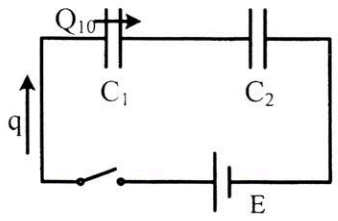
Elektrotehnika

06. jul 2016.

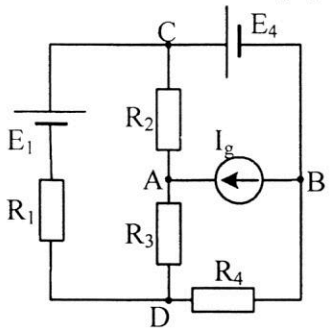
1. a) Pločasti kondenzator nepoznate kapacitivnosti C_1 optrećen je količinom naelektrisanja $Q_{10} = 10\,\text{pC}$. Ploče kondenzatora su oblika kvadrata stranice $a = 1\,\text{cm}$, a rastojanje između njih iznosi $d = 1\,\text{mm}$. Dielektrična konstanta dielektrika između ploča kondenzatora je $\varepsilon = 5 \cdot 10^{-11}\,\text{F/m}$. Odrediti:

- nepoznatu kapacitivnost kondenzatora, C_1; **(3 poena)**
- napon na krajevima kondenzatora; **(2 poena)**
- intenzitet vektora električnog polja u dielektriku; **(2 poena)**
- intenzitet sile kojom se privlače ploče. **(3 poena)**

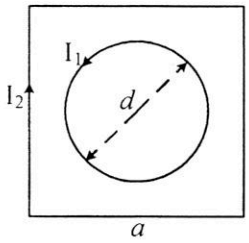
b) Ako se kondenzator iz tačke **a)** poveže u kolo na Slici 1, sa neopterećenim kondenzatorom kapacitivnosti $C_2 = 20\,\text{pF}$ i generatorom elektromotorne sile $E = 12\,\text{V}$, odrediti kolika će količina naelektrisanja proteći kroz kolo u naznačenom smeru nakon zatvaranja prekidača. **(10 poena)**



Slika 1



Slika 2

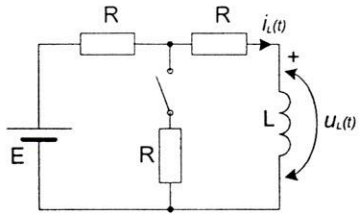


Slika 3

2. U kolu na Slici 2 poznate su vrednosti $R_1 = R_2 = R_3 = 3\Omega$, $R_4 = 8\Omega$, $I_g = 1\text{A}$, $E_1 = 6\text{V}$, $E_4 = 8\text{V}$. Primenom Tevenenove teoreme odrediti napon U_{DB} i snagu otpornika R_4. **(25 poena)**

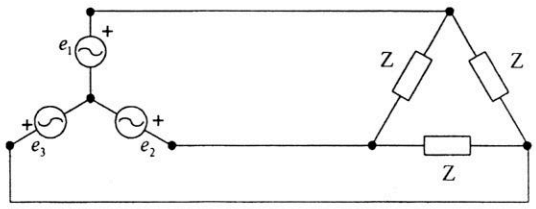
3. Kružna kontura prečnika $d = 4\,\text{cm}$ sa strujom intenziteta $I_1 = 1\text{A}$ i kvadratna kontura stranice $a = 3\,\text{cm}$ sa strujom intenziteta $I_2 = 4\text{A}$ nalaze se u istoj ravni u vazduhu (Slika 3). Odrediti i nacrtati vektor jačine magnetnog polja u centru kružne konture.**(15 poena)**

4. U kolu na Slici 4 poznate su vrednosti elemenata: $E = 60\,\text{V}$, $R = 2\Omega$, $L = 24\,\mu\text{H}$. Prekidač je otvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se zatvara. a) Odrediti izraze za struju i napon kalema nakon zatvaranja prekidača **(12 p)** i nacrtati odgovarajuće vremenske dijagrame **(4 p)**. b) Odrediti energiju magnetnog polja kalema u trenutku $t_1 = 12\,\mu\text{s}$. **(4 p)**



Slika 4

5. Na Slici 5 prikazan je trofazni sistem generator potrošač. Efektivna vrednost elektromotornih sila iznosi $E = 100\,\text{V}$, aktivna snaga potrošača $P = 27\,\text{W}$, a reaktivna snaga $Q = -36\,\text{var}$. Odrediti: efektivnu vrednost linijske struje, faktor snage potrošača i kompleksnu impedansu potrošača. **(20 poena)**



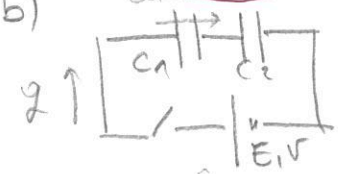
Slika 5

ELEKTROTEHNIKA – PISMENI JUL

1. a) $C_1 = \varepsilon \frac{S}{d} = 5\cdot10^{-11}\frac{10^{-4}}{10^{-3}} = 5\cdot10^{-12} = 5\,pF$

