

1. U kolu na slici 1, metodom Kirhofovih zakona (metod struja u granama) odrediti struje u svim granama, potencijale tačaka A, B, C i D, napon U_{BD} , kao i snage svih elemenata u kolu. Poznato je: $E_1 = 6\text{ V}$, $E_2 = 2\text{ V}$, $E_3 = 10\text{ V}$, $R_1 = 10\Omega$ i $R_2 = 20\Omega$.

Rešenje:

Kolo na slici 1 ima $n_C=2$ čvora (A i C) i $n_G=3$ grane (granu sa idealnim naponskim generatorom E_3 , granu sa generatorom E_2 i granu sa generatorom E_1).

Stoga postoji $n_C-1=1$ jednačina po I KZ i $n_G - n_C + 1 = 3-2+1=2$ jednačine po II KZ.

IKZ: $A: I_3 + I_1 - I_2 = 0$.

IIKZ: $k_1: R_1 I_1 - E_1 + E_3 = 0$

$k_2: E_2 - R_2 I_2 + E_3 = 0$.

Iz II KZ $\rightarrow I_1 = \frac{E_1 - E_3}{R_1} = \frac{6 - 10}{10} = -0,4\text{ A}$ i $I_2 = \frac{E_2 + E_3}{R_2} = \frac{2 + 10}{20} = 0,6\text{ A}$.

Iz I KZ $\rightarrow I_3 = I_2 - I_1 = 0,6 - (-0,4) = 1\text{ A}$.

Da bi se odredio potencijal potrebno je da neku tačku označimo nultim potencijalom.

Na primer neka to bude tačka C, označena 0 na slici 1a). Tada je:

$V_A = E_3 = R_2 I_2 - E_2 = E_1 - R_1 I_1 = 10\text{ V}$

$V_B = R_2 I_2 = E_3 + E_2 = E_1 - R_1 I_1 + E_2 = 12\text{ V}$

$V_D = E_3 + R_1 I_1 = R_2 I_2 - E_2 + R_1 I_1 = E_1 = 6\text{ V}$

Napon U_{BD} : $U_{BD} = V_B - V_D = 6\text{ V}$, ali može se izračunati i kao:

$U_{BD} = -E_1 + R_2 I_2 = -6 + 20 \cdot 0,6 = 6\text{ V}$ ili $U_{BD} = -R_1 I_1 + E_2 = -E_1 + E_3 + E_2$.

Snaga generatora E_1 iznosi: $P_{E1} = E_1 I_1 = -2,4\text{ W}$

Snaga generatora E_2 iznosi: $P_{E2} = E_2 I_2 = 1,2\text{ W}$

Snaga generatora E_3 iznosi: $P_{E3} = E_3 I_3 = 10\text{ W}$

Snaga otpornika R_1 iznosi: $P_{R1} = R_1 I_1^2 = \frac{U_{R1}^2}{R_1} = \frac{U_{AD}^2}{R_1} = \frac{(V_A - V_D)^2}{R_1} = 1,6\text{ W}$

Snaga otpornika R_2 iznosi: $P_{R2} = R_2 I_2^2 = \frac{U_{R2}^2}{R_2} = \frac{U_{BC}^2}{R_2} = \frac{(V_B - V_C)^2}{R_2} = \frac{V_B^2}{R_2} = 7,2\text{ W}$

2. Odrediti struje u svim granama kola prikazanog na Slici 2. Poznato je:

$E = 4\text{ V}$, $r = 0,4\Omega$, $R = 1,4\Omega$.

Rešenje:

Kolo na slici 2 ima $n_C=4$ čvora (A, B, C i D) i $n_G=6$ grana (granu sa naponskim generatorom E i pet grana sa otpornicima).

Stoga postoji $n_C-1=3$ jednačina po I KZ i $n_G - n_C + 1 = 6-4+1=3$ jednačina po II KZ.

