DRUGI KOLOKVIJUM IZ ELEKTROTEHNIKE
29.12.2014.

GRUPA 3

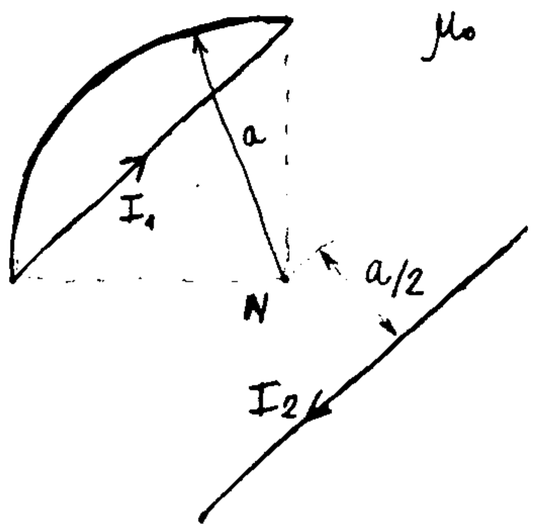
1. U istoj ravni u vazduhu nalaze se kontura kroz koju protiče struja intenziteta I_1 i beskonačno dugačak pravolinijski provodnik sa strujom I_2 (Slika 1). Kontura se sastoji od kružnog luka poluprečnika a (četvrtina kružnice) i odgovarajuće tetive. Odrediti vektor magnetne indukcije u tački N, koja se nalazi u centru kružnog luka, i na rastojanju $a/2$ od provodnika. **(5 poena)**

2. U kolu na Slici 2 prikazano je magnetno kolo sa dva namotaja. Namotaj sa N_2 navojaka je otvorenih krajeva, a kroz namotaj sa N_1 navojaka protiče struja intenziteta $i(t) = I_m \sin(\omega t)$. Jezgro je homogeno i sastoji se iz dva dela površina poprečnog preseka S_1 i S_2, čije dužine srednjih linija iznose l_1 i l_2. Relativna magnetna permeabilnost jezgra iznosi μ_r.

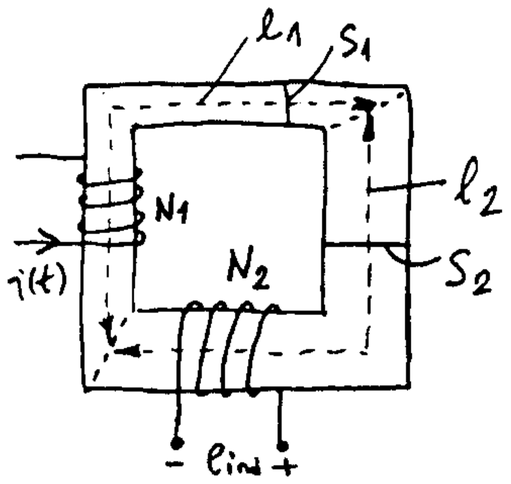
a) Odrediti izraz za fluks vektora magnetne indukcije u jezgru. **(3 poena)**

b) Odrediti izraz za induktivnost namotaja sa N_1 navojaka. **(2 poena)**

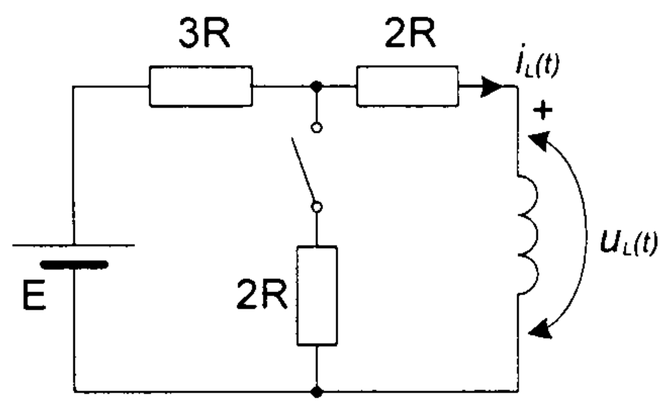
c) Odrediti izraz za elektromotornu silu indukovanu na krajevima namotaja sa N_2 navojaka. **(3 poena)**



Slika 1



Slika 2



Slika 3

3. U kolu na Slici 3 poznato je E, R, i L. Prekidač je zatvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se otvara. Odrediti izraz za intenzitet struje $i_L(t)$ i napon kalema $u_L(t)$ nakon otvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame. Odrediti minimalnu i maksimalnu vrednost energije magnetnog polja kalema u toku prelaznog procesa. **(7 poena)**

