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29.12.2014.

GRUPA 4

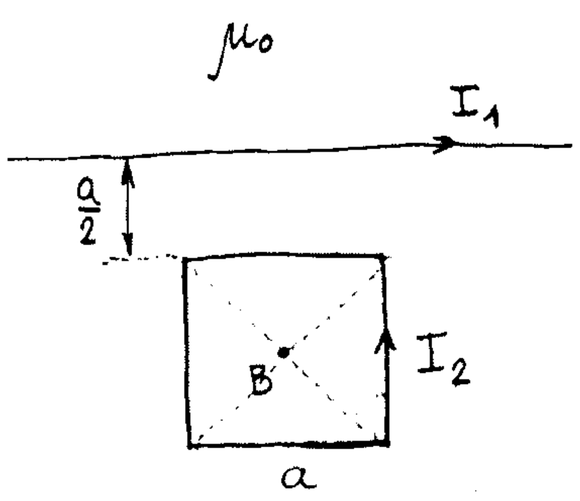
1. U istoj ravni u vazduhu nalaze se kontura kroz koju protiče struja intenziteta I_2 i beskonačno dugačak pravolinijski provodnik sa strujom I_1 (Slika 1). Kontura je oblika kvadrata stranice a. Rastojanje provodnika od bliže stranice konture (koji su, pritom, paralelni) iznosi $a/2$. Odrediti vektor magnetne indukcije u tački B, koja se nalazi u centru konture. **(5 poena)**

2. U kolu na Slici 2 prikazano je magnetno kolo sa dva namotaja. Namotaj sa N_1 navojaka je otvorenih krajeva, a kroz namotaj sa N_2 navojaka protiče struja intenziteta $i(t) = I_m \cos(\omega t)$. Jezgro je površine poprečnog preseka S i dužine srednje linije b, i sadrži vazdušni procep debljine b_0. Magnetna permeabilnost jezgra iznosi μ.

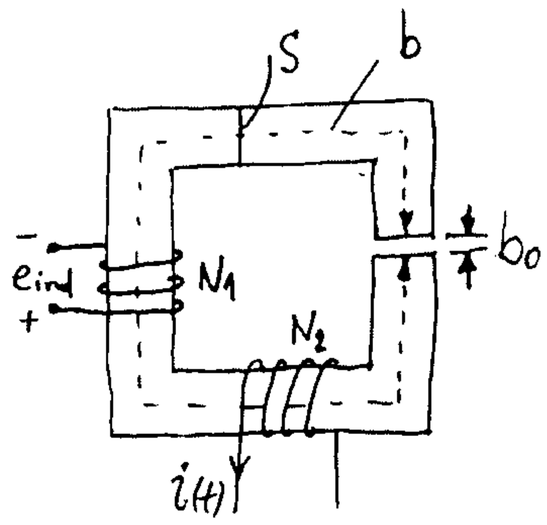
a) Odrediti izraz za fluks vektora magnetne indukcije u jezgru. **(3 poena)**

b) Odrediti izraz za induktivnost namotaja sa N_2 navojaka. **(2 poena)**

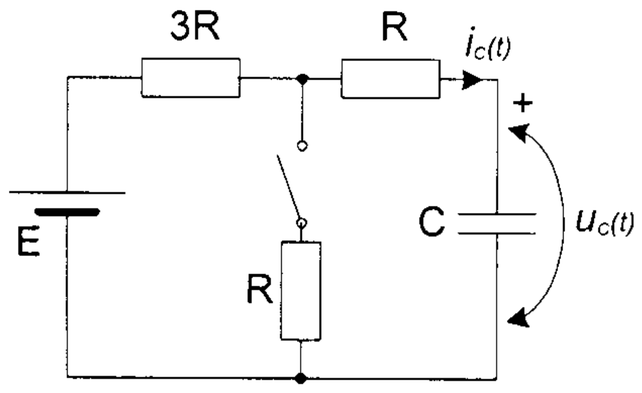
c) Odrediti izraz za elektromotornu silu indukovanu na krajevima namotaja sa N_1 navojaka. **(3 poena)**