IKZ

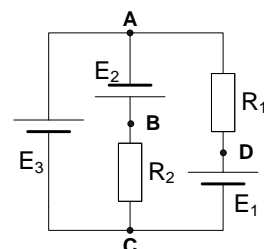
$A: I_E - I_5 - I_1 = 0$

$B: I_1 - I_2 - I_3 = 0 \Rightarrow I_1 = I_2 + I_3$

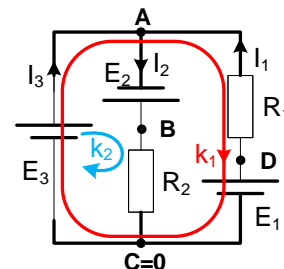
$C: I_2 + I_3 - I_4 = 0 \Rightarrow I_4 = I_2 + I_3$, znači I_1 i I_4 su iste struje.

IIKZ: $k_1: E - r I_E - 2R I_5 = 0$

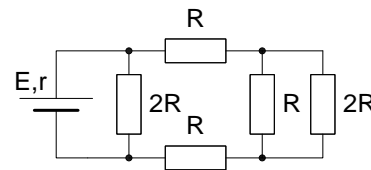
$k_2: 2R I_5 - R I_1 - R I_2 - R I_4 = 0$



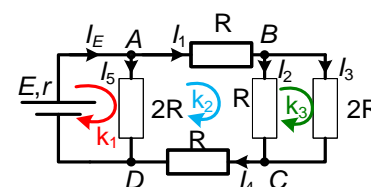
Slika 1



Slika 1a)



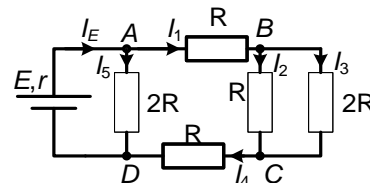
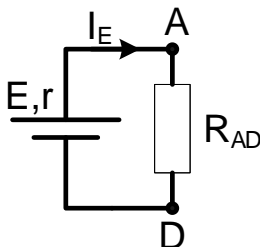
Slika 2



$$k_3: RI_2 - 2RI_3 = 0 \Rightarrow I_2 = 2I_3$$

Međutim, ovakvo rešavanje zadataka je suviše komplikovano i ima previše jednačina. Jednostavnije je da se nađe ekvivalentna otpornost između tačaka A i D, Re:

$$R_{AD} = \frac{1}{\frac{1}{2R} + \frac{1}{R + \frac{1}{\frac{1}{R} + \frac{1}{2R}} + R}} = \frac{8R}{7} = 1,6\Omega$$



$$I_E = \frac{E}{R_{AD} + r} = 2A, U_{AD} = R_{AD}I_E = 3,2V, I_5 = \frac{U_{AD}}{2R} = \frac{8}{7}A.$$

$$\text{IKZ A: } I_E - I_5 - I_1 = 0 \Rightarrow I_1 = I_E - I_5 = \frac{6}{7}A$$

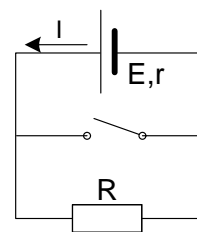
$$\text{IKZ B: } I_1 - I_2 - I_3 = 0 \Rightarrow I_1 = I_2 + I_3$$

$$\text{IKZ C: } I_2 + I_3 - I_4 = 0 \Rightarrow I_4 = I_2 + I_3, \text{ znači } I_1 \text{ i } I_4 \text{ su iste struje, pa je } I_4 = I_1 = \frac{6}{7}A.$$

$$U_{BC} = R_{BC}I_1 = \frac{1}{\frac{1}{R} + \frac{1}{2R}}I_1 = 0,8V, I_2 = \frac{U_{BC}}{R} = \frac{4}{7}A, I_3 = \frac{U_{BC}}{2R} = \frac{2}{7}A.$$

$$\text{Provera: } C: I_2 + I_3 - I_4 = \frac{4}{7} + \frac{2}{7} - \frac{6}{7} = 0$$

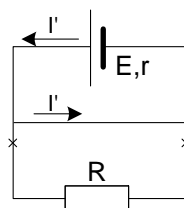
3. U kolu na slici 3, pri zatvorenom prekidaču, izmerena je struja u grani sa generatorom $I' = 2A$. Kada se prekidač otvori, kroz generator teče struja $I'' = 1A$. Poznata je otpornost $R = 20\Omega$. Odrediti parametre naponskog generatora E i r .