4. Dva prijemnika vezana su paralelno i priključena na naizmeničan napon efektivne vrednosti $U = 300\,\text{V}$. Prvi prijemnik je pretežno kapacitivan i ima aktivnu otpornost $R_1 = 24\,\Omega$ i faktor snage $\cos\varphi_1 = 0.8$. Aktivna i reaktivna snaga drugog prijemnika iznose $P_2 = 600\,\text{W}$ i $Q_2 = -450\,\text{var}$.

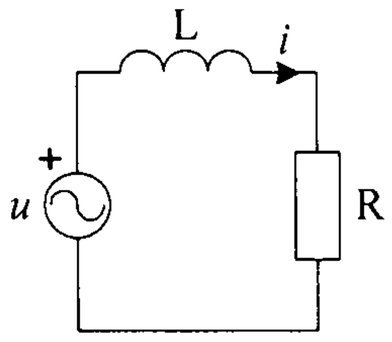
a) Odrediti kompleksnu impedansu prvog prijemnika. **(1 poen)**

b) Odretiti efektivne vrednosti struja I_1 i I_2 u prijemnicima. **(2 poena)**

c) Odrediti ukupnu aktivnu, ukupnu reaktivnu i ukupnu kompleksnu prividnu snagu celokupnog potrošača. **(3 poena)**

5. Na Slici 4 je prikazano kolo naizmenične struje koje se napaja naponom trenutne vrednosti: $u(t) = 10\sin(\omega t + \pi/2)\,\text{V}$, gde je $\omega = 100\,\text{rad/s}$. Poznate su sledeće vrednosti elemenata u kolu: $R = 1\,\Omega$, $L = 10\,\text{mH}$.

a) Odrediti kompleksne izraze struje u kolu i napona na otporniku i kalemu; **(3 p.)**
b) Predstaviti na fazorskom dijagramu struju i napone na svim elementima; **(1 p.)**
c) Odrediti aktivnu, reaktivnu i prividnu snagu celokupnog potrošača; **(2 p.)**
d) Odrediti trenutnu vrednost napona na kalemu i napona na otporniku. **(3 p.)**

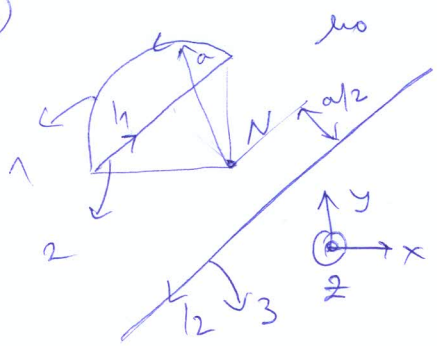


Slika 4

6. Na sistem trofaznog napona $3\times 3\,\text{kV}$ priključen je trofazni potrošač povezan u trougao. Impedansa svake faze potrošača iznosi $\overline{Z}_f = 60 - j80\,\Omega$. Odrediti: efektivnu vrednost linijske struje, aktivnu, reaktivnu i prividnu snagu potrošača. **(5 poena)**

GRUPA 3

1.



$$\vec{B}_N = \vec{B}_{N1} + \vec{B}_{N2} + \vec{B}_{N3}$$

$$\vec{B}_{N1} = \frac{1}{4}\,\frac{\mu_0 I_1}{2a}\,\vec{k} = \frac{\mu_0 I_1}{8a}\,\vec{k}$$

$$\vec{B}_{N2} = \frac{\mu_0 I_1}{4\pi\left(\frac{a\sqrt{2}}{2}\right)}\left(\cos 45^\circ + \cos 45^\circ\right)(\vec{k})$$

$$= \frac{\mu_0 I_1}{2\pi a\sqrt{2}}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)(-\vec{k}) = -\frac{\mu_0 I_1}{2\pi a\sqrt{2}}\cdot\sqrt{2}\,\vec{k}$$