Slika 1



Slika 2



Slika 3

3. U kolu na Slici 3 poznato je E, R, i C. Prekidač je zatvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se otvara. Odrediti izraz za napon kondenzatora $u_c(t)$ i intenzitet struje $i_c(t)$ nakon otvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame. Odrediti minimalnu i maksimalnu vrednost energije električnog polja kondenzatora u toku prelaznog procesa. **(7 poena)**

4. Dva prijemnika vezana su paralelno i priključena na naizmenični napon efektivne vrednosti $U = 200\,\mathrm{V}$. Kompleksna impedansa prvog prijemnika iznosi $\overline{Z}_1 = 10 - j10\,\Omega$. Drugi prijemnik je pretežno induktivan, faktora snage $\cos\varphi_2 = 0.5$ i prividne snage $S_2 = 4\,\mathrm{kVA}$.

a) Odrediti aktivnu i reaktivnu snagu drugog prijemnika. **(1 poen)**

b) Odretiti efektivne vrednosti struja I_1 i I_2 u prijemnicima. **(2 poena)**

c) Odrediti ukupnu aktivnu, ukupnu reaktivnu i ukupnu kompleksnu prividnu snagu celokupnog potrošača. **(3 poena)**

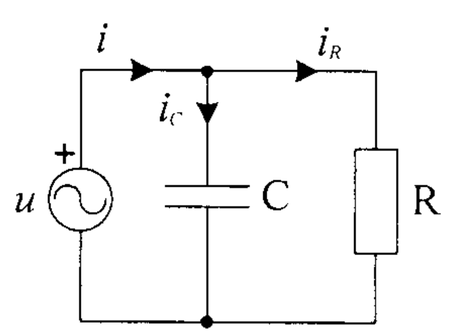
5. Na Slici 4 je prikazano kolo naizmenične struje koje se napaja naponom trenutne vrednosti: $u(t) = 20\sin(\omega t - \pi/2)\,\mathrm{V}$, gde je $\omega = 200\,\mathrm{rad/s}$. Poznate su sledeće vrednosti elemenata u kolu: $R = 10\,\Omega$, $C = 0.5\,\mathrm{mF}$.

a) Odrediti kompleksne izraze za označene struje; **(3p.)**

b) Predstaviti na fazorskom dijagramu napon generatora i struje u granama; **(1 p.)**

c) Odrediti aktivnu, reaktivnu i prividnu snagu celokupnog potrošača; **(2 p.)**

d) Odrediti trenutnu vrednost struje generatora i otpornika. **(3 p.)**

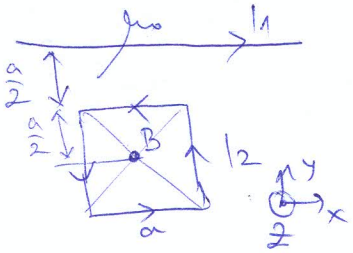


Slika 4

6. Na sistem trofaznog napona $3\times 3\,\mathrm{kV}$ priključen je trofazni potrošač povezan u zvezdu. Impedansa svake faze potrošača iznosi $\overline{Z}_f = 300 + j400\,\Omega$. Odrediti: efektivnu vrednost linijske struje, aktivnu, reaktivnu i prividnu snagu potrošača. **(5 poena)**

GRUPA 4

1.



$$\vec{B_B} = \vec{B_1} + 4\vec{B_2}$$

$$\vec{B_1} = \frac{\mu_0 I_1}{2\pi\left(\frac{a}{2}+\frac{a}{2}\right)}(-\vec{k}) = \frac{-\mu_0 I_1}{2\pi a}\vec{k}$$

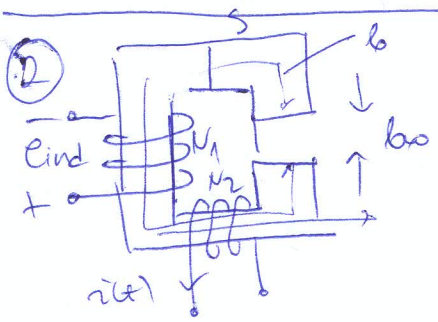
$$\vec{B_2} = \frac{\mu_0 I_2}{4\pi\frac{a}{2}}(\cos 45^\circ + \cos 45^\circ)\vec{k}$$

$$= \frac{\mu_0 I_2}{2\pi a} 2\cdot\frac{\sqrt{2}}{2}\vec{k} = \frac{\mu_0 I_2\sqrt{2}}{2\pi a}\vec{k}$$

$$\vec{B_B} = -\frac{\mu_0 I_1}{2\pi a}\vec{k} + 4\cdot\frac{\mu_0 I_2\sqrt{2}}{2\pi a}\vec{k} = \boxed{\frac{\mu_0}{2\pi a}(-I_1 + 4I_2\sqrt{2})\vec{k}}$$