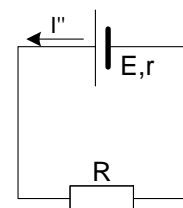
Slika 3

Rešenje:

Kada je prekidač zatvoren, struja ne prolazi kroz granu sa otpornikom jer je on kratko spojen, slika 3a), pa je struja kroz generator: $I' = \frac{E}{r} = 2A$, Kada je prekidač otvoren struja teče kroz otpornik R , slika 3b).

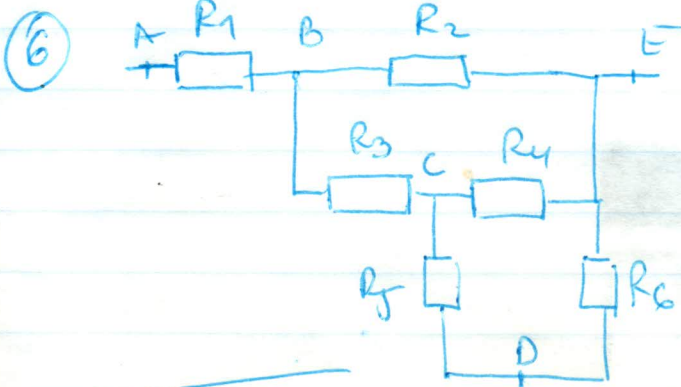


Slika 3a)



Slika 3b)

Tada je struja u kolu $I'' = \frac{E}{R+r} = 1A$: Odatle se dobija vrednost unutrašnje otpornosti generatora r i elektromotorne sile E : $r = R \frac{I'}{I' - I''} = 20 \cdot \frac{1}{2-1} = 20\Omega$ i $E = I' r = 2 \cdot 20 = 40V$.

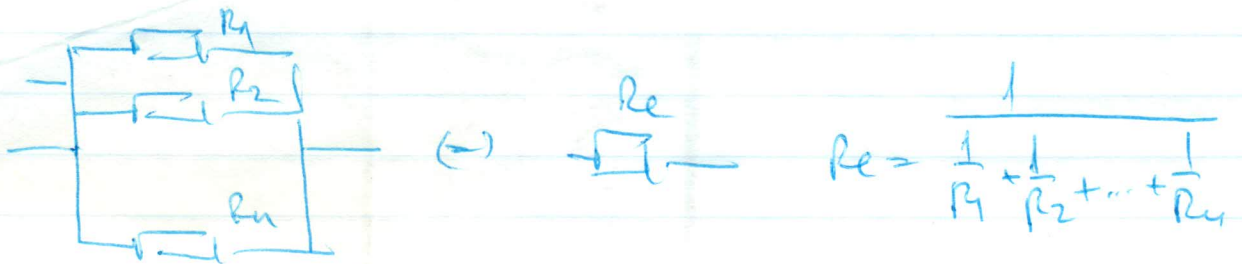


$$R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = R = 16 \Omega$$

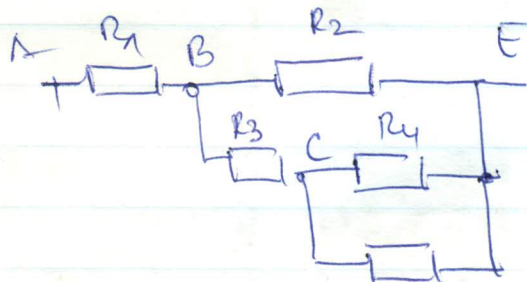
$$R_{AE} = ?$$

$$R_{DE} = ?$$

$$R_{CD} = ?$$



Resonanz:

