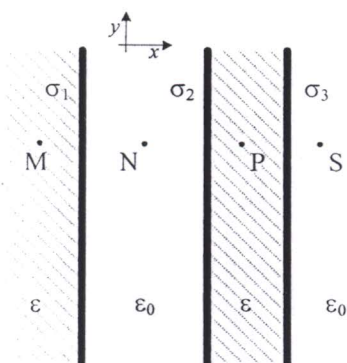


PRVI KOLOKVIJUM IZ ELEKTROTEHNIKE

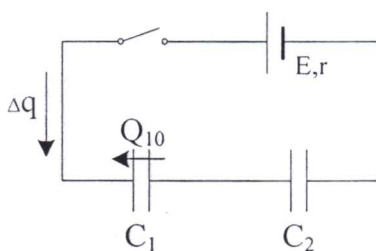
24. novembar 2016.

GRUPA 3

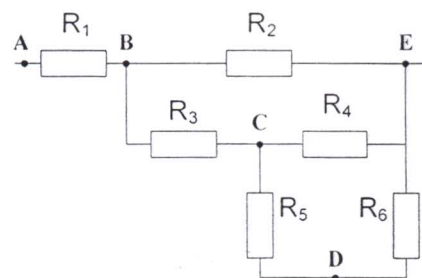
1. Na Slici 1 prikazane su tri paralelne, veoma velike, ravnomerno naelektrisane površi, površinskih gustina naelektrisanja $\sigma_1 = \sigma = 20 \text{ nC/m}^2$, $\sigma_2 = -\sigma$ i $\sigma_3 = -2\sigma$. Levo od prve ravni i između druge i treće ravni nalazi se dielektrik dielektrične konstante $\epsilon = 10^{-10} \text{ F/m}$, dok je u ostalom delu prostora vazduh ($\epsilon_0 = 8.85 \cdot 10^{-12} \text{ F/m}$). Odrediti **vektor** električnog polja u tački P. (4 poena)



Slika 1

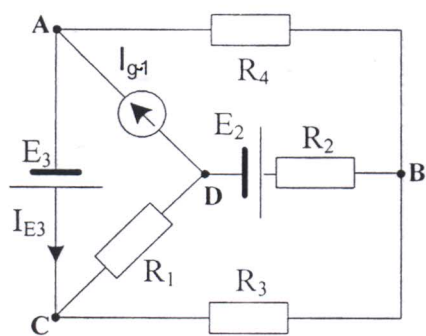


Slika 2

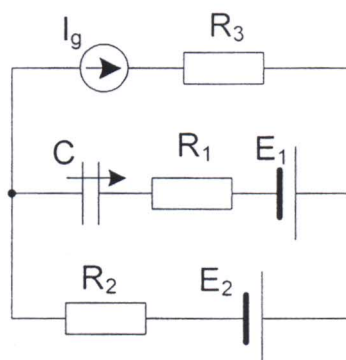


Slika 3

3. Na Slici 3 prikazana je grupa od šest jednakih otpornika, $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = R = 16 \Omega$. Odrediti ekvivalentnu otpornost između tačaka D-E. (6 poena)



Slika 4



Slika 5

4. U kolu na Slici 4 poznato je: $R_1 = R_2 = R_3 = R_4 = 30 \Omega$, $E_2 = 10 \text{ V}$, $E_3 = 10 \text{ V}$, $I_{g1} = 3 \text{ A}$.

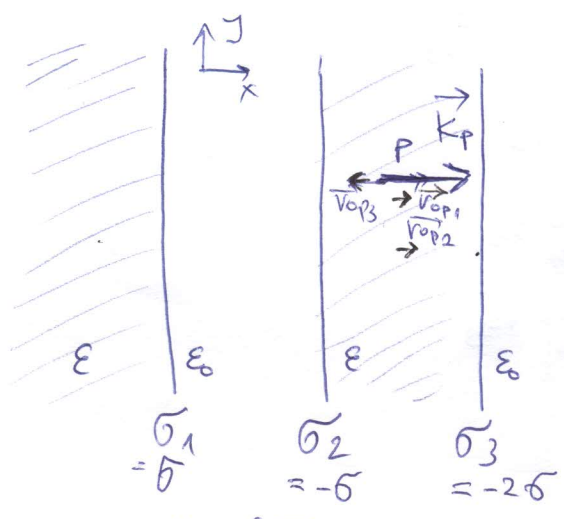
a) Primenom metode napona između čvorova ili metode konturnih struja odrediti intenzitet struje i snagu otpornika R_2 . (7 poena)

b) Primenom Tevenenove teoreme odrediti intenzitet struje I_{E3} kroz naponski generator E_3 . (10 poena)