5p

2.



a) $\oint \vec{H}(t)\,d\vec{l} = \Sigma I$

$$H_i(t)\,b + H_0(t)\,b_0 = N_2 i(t)$$

$$\frac{B(t)\,b}{\mu} + \frac{B(t)\,b_0}{\mu_0} = N_2 i(t)$$

$$\frac{\Phi(t)\,b}{\mu S} + \frac{\Phi(t)\,b_0}{\mu_0 S} = N_2 i(t)$$

$$\boxed{\Phi(t) = \frac{S N_2 i(t)}{b/\mu + b_0/\mu_0}}$$

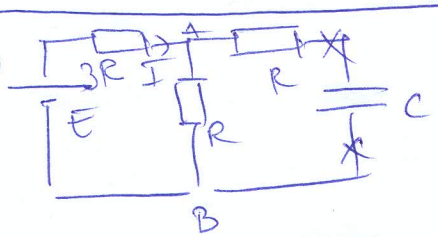
b) $L_2 = \frac{N_2\Phi(t)}{i(t)} \Rightarrow$ $\boxed{L_2 = \frac{N_2^2 S}{b/\mu + b_0/\mu_0}}$

c) $e_{ind} = -N_1\frac{d\Phi(t)}{dt} = -N_1 N_2 \frac{S}{b/\mu + b_0/\mu_0}\frac{di(t)}{dt} = -\frac{N_1 N_2 S}{b/\mu + b_0/\mu_0}\omega I_m(-\sin\omega t)$

$$\boxed{e_{ind} = \frac{N_1 N_2 S I_m \omega}{b/\mu + b_0/\mu_0}\sin\omega t}$$

a) 3p
b) 2p
c) 3p

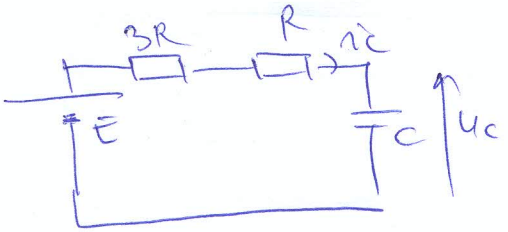
3.



STAȚIONAR STARE $t \to \infty$

$$I = \frac{E}{3R+R} = \frac{E}{4R}$$

$$U_{C0} = U_{AB} = RI = \frac{E}{4}$$



PRELAZAK PROMJENE $t \to 0$

$$E - 4Ri(t) - u_c(t) = 0$$

$$i_c(t) = C\frac{du_c}{dt}$$

$$E - 4RC\frac{du_c(t)}{dt} - u_c(t) = 0$$

$$\frac{du_c(t)}{dt} + \frac{u_c(t)}{4RC} = \frac{E}{4RC} \qquad \tau = 4RC \qquad K = \frac{E}{4RC}$$

