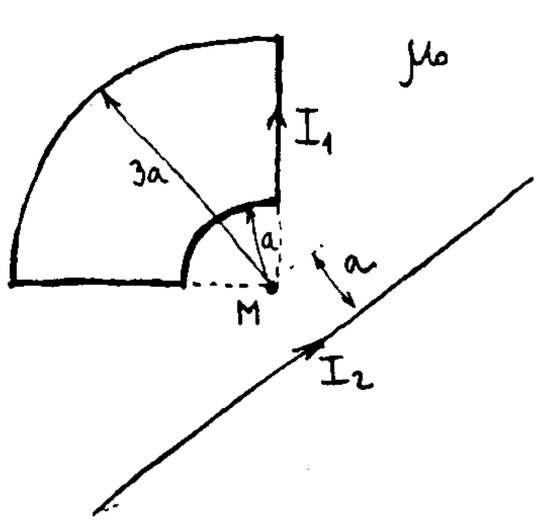
1. U istoj ravni u vazduhu nalaze se kontura kroz koju protiče struja intenziteta I_1 i beskonačno dugačak pravolinijski provodnik sa strujom I_2 (Slika 1). Kontura se sastoji od dva kružna luka (četvrtine kružnica) poluprečnika a i $3a$ i dva pravolinijska segmenta. Odrediti vektor magnetne indukcije u tački M, koja se nalazi u centru kružnih lukova, i na rastojanju a od provodnika. **(5 poena)**

2. U kolu na Slici 2 prikazano je magnetno kolo sa dva namotaja. Namotaj sa N_2 navojaka je otvorenih krajeva, a kroz namotaj sa N_1 navojaka protiče struja intenziteta $i(t) = I_m \sin(\omega t)$. Jezgro je površine poprečnog preseka S i dužine srednje linije l, i sadrži vazdušni procep debljine l_0. Magnetna permeabilnost jezgra iznosi μ.

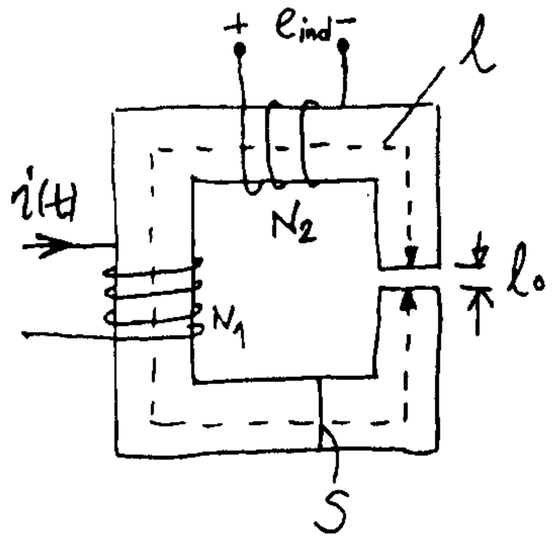
a) Odrediti izraz za fluks vektora magnetne indukcije u jezgru. **(3 poena)**

b) Odrediti izraz za induktivnost namotaja sa N_1 navojaka. **(2 poena)**

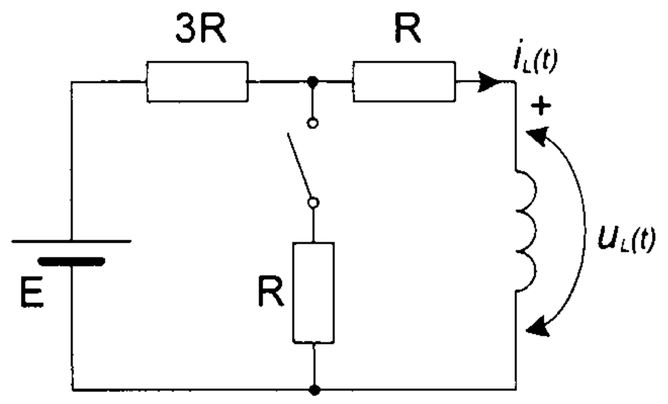
c) Odrediti izraz za elektromotornu silu indukovanu na krajevima namotaja sa N_2 navojaka. **(3 poena)**