5. U kolu na Slici 5 poznato je: $R_1 = 10 \Omega$, $R_2 = 5 \Omega$, $R_3 = 20 \Omega$, $E_1 = 23 \text{ V}$, $E_2 = 5 \text{ V}$, $I_g = 2 \text{ A}$, $C = 10 \mu\text{F}$. Odrediti količinu naelektrisanja kondenzatora u naznačenom smeru i elektrostatičku energiju kondenzatora. (7 poena)

Izrada kolokvijuma traje 90 minuta. Nije dozvoljena upotreba digitrona. Na vežbanci napisati broj grupe zadatka. Papir sa tekstom zadatka predaje se u vežbanci tj. ne sme se izneti.

1.



$$\vec{K}_P = \vec{K}_{P1} + \vec{K}_{P2} + \vec{K}_{P3}$$

$$\vec{K}_{P1} = \frac{\sigma_1}{2\epsilon} \vec{v}_{op1} = \frac{\sigma}{2\epsilon} (+\vec{i})$$

$$\vec{K}_{P2} = \frac{\sigma_2}{2\epsilon} \vec{v}_{op2} = \frac{-\sigma}{2\epsilon} (-\vec{j})$$

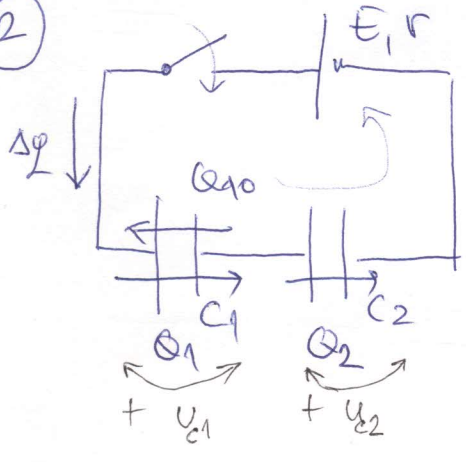
$$\vec{K}_{P3} = \frac{\sigma_3}{2\epsilon} \vec{v}_{op3} = \frac{-2\sigma}{2\epsilon} (-\vec{i})$$

$$\sigma = 20 \frac{\text{nC}}{\text{m}^2}$$

$$\epsilon = 10^{-10} \frac{\text{F}}{\text{m}}$$

$$\vec{K}_P = \left(\frac{\sigma}{2\epsilon} - \frac{\sigma}{2\epsilon} + \frac{2\sigma}{2\epsilon} \right) \vec{i} = \frac{\sigma}{\epsilon} \vec{i} = \frac{20 \cdot 10^{-9} \frac{\text{C}}{\text{m}^2}}{10^{-10} \frac{\text{F}}{\text{m}}} \vec{i} = 200 \frac{\text{V}}{\text{m}} \vec{i}$$

2.



$$C_1 = C_2 = 20 \mu\text{F}$$

$$E = 6\text{V}$$

$$r = 1\Omega$$

$$Q_{10} = 40 \mu\text{C}$$

$$Q_1 = -Q_{10} + \Delta q$$

$$Q_2 = \Delta q$$

$$U_{c1} + U_{c2} = E$$

$$\frac{Q_1}{C_1} + \frac{Q_2}{C_2} = E$$

$$\frac{\Delta q - Q_{10}}{C_1} + \frac{\Delta q}{C_2} = E$$

$$\Delta q \left(\frac{1}{C_1} + \frac{1}{C_2} \right) = E + \frac{Q_{10}}{C_1}$$

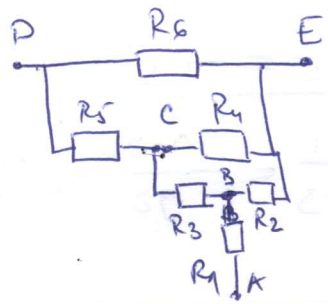
$$\Delta q = 80 \mu\text{C}$$

$$\Delta q = \frac{E + Q_{10}/C_1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{6 + \frac{40 \mu\text{C}}{20 \mu\text{F}}}{\frac{1}{20 \mu\text{F}} + \frac{1}{20 \mu\text{F}}} = \frac{8}{\frac{2}{20 \mu\text{F}}} = 80 \mu\text{C}$$

$$U_{c1} = \frac{Q_1}{C_1} = \frac{\Delta q - Q_{10}}{C_1} = \frac{80 \mu\text{C} - 40 \mu\text{C}}{20 \mu\text{F}} = \frac{40 \mu\text{C}}{20 \mu\text{F}} = 2\text{V}$$

$$Q_2 = \Delta q = 80 \mu\text{C}$$

3.