$$u_c(t) = Ae^{-t/\tau} + B$$

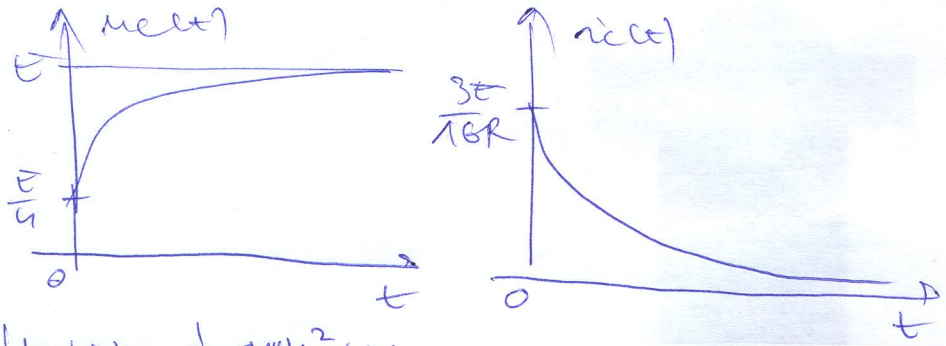
$$B = K\tau = E$$

$$A + B = U_{c0} \Rightarrow A = U_{c0} - B = -\frac{3E}{4}$$

$$\boxed{u_c(t) = -\frac{3E}{4}e^{-t/\tau} + E = E\left(1 - \frac{3}{4}e^{-t/\tau}\right)}$$

$$i_c(t) = C\frac{du_c}{dt} = CE\left(-\frac{3}{4}\cdot\frac{-1}{\tau}e^{-t/\tau}\right) = CE\,\frac{3}{4}\cdot\frac{1}{4RC}e^{-t/\tau}$$

$$\boxed{i_c(t) = \frac{3E}{16R}e^{-t/\tau}}$$



$$W_c(t) = \frac{1}{2}Cu_c^2(t)$$

$$W_{cMIN}(t) = \frac{1}{2}Cu_{cMIN}^2(t) = \frac{1}{2}C\frac{E^2}{16} = \boxed{\frac{CE^2}{32}}$$

$$W_{cMAX}(t) = \frac{1}{2}Cu_{cMAX}^2(t) = \boxed{\frac{1}{2}C\cdot E^2}$$

7p

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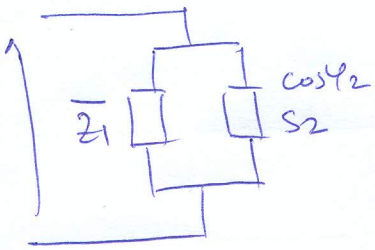
4)



a) $\sin\varphi_2 = +\sqrt{1-\cos^2\varphi_2} = \frac{\sqrt{3}}{2}$ (inductiv), $S_2 = 4$ kVA

$$P_2 = S_2\cos\varphi_2 = 4k\cdot 0{,}5 = \boxed{2\text{ kW}}$$

$$Q_2 = S_2\sin\varphi_2 = 4k\cdot\frac{\sqrt{3}}{2} = 2\sqrt{3}\text{ kVAr} = \boxed{3{,}46\text{ kVAr}}$$

b) $I_1 = \frac{U}{Z_1} = \frac{200}{10\sqrt{2}} = \frac{20\sqrt{2}}{2} = \boxed{10\sqrt{2}\text{ A}}$

$Z_1 = 10\sqrt{2}$

$$I_2 = \frac{S_2}{U} = \frac{4000}{200} = \frac{200}{10} = \boxed{20\text{ A}}$$

a) 1p
b) 2p
c) 3p

c) $P_1 = R_1 I_1^2 = 10\cdot(10\sqrt{2})^2 = 10\cdot 200 = 2000\text{ W} = 2\text{ kW}$

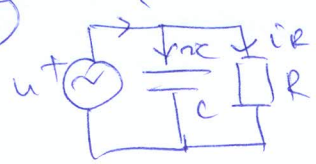
$$\boxed{P = P_1 + P_2 = 2 + 2 = 4\text{ kW}}$$

$Q_1 = X_1 I_1^2 = -10\cdot(10\sqrt{2})^2 = -10\cdot 200 = -2000\text{ VAr} = -2\text{ kVAr}$

$$\boxed{Q = Q_1 + Q_2 = 2\sqrt{3} - 2 = 2(\sqrt{3} - 1) = 2(1{,}73 - 1) = 1{,}46\text{ kVAr}}$$

$$\boxed{\bar{S} = P + jQ = (4 + j1{,}46)\text{ kVA}}$$

(5)



$\bar{U} = \frac{20}{\sqrt{2}} e^{-j\pi/2} = -j10\sqrt{2}\ V$

$\bar{Z}_C = -j\frac{1}{\omega C} = -j\frac{1}{200 \cdot 0{,}1\sqrt{} \cdot 10^{-3}} = -j10\ \Omega$