Slika 1



Slika 2



Slika 3

3. U kolu na Slici 3 poznato je E, R, i L. Prekidač je zatvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se otvara. Odrediti izraz za intenzitet struje $i_L(t)$ i napon kalema $u_L(t)$ nakon otvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame. Odrediti minimalnu i maksimalnu vrednost energije magnetnog polja kalema u toku prelaznog procesa. **(7 poena)**

4. Dva prijemnika vezana su paralelno i priključena na naizmeničan napon efektivne vrednosti $U = 500\,\mathrm{V}$. Prvi prijemnik je pretežno kapacitivan i ima aktivnu otpornost $R_1 = 25\,\Omega$ i faktor snage $\cos\varphi_1 = \sqrt{2}/2$. Drugi prijemnik je pretežno induktivan, aktivne i prividne snage $P_2 = 600\,\mathrm{W}$ i $S_2 = 750\,\mathrm{VA}$.

a) Odrediti kompleksnu impedansu prvog prijemnika. **(1 poen)**

b) Odretiti efektivne vrednosti struja I_1 i I_2 u prijemnicima. **(2 poena)**

c) Odrediti ukupnu aktivnu, ukupnu reaktivnu i ukupnu kompleksnu prividnu snagu celokupnog potrošača. **(3 poena)**

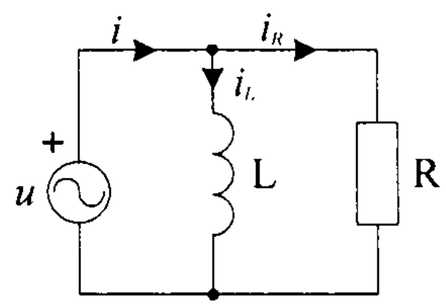
5. Na Slici 4 je prikazano kolo naizmenične struje koje se napaja naponom trenutne vrednosti: $u(t) = 20\sin(\omega t + \pi/2)\,\mathrm{V}$, gde je $\omega = 1000\,\mathrm{rad/s}$. Poznate su sledeće vrednosti elemenata u kolu: $R = 10\,\Omega$, $L = 10\,\mathrm{mH}$.

a) Odrediti kompleksne izraze za označene struje; **(3 p.)**

b) Predstaviti na fazorskom dijagramu napon generatora i struje u granama; **(1 p.)**

c) Odrediti aktivnu, reaktivnu i prividnu snagu celokupnog potrošača; **(2 p.)**

d) Odrediti trenutnu vrednost struje otpotnika i struje generatora. **(3 p.)**

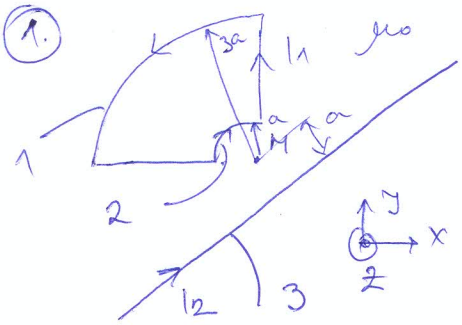


Slika 4

6. Na sistem trofaznog napona $3\times 5\,\mathrm{kV}$ priključen je trofazni potrošač povezan u zvezdu. Impedansa svake faze potrošača iznosi $\overline{Z}_f = 300 - j400\,\Omega$. Odrediti: efektivnu vrednost linijske struje, aktivnu, reaktivnu i prividnu snagu potrošača. **(5 poena)**

GRUPA 2

1.



$$\vec{B_M} = \vec{B_{M1}} + \vec{B_{M2}} + \vec{B_{M3}}$$

$$\vec{B_{M1}} = \frac{1}{4} \cdot \frac{\mu_0 I_1}{2 \cdot 3\pi} \vec{k} = \frac{\mu_0 I_1}{24a} \vec{k}$$

$$\vec{B_{M2}} = \frac{1}{4} \frac{\mu_0 I_1}{2 \cdot a} (-\vec{k}) = -\frac{\mu_0 I_1}{8a} \vec{k}$$