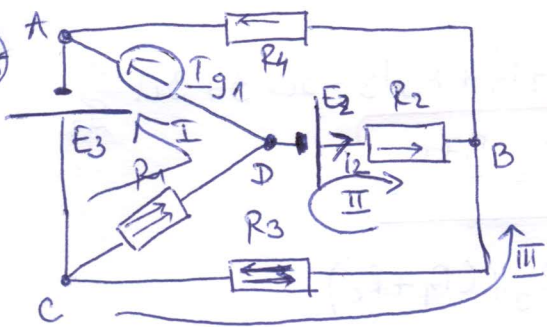
$$R_{DE} = R_6 \parallel (R_5 + R_4 \parallel (R_3 + R_2))$$

$$R_{DE} = R \parallel (R + R \parallel 2R) = R \parallel (R + \frac{R \cdot 2R}{3R}) = R \parallel \frac{5R}{3}$$

$$R_{DE} = \frac{R \cdot \frac{5R}{3}}{\frac{5R}{3} + R} = \frac{5R^2}{8R} = \frac{5}{8} R = \frac{5}{8} \cdot 16\Omega = 10\Omega$$

$R_{DE} = 10\Omega$

4A



$n_C = 4, n_g = 6 \Rightarrow n_g - n_C + 1 = 3$ jednačine po metodi KS, $u_{sg} = 1 \Rightarrow$ Realno 2 jednačine
 $n_C = 4 \Rightarrow n_C - 1 = 3$ jed. po metodi napona između čvorova, $n_{ng} = 1 \Rightarrow$ Realno 2 jednačine

$$I_I = I_{g1}$$

$$R_1 I_I + (R_1 + R_2 + R_3) I_{II} - R_3 I_{III} = E_2$$

$$-R_3 I_{II} + (R_3 + R_4) I_{III} = E_3$$

$$3R I_{II} - R I_{III} = E_2 - R_1 I_{g1} / 2$$

$$-R I_{II} + 2R I_{III} = E_3$$

$$-R I_{II} + 2R I_{III} = E_3$$

$$6R I_{II} - 2R I_{III} - R I_{II} + 2R I_{III} = 2E_2 - 2R I_{g1} + E_3 \Rightarrow 5R I_{III} = 2E_2 + E_3 - 2R I_{g1}$$

$$I_{III} = \frac{2E_2 + E_3 - 2R I_{g1}}{5R} = \frac{20 + 10 - 2 \cdot 30 \cdot 3}{5 \cdot 30} = \frac{30 - 6 \cdot 30}{5 \cdot 30} = \frac{1 - 6}{5} = -1A$$

$I_{II} = -1A$

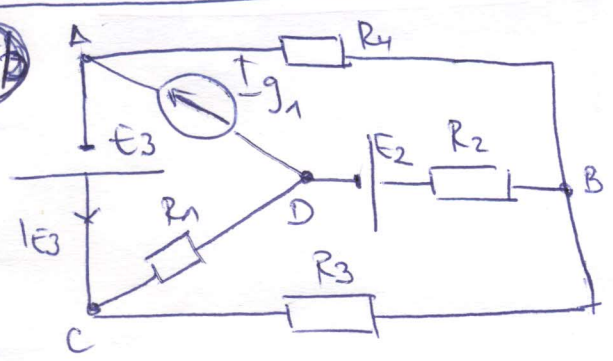
$I_{III} = -\frac{1}{3}A$

$$I_{II} = \frac{E_3 + R I_{III}}{2R} = \frac{10 + 30(-1)}{2 \cdot 30} = \frac{10 - 30}{60} = \frac{-20}{60} = -\frac{1}{3}A$$