$$\vec{B}_{N2} = -\frac{\mu_0 I_1}{2\pi a}\,\vec{k}$$

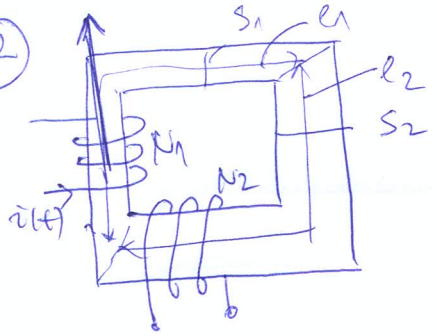
$$\vec{B}_{N3} = \frac{\mu_0 I_2}{2\pi a/2}(-\vec{k}) = -\frac{\mu_0 I_2}{\pi a}\,\vec{k}$$

$$\vec{B}_N = \left(\frac{\mu_0 I_1}{8a} - \frac{\mu_0 I_1}{2\pi a} - \frac{\mu_0 I_2}{\pi a}\right)\vec{k} = \frac{\mu_0}{8a}\left(I_1\left(1-\frac{4}{\pi}\right) - \frac{8I_2}{\pi}\right)\vec{k}$$

$$\boxed{\vec{B}_N = -\frac{\mu_0}{8a}\left(I_1\left(\frac{4}{\pi}-1\right) + \frac{8I_2}{\pi}\right)\vec{k}}$$

5p

2



a) $\oint \vec{H}(t)\,d\vec{l} = \sum I$

$$H_1(t)\,l_1 + H_2(t)\,l_2 = N_1 i(t)$$

$$\frac{B_1(t)\,l_1}{\mu_0\mu_r} + \frac{B_2(t)\,l_2}{\mu_0\mu_r} = N_1 i(t)$$

$$\frac{\Phi(t)\,l_1}{\mu_0\mu_r S_1} + \frac{\Phi(t)\,l_2}{\mu_0\mu_r S_2} = N_1 i(t)$$

$$\boxed{\Phi(t) = \frac{N_1\mu_0\mu_r i(t)}{l_1/S_1 + l_2/S_2}}$$

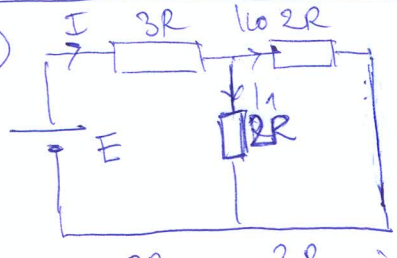
b) $$\boxed{L_1 = \frac{N_1\,\Phi(t)}{i(t)} = \frac{N_1^2\mu_0\mu_r}{l_1/S_1 + l_2/S_2}}$$

c) $$e_{ind} = -N_2\frac{d\Phi(t)}{dt} = -\frac{N_1 N_2\mu_0\mu_r}{l_1/S_1 + l_2/S_2}\frac{di(t)}{dt} = -\frac{N_1 N_2\mu_0\mu_r}{l_1/S_1 + l_2/S_2} I_m\cos(\omega t)\cdot\omega$$

$$\boxed{e_{ind} = -\frac{N_1 N_2\mu_0\mu_r I_m\omega}{l_1/S_1 + l_2/S_2}\cos\omega t}$$

a) 3p
b) 2p
c) 3p

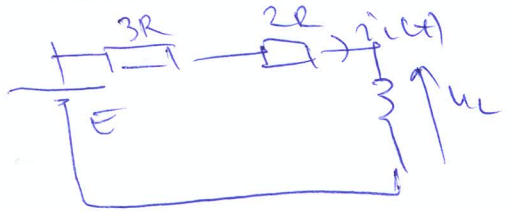
3



$$I = \frac{E}{3R+R} = \frac{E}{4R}$$

$$I_{Lo} = I_1 = \frac{I}{2} = \frac{E}{8R}$$

СТАНЦИОНИРО СТАНЕ, ПРЕКЪСВАМ ЗАТВОРЕН



Преходен процес:

$$E - 5Ri(t) - u_L(t) = 0$$

$$u_L(t) = L\frac{di(t)}{dt}$$

$$E - 5R\,i_L(t) - L\frac{di_L(t)}{dt} = 0$$

$$\tau = L/5R \qquad K = E/L$$

$$\frac{di_L(t)}{dt} + \frac{i_L(t)}{L/5R} = \frac{E}{L}$$

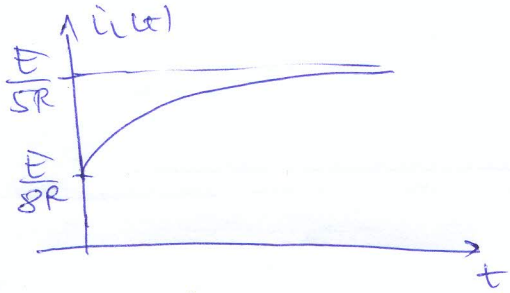
$$i_L(t) = Ae^{-t/\tau} + B \qquad B = K\tau = \frac{L}{5R}\cdot\frac{E}{L} = \frac{E}{5R}$$