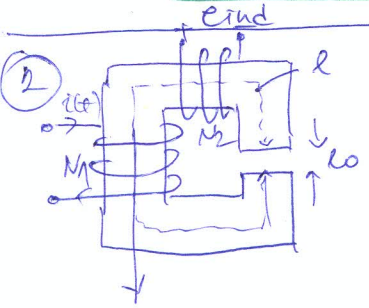
$$\vec{B_{M3}} = \frac{\mu_0 I_2}{2\pi a} \vec{k}$$

$$\vec{B_M} = \left(\frac{\mu_0 I_1}{24a} - \frac{3\mu_0 I_1}{3 \cdot 8a} + \frac{\mu_0 I_2}{2\pi a} \right) \vec{k} = \left(\frac{-2\mu_0 I_1}{24a} + \frac{\mu_0 I_2}{2\pi a} \right) \vec{k}$$

$$\vec{B_M} = \frac{\mu_0}{12a} \left(-I_1 + \frac{6 I_2}{\pi} \right) \vec{k}$$

5p

2.



a) $\oint \vec{H} d\vec{l} = \Sigma I$

$$H_1(t) l + H_0(t) l_0 = N_1 i(t)$$

$$\frac{B(t) l}{\mu} + \frac{B(t) l_0}{\mu_0} = N_1 i(t)$$

$$\frac{\phi(t) l}{S\mu} + \frac{\phi(t) l_0}{S\mu_0} = N_1 i(t)$$

$$\phi(t) = \frac{N_1 i(t) S}{l/\mu + l_0/\mu_0}$$

a) 3p
b) 2p
c) 3p

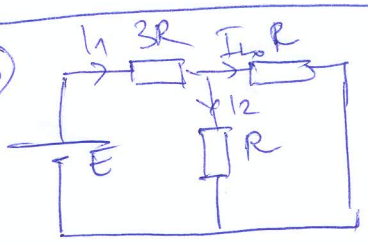
b) $L_1 = \frac{N_1 \phi(t)}{i(t)} \Rightarrow$

$$L_1 = \frac{N_1^2 S}{l/\mu + l_0/\mu_0}$$

c) $e_{ind}(t) = -\frac{N_2 d\phi(t)}{dt} = -N_2 \frac{N_1 S}{l/\mu + l_0/\mu_0} \frac{di(t)}{dt} = -\frac{N_1 N_2 S}{l/\mu + l_0/\mu_0} I_m \omega \cos \omega t$

$$e_{ind} = -\frac{N_1 N_2 S I_m \omega}{l/\mu + l_0/\mu_0} \cos \omega t$$

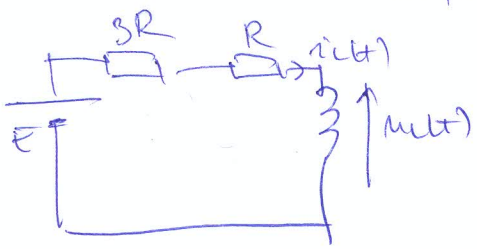
3.



CTAH. CTATBE n°3

$$I_{l_0} = I_2 = \frac{1}{2} I_1 = \frac{1}{2} \frac{E}{3R + R/2} = \frac{1}{2} \frac{2E}{7R} = \frac{E}{7R}$$

Преназим процеса по



$$E - 4R i(t) - u_L(t) = 0$$

$$u_L(t) = L \frac{di_L}{dt}$$

$$E - 4R i_L(t) - L \frac{di_L}{dt} = 0$$

$$\frac{di_L}{dt} + \frac{i_L(t)}{L/4R} = \frac{E}{L}$$

$$\frac{di_L(t)}{dt} + \frac{i_L(t)}{L/4R} = \frac{E}{L} \Rightarrow \tau = \frac{L}{4R}$$