- $U = \frac{Q_{10}}{C_1} = 2\,V$
- $k = \frac{U}{d} = 2\,\frac{kV}{m}$
- $F = \frac{1}{2}Q_{10}k = \frac{Q_{10}^2}{2\varepsilon S} = 10^{-8}\,N$

b)



$Q_1 = Q_{10} + q$

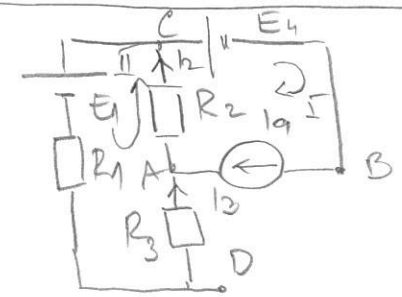
$Q_2 = q$

$E - U_{C1} - U_{C2} = 0$

$E - \frac{Q_{10}+q}{C_1} - \frac{q}{C_2} = 0$

$q = \frac{E - \frac{Q_{10}}{C_1}}{\frac{1}{C_1}+\frac{1}{C_2}}$ $\quad q = 40\,pC$

2.



$E_T = U_{BD}^{(0V)} = R_3 I_3 + R_2 I_2 + E_4$

$I_I = I_9$

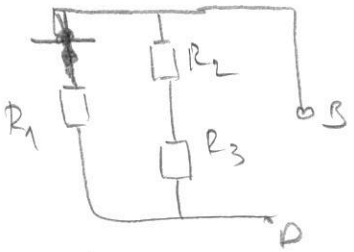
$I_{II}(R_1+R_2+R_3) + R_2 I_I = -E_1$

$I_{II} = \frac{-E_1 - R_2 I_9}{R_1+R_2+R_3} = \frac{-6-3\cdot 9}{3+3+3} = \frac{-9}{9} = -1\,A$

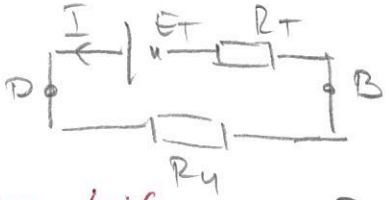
$I_3 = I_{II} = -1\,A \qquad I_2 = I_I + I_{II} = 0$

$E_T = 3\cdot(-1) + 3\cdot 0 + 9 = 8 - 3 = 5\,V$

$E_T = 5\,V$



$R_T = R_1 \| (R_2+R_3) = 3 \| (3+3) = 3 \| 6 = \frac{3\cdot6}{9} = 2\,\Omega$



$I = \frac{E_T}{R_T+R_4} = \frac{5}{8+2} = \frac{1}{2}\,A$

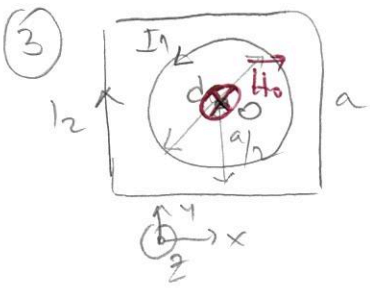
$U_{DB} = R_4 I = 8\cdot\frac{1}{2} = 4\,V$

$P_{R_4} = R_4 I^2 = 8\left(\frac{1}{2}\right)^2 = 2\,W$

$E_T = 5\,V \quad R_T = 2\,\Omega \quad U_{DB} = 4\,V$

$P_{R_4} = 2\,W$

3.



$\vec{B_0} = \vec{B_{01}} + \vec{B_{02}}$

$\vec{B_{01}} = \frac{\mu_0 I_1}{2\cdot d/2}\vec{k} = \frac{\mu_0 I_1}{d}\vec{k}$

$\vec{B_{02}} = 4\cdot\frac{\mu_0 I_2}{4\pi\frac{a}{2}}(\cos 45^\circ + \cos 45^\circ)(-\vec{k}) = -\frac{2\mu_0 I_2}{\pi a}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)\vec{k}$

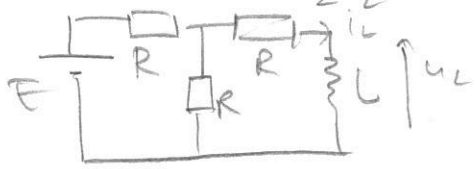
$= -\frac{2\mu_0 I_2\sqrt{2}}{\pi a}\vec{k}$

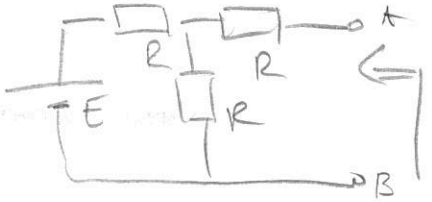
$\vec{B_0} = \mu_0\left(\frac{I_1}{d} - \frac{2I_2\sqrt{2}}{\pi a}\right)\vec{k}$

$\vec{H_0} = \frac{\vec{B_0}}{\mu_0} = \left(\frac{I_1}{d} - \frac{2I_2\sqrt{2}}{\pi a}\right)\vec{k} = -95{,}59\,\frac{A}{m}\,\vec{k}$

(4) S.S. $I_{L0} = \frac{E}{2R} = \frac{60}{2 \cdot 2} = 15\,A$ $\quad u_{L0} = 0$

P.P.