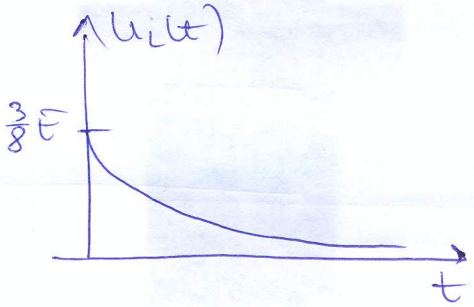
$$A + B = I_0 \Rightarrow A = I_0 - B = \frac{E}{8R} - \frac{E}{5R} = \frac{-3E}{40R}$$

$$i_L(t) = -\frac{3E}{40R}e^{-t/\tau} + \frac{E}{5R} = \boxed{\frac{E}{5R}\left(1 - \frac{3}{8}e^{-t/\tau}\right)}$$

$$u_L(t) = L\frac{di_L}{dt} = \frac{LE}{5R}\left(-\frac{3}{8}\cdot\frac{-1}{\tau}\cdot e^{-t/\tau}\right) = \frac{LE}{5R}\cdot\frac{3}{8}\cdot\frac{5R}{L}e^{-t/\tau}$$

$$\boxed{u_L(t) = \frac{3}{8}Ee^{-t/\tau}}$$





$$W_L(t) = \frac{1}{2}L\,i_L^2(t)$$

$$W_{L\,MIN}(t) = \frac{1}{2}L\,i_{L\,MIN}^2(t) = \frac{1}{2}L\cdot\frac{E^2}{64R^2} = \boxed{\frac{E^2L}{128R^2}}$$

$$W_{L\,MAX}(t) = \frac{1}{2}L\,i_{MAX}^2(t) = \frac{1}{2}L\frac{E^2}{25R^2} = \boxed{\frac{E^2L}{50R^2}}$$

7p

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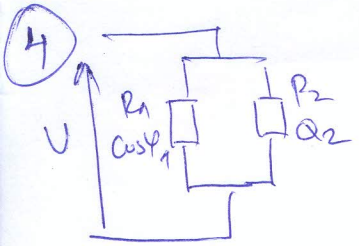
4)



a) $R_1 = 24\Omega$, $\cos\varphi_1 = 0{,}8$ – capacitiv

$$\sin\varphi_1 = -\sqrt{1-\cos^2\varphi_1} = -\sqrt{1-0{,}8^2} = -0{,}6$$

$$Z_1 = \frac{R_1}{\cos\varphi_1} = 30\Omega \qquad X_1 = Z_1\sin\varphi_1 = -18\Omega$$

$$\underline{Z}_1 = Z_1\cos\varphi_1 + jZ_1\sin\varphi_1 = \boxed{(24 - j18)\Omega = \underline{Z}_1}$$

b) $I_1 = \frac{U}{Z_1} = \frac{300}{3\Phi}\,A = 10A$

$$S_2 = \sqrt{P_2^2 + Q_2^2} = 750\,VA \Rightarrow I_2 = \frac{S_2}{U} = \frac{750\,VA}{300\,V} = 2{,}5A$$

c) $\boxed{P = P_1 + P_2 = 3000\ W}$

$P_1 = R_1 I_1^2 = 24\cdot 10^2 = 2400\ W$

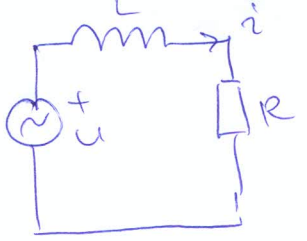
$Q_1 = X_1 I_1^2 = -18\cdot 10^2 = -1800\ VAR$

$\boxed{Q = Q_1 + Q_2 = -2250\ VAR}$

$\boxed{\underline{S} = P + jQ = (3000 - j2250)\,VA}$

a) 1p
b) 2p
c) 3p

(5)