$$K = \frac{E}{L}$$

$$i_L(t) = Ae^{-t/\tau} + B$$

$$B = K\tau = \frac{K}{4R} \cdot \frac{E}{K} = \frac{E}{4R}$$

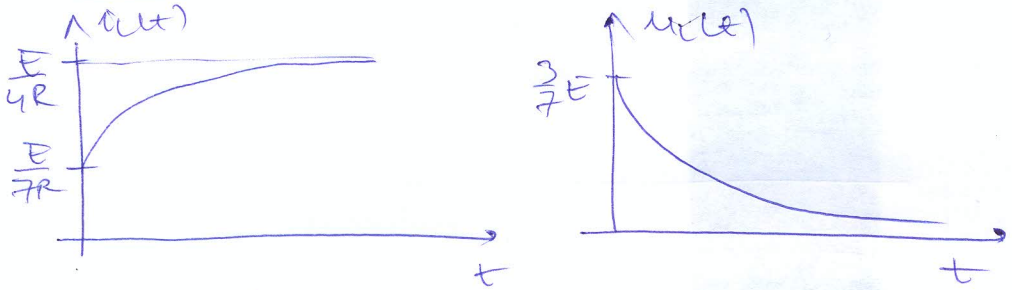
$$A + B = I_{L0} \Rightarrow A = I_{L0} - B = \frac{E}{7R} - \frac{E}{4R} = \frac{-3E}{28}$$

$$i_L(t) = -\frac{3E}{28} e^{-t/\tau} + \frac{E}{4R} = \frac{E}{4R}\left(1 - \frac{3}{7}e^{-t/\tau}\right)$$

$$\boxed{i_L(t) = \frac{E}{4R}\left(1 - \frac{3}{7}e^{-t/\tau}\right)}$$

$$u_L(t) = L\frac{di_L}{dt} = \frac{LE}{4R}\left(-\frac{3}{7}\right)\cdot\frac{-1}{\tau}e^{-t/\tau} \Rightarrow \frac{3\cancel{L}E}{28\cancel{R}} \cdot \frac{\cancel{4R}}{\cancel{L}}e^{-t/\tau}$$

$$\boxed{u_L(t) = \frac{3}{7}Ee^{-t/\tau}}$$



$$W_L(t) = \frac{1}{2}L i_L^2(t)$$

$$\boxed{W_{L\,MIN} = \frac{1}{2}L i_{L\,MIN}^2(t) = \frac{1}{2}L \cdot \frac{E^2}{49R^2} = \frac{E^2 L}{98R^2}}$$

$$\boxed{W_{L\,MAX} = \frac{1}{2}L i_{L\,MAX}^2(t) = \frac{1}{2}L \frac{E^2}{16R^2} = \frac{E^2 L}{32R^2}}$$

7p

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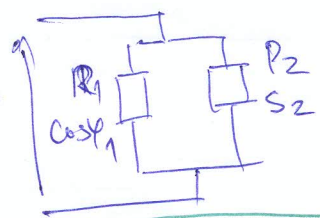
2)



a) $\bar{Z}_1 = R_1 + jX_1$ → prezentă condensatoare

$$\sin\varphi_1 = -\sqrt{1 - \cos^2\varphi_1} = -\sqrt{2}/2$$

$$Z_1 = R_1/\cos\varphi_1 = 25\sqrt{2} = 35{,}21\ \Omega$$

$$\bar{Z}_1 = Z_1\cos\varphi_1 + jZ_1\sin\varphi_1 \Rightarrow \boxed{\bar{Z}_1 = (25 - j25)\ \Omega}$$

b) $\boxed{I_1 = \frac{U}{Z_1} = 10\sqrt{2}\ A}$ $\boxed{I_2 = \frac{S_2}{U} = 1{,}5\ A}$

c) $P_1 = R_1 I_1^2 = 5000\ W$ $\boxed{P = P_1 + P_2 = 5600\ W}$

$Q_2 = -\sqrt{S_2^2 - P_2} = -450\ VAr$ $\boxed{Q = Q_1 + Q_2 = -4550\ VAr}$

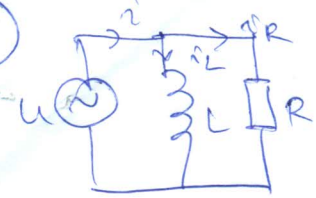
→ prezentă inductivă

$Q_1 = X_1 I_1^2 = -5000\ VAr$

$$\boxed{\bar{S} = P + jQ = (5600 - j4550)\ VA}$$

a) 1p
b) 2p
c) 3p

(5)