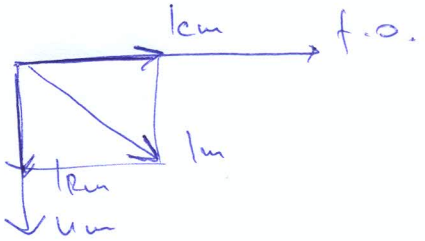
$\bar{Z}_R = R = 10\ \Omega$

$\boxed{\bar{I}_C = \frac{\bar{U}}{\bar{Z}_C} = \frac{-j10\sqrt{2}}{-j10} = \sqrt{2}\ A}$

$\boxed{\bar{I}_R = \frac{\bar{U}}{\bar{Z}_R} = \frac{-j10\sqrt{2}}{10} = -j\sqrt{2}\ A}$

$\boxed{\bar{I} = \bar{I}_R + \bar{I}_C = \sqrt{2}(1-j)\ A}$

b)



c) $\bar{S} = \bar{U}\bar{I}^* = -j10\sqrt{2}\sqrt{2}(1+j)$

$= 20(-j-j^2) = (1-j)20\ VA$

$\boxed{P = 20\ W}$ $\boxed{Q = -20\ VAr}$ $\boxed{S = 20\sqrt{2}\ VA}$

d) $I_R = \sqrt{2} \Rightarrow I_{Rm} = \sqrt{2}\cdot\sqrt{2} = 2\ A$ $\quad \psi_R = -\pi/2$

$\boxed{i_R(t) = 2\sin(\omega t - \pi/2)\ A}$

$I_\bullet = \sqrt{2}\cdot\sqrt{2} = 2\ A \Rightarrow I_m = 2\sqrt{2}\ A$

$\psi = \operatorname{arctg}\frac{-\sqrt{2}}{\sqrt{2}} = \operatorname{arctg}(-1) = -\pi/4$

$\boxed{i(t) = 2\sqrt{2}\sin(\omega t - \pi/4)\ A}$

a) 3p
b) 1p
c) 2p
d) 3p

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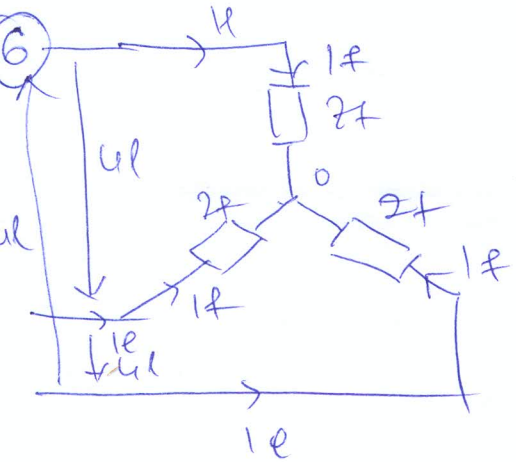
(6)



$U_l = 3kV$

$\lambda \Rightarrow U_f = \frac{U_l}{\sqrt{3}} = \frac{3kV}{\sqrt{3}} = \sqrt{3}\ kV$

$Z_f = \sqrt{300^2 + 400^2} = 500\ \Omega$

$I_f = \frac{U_f}{Z_f} = \frac{\sqrt{3}1000\ V}{500\ \Omega} = 2\sqrt{3}\ A$

$\boxed{I_l = I_f = 2\sqrt{3}\ A}$

$P = 3U_f I_f \cos\varphi = 3\cdot\sqrt{3}\,kV\cdot 2\sqrt{3}\cdot 0{,}6$

$P = 3\cdot 3\cdot 2{,}96\cdot 10^3 = 9\cdot 1200 = \boxed{10{,}8\ kW}$

$\cos\varphi = \frac{R}{Z_f} = 0{,}6$

$\sin\varphi = \frac{X}{Z_f} = 0{,}8$

$Q = 3U_f I_f \sin\varphi = 3\sqrt{3}\cdot 2\sqrt{3}\cdot 0{,}8\ k$

$= \boxed{14{,}4\ kVAr}$

$S = 3U_f I_f = 3\cdot\sqrt{3}\cdot 2\sqrt{3} = \boxed{18\ kVA}$

5p