$\bar{U} = \frac{10}{\sqrt{2}} e^{j\pi/2} = j5\sqrt{2}\ V$

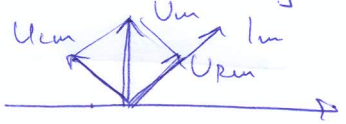
$\bar{Z}_L = j\omega L = j \cdot 100 \cdot 10 \cdot 10^{-3} = j\Omega$

$\bar{Z}_R = R = 1\Omega$

$\bar{I}_R = \bar{I}_L = \bar{I} = \frac{\bar{U}}{\bar{Z}_R + \bar{Z}_L} = \frac{j5\sqrt{2}}{1+j} \cdot \frac{1-j}{1-j} = \frac{5\sqrt{2}(j-j^2)}{1-j^2} = \frac{5\sqrt{2}(1+j)}{2}$

$\boxed{\bar{I} = \bar{I}_L = \bar{I}_R = \frac{5\sqrt{2}}{2}(1+j)\ A}$

b)



a) 3p
b) 1p
c) 2p
d) 3p

$\bar{U}_R = \bar{Z}_R \bar{I} = \boxed{\frac{5\sqrt{2}}{2}(1+j)\ V}$

$\bar{U}_L = \bar{Z}_L \bar{I} = \frac{5\sqrt{2}}{2} j(1+j) = \frac{5\sqrt{2}}{2}(j+j^2) = \frac{5\sqrt{2}}{2}(j-1) = \boxed{\frac{5\sqrt{2}}{2}(1j)\ V}$

c) $\bar{S} = \bar{U}\bar{I}^* = 5\sqrt{2} \cdot \frac{5\sqrt{2}}{2}(1-j) = 25(j-j^2) = 25(1+j)\ VA$

$\boxed{P = 25W}$ $\boxed{Q = 25VAr}$ $\boxed{S = 25\sqrt{2}\ VA}$

d) $U_L = \frac{5\sqrt{2}}{2} \cdot \sqrt{2} = 5V \Rightarrow U_{Lm} = 5\sqrt{2}\ V$

$\varphi_L = \operatorname{arctg} \frac{-5\sqrt{2}/2}{5\sqrt{2}/2} = \operatorname{arctg}(-1) = \frac{3\pi}{4}$

$\boxed{u_L(t) = 5\sqrt{2}\sin(\omega t + \frac{3\pi}{4})\ V}$

$U_R = \frac{5\sqrt{2}}{2} \cdot \sqrt{2} = 5V \Rightarrow U_{Rm} = 5\sqrt{2}\ V$

$\varphi_R = \operatorname{arctg} \frac{5\sqrt{2}/2}{5\sqrt{2}/2} = \operatorname{arctg} 1 = \frac{\pi}{4}$

$\boxed{u_R(t) = 5\sqrt{2}\sin(\omega t + \frac{\pi}{4})\ V}$

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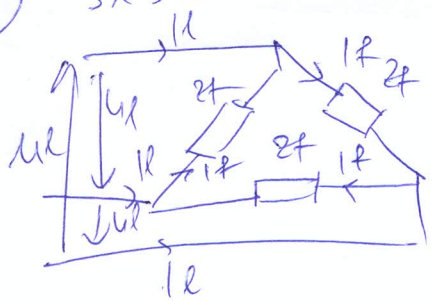
(6) 3x3kV



$\bar{Z}_f = (60 - j80)\Omega \Rightarrow Z_f = 100\Omega$

$\Delta \Rightarrow U_l = U_f = 3kV$

$I_f = \frac{U_f}{Z_f} = \frac{3000V}{100\Omega} = 30A$

$\Delta \Rightarrow \boxed{I_l = I_f \cdot \sqrt{3} = 30\sqrt{3}A}$

$P = 3U_f I_f \cos\varphi = 3 \cdot 3k \cdot 30 \cdot 0{,}6$

$P = 270 \cdot 0{,}6\ kW = 27 \cdot 6\ kW$

$\boxed{P = 162\ kW}$

$\cos\varphi = \frac{R}{Z_f} = \frac{60}{100} = 0{,}6$

$\sin\varphi = \frac{X}{Z_f} = \frac{-80}{100} = -0{,}8$

$Q = 3U_f I_f \sin\varphi = 3 \cdot 3k \cdot 30(-0{,}8)$

$Q = -270 \cdot 0{,}8\ KVAr$

$\boxed{Q = -216\ KVAr}$

$S = 3U_f I_f = 3 \cdot 30 \cdot 3k = \boxed{270KVA}$

57