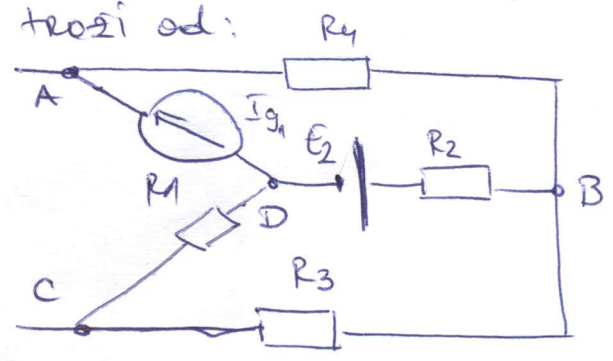
$I_2 = I_{II} = -1A$

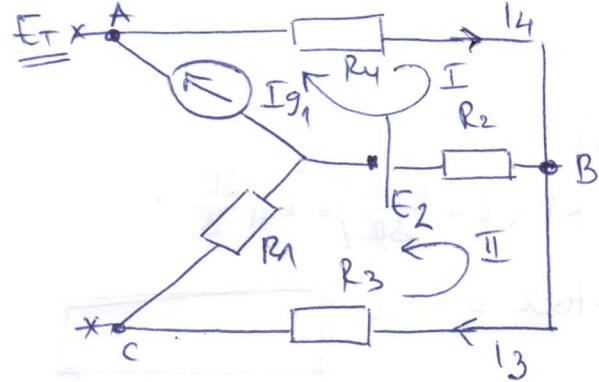
$P_{R2} = R_2 I_2^2 = 30W$

4B



Trži se struj kroz E_3 pa se on "čuva", a Theveninov generator se trži od:





$$I_I = I_{g1}$$

$$R_2 I_{g1} + (R_1 + R_2 + R_3) I_{II} = -E_2$$

$$I_{II} = \frac{-E_2 - R_2 I_{g1}}{R_1 + R_2 + R_3} = \frac{-10 - 90}{90} = \frac{-100}{90}$$

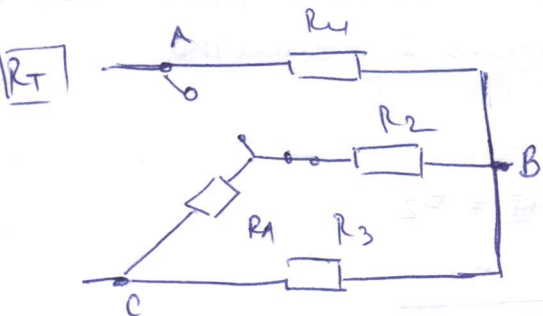
$$I_{g1} = \frac{-10}{9} \text{ A}$$

$$I_4 = I_I = I_{g1} = 3 \text{ A}$$

$$I_3 = -I_{II} = \frac{10}{9} \text{ A}$$

$$E_T = U_{AC}^{(ov)} = R_4 I_4 + R_3 I_3 = 30 \cdot 3 + 30 \cdot \frac{10}{9}$$

$$E_T = 90 + \frac{100}{3} = \frac{370}{3} \text{ V}$$

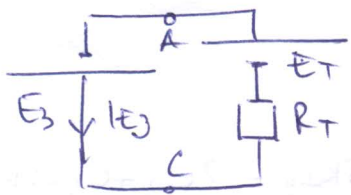


$$R_T = R_{AC} = R_4 + R_3 \parallel (R_1 + R_2)$$

$$R_T = R + R \parallel 2R = R + \frac{R \cdot 2R}{3R} = \frac{5R}{3}$$

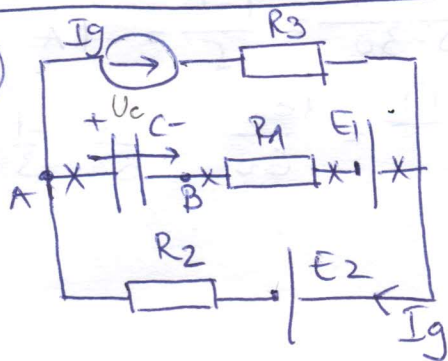
$$R_T = \frac{5 \cdot 30}{3} = 50 \text{ } \Omega$$

Kad se vrati u osnovnu ložu:



$$I_{E3} = \frac{E_3 + E_T}{R_T} = \frac{10 + \frac{370}{3}}{50} = \frac{30 + 370}{3 \cdot 50} = \frac{400}{150}$$

$$I_{E3} = \frac{8}{3} \text{ A}$$



$$U_c = U_{AB} = E_1 - E_2 - R_2 I_g = 23 - 5 - 5 \cdot 2$$

$$U_c = 23 - 15 = 8 \text{ V}$$

$$Q = U_c \cdot C = 80 \text{ } \mu\text{C}$$

$$W_c = \frac{1}{2} Q U_c = \frac{1}{2} \cdot 80 \text{ } \mu\text{C} \cdot 8 \text{ V} = 320 \text{ } \mu\text{J}$$