a) $u(t) = 20\sin(\omega t + \pi/2)$ $\omega = 1000$ rad/s

$\bar{U} = \frac{20}{\sqrt{2}} e^{j\pi/2} = j10\sqrt{2}$ V

$\bar{Z}_L = j\omega L = j \cdot 1000 \cdot 10 \cdot 10^{-3} = j10\,\Omega$

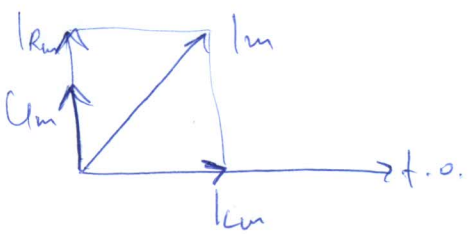
$\bar{Z}_R = R = 10\,\Omega$

$\bar{I}_L = \frac{\bar{U}}{\bar{Z}_L} = \frac{j10\sqrt{2}}{j10} = \sqrt{2}$ A

$\bar{I}_R = \frac{\bar{U}}{\bar{Z}_R} = \frac{j10\sqrt{2}}{10} = j\sqrt{2}$ A

$\bar{I} = \bar{I}_R + \bar{I}_L = \sqrt{2}(1+j)$ A

b)



c) $\bar{S} = \bar{U}\bar{I}^* = j10\sqrt{2}\,\sqrt{2}(1-j) = 20(j-j^2) = 20(1+j)$ VA

$P = 20$ W $\quad Q = 20$ VAR $\quad S = 20\sqrt{2}$ VA

d) $I = \sqrt{2} \cdot \sqrt{2} = 2$ A $\Rightarrow I_m = 2\sqrt{2}$ A $\quad \varphi = \operatorname{arctg}\frac{\sqrt{2}}{\sqrt{2}} = \operatorname{arctg} 1 = \pi/4$

$i(t) = 2\sqrt{2}\sin(\omega t + \pi/4)$ A $\quad \omega = 1000$ rad/s

$I_R = \sqrt{2}$ A $\Rightarrow I_{Rm} = 2$ A $\quad \varphi_R = \pi/2$

$i_R(t) = 2\sin(\omega t + \pi/2)$ A $\quad \omega = 1000$ rad/s

a) 3p
b) 1p
c) 2p
d) 3p

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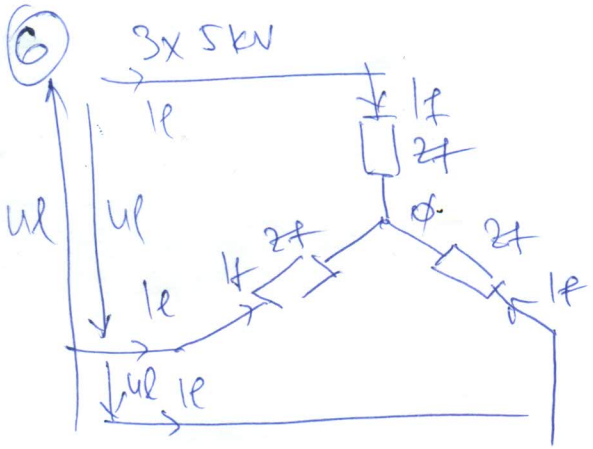
(6)



$U_l = 5$ kV

$\Delta \Rightarrow U_f = \frac{U_l}{\sqrt{3}} = \frac{5}{\sqrt{3}}$ kV

$Z_f = \sqrt{300^2 + (-400)^2} = 500\,\Omega$

$I_f = \frac{U_f}{Z_f} = \frac{5 \cdot 10^3}{\sqrt{3} \cdot 500} = \frac{10\sqrt{3}}{3}$ A

$I_l = I_f = \frac{10\sqrt{3}}{3}$ A $= 5{,}77$ A

$\cos\varphi = \frac{R}{Z_f} = \frac{300}{500} = 0{,}6$

$\sin\varphi = \frac{X}{Z_f} = \frac{-400}{500} = -0{,}8$

$P = 3U_f I_f \cos\varphi = 30$ kW

$Q = 3U_f I_f \sin\varphi = -40$ kVAR

$S = 50$ kVA $= 3U_f I_f$

5p