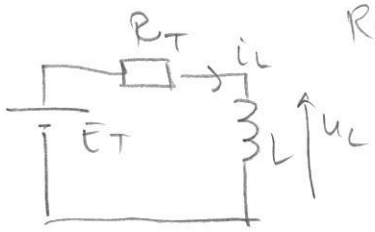




E_T, R_T

$E_T = U_{AB}{}^{OV} = R \cdot \frac{E}{2R} = \frac{E}{2} = 30\,V$

$R_T = R + (R \| R) = R + \frac{R}{2} = \frac{3R}{2} = 3\,\Omega$



$E_T - R_T i_L - u_L = 0$

$u_L = L \frac{di_L}{dt}$

$E_T - R_T i_L - L \frac{di_L}{dt} = 0$

$\frac{di_L}{dt} + \frac{i_L}{L/R_T} = \frac{E_T}{L} = K$

$\tau = \frac{L}{R_T} = \frac{24\,\mu H}{3\,\Omega} = 8\,\mu s$

$K = \frac{E_T}{L}$

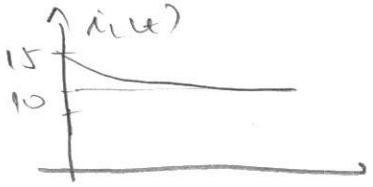
$B = K\tau = \frac{E_T}{R_T} = 10\,A$

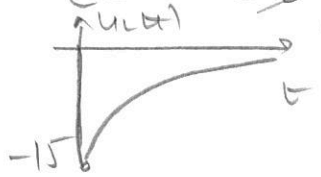
$i_L(t) = Ae^{-t/\tau} + B = (5e^{-t/\tau} + 10)\,A$

$A + B = I_{L0} \Rightarrow$

$A = I_{L0} - B = 5\,A$

$u_L(t) = L \frac{di_L}{dt} = LA \left(-\frac{1}{\tau}\right) e^{-t/\tau} = 24\mu \cdot 5 \left(\frac{-1}{8\mu}\right) e^{-t/\tau} = -15e^{-t/\tau}\,V$





b) $W_m(t_1 = 12\mu s) = \frac{1}{2} L i_L^2(t_1 = 12\mu s) = \frac{1}{2} \cdot 24\mu \left(5e^{-\frac{12}{8}} + 10\right)^2 = 12\left(10 + 5e^{-1,5}\right)^2\,\mu J$

$W_m(t_1 = 12\mu s) = 1482{,}69\,\mu J = 1{,}48\,mJ$

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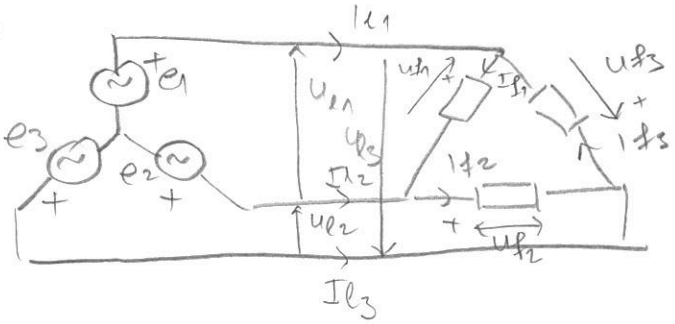
(5)



$U_l = U_f = E\sqrt{3} = 100\sqrt{3}\,V$

$P = 27\,kW$, $Q = -36\,kVAR$ $\Rightarrow$ $S = \sqrt{P^2 + Q^2} = 45\,kVA$

$S = 3U_f I_f \Rightarrow I_f = \frac{S}{3U_f} = \frac{45}{300\sqrt{3}}$

$I_f = \frac{150\sqrt{3}}{3}\,mA = 50\sqrt{3}\,mA$

$I_l = I_f \sqrt{3} = 150\,mA$

$\cos\varphi = \frac{P}{S} = \frac{27}{45} = 0{,}6$ $\quad \sin\varphi = \frac{Q}{S} = -0{,}8$

$Z_f = \frac{U_f}{I_f} = \frac{100\sqrt{3}\,V}{50\sqrt{3}\,mA} = 2\,k\Omega$ $\quad \bar{Z}_f = Z_f \cos\varphi + jZ_f \sin\varphi = 2(0{,}6 - j0{,}8)\,k\Omega$

$\bar{Z}_f = (1{,}2 - j1{,}6)\,k\Omega = (1200 - j1600)\,\Omega$