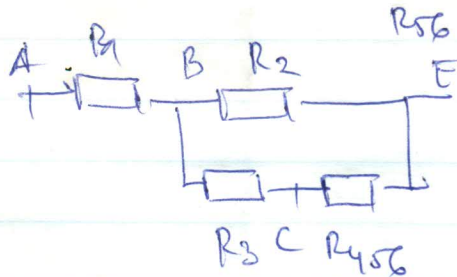


$$R_{56} = R_5 + R_6 = 2R$$

$$R_{456} = \frac{1}{\frac{1}{R_4} + \frac{1}{R_{56}}} = \frac{1}{\frac{1}{R} + \frac{1}{2R}} = \frac{2}{3}R$$

$$R_{3456} = R_3 + R_{456} = \frac{5}{3}R$$

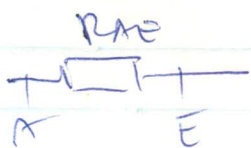
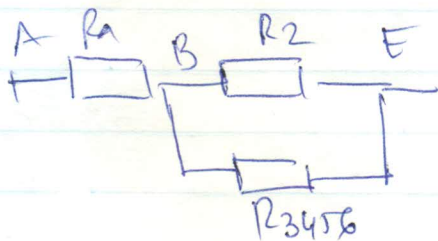
$$\rightarrow = \frac{1}{\frac{1}{2} + \frac{3}{5R}} = \frac{5R}{8}$$



$$R_{23456} = \frac{1}{\frac{1}{R_2} + \frac{1}{R_{3456}}}$$

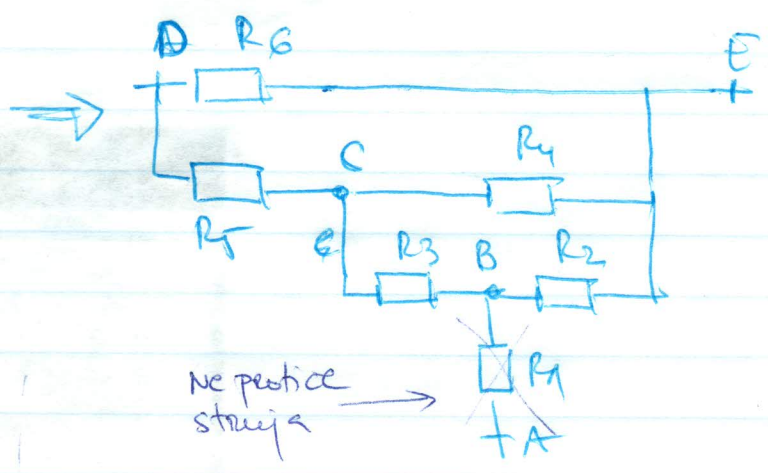
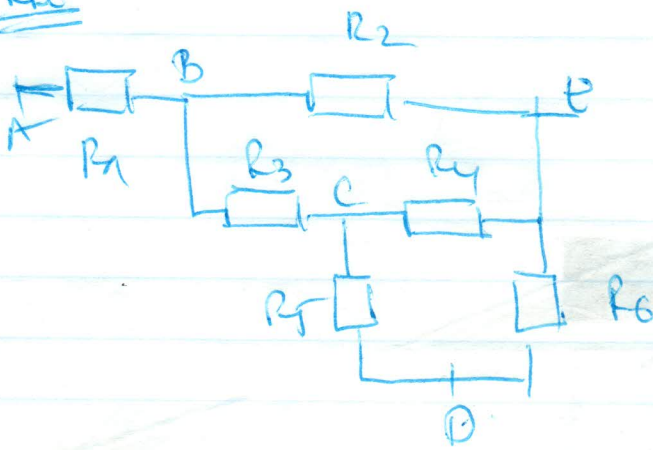
$$R_{AE} = R_1 + R_{23456} = R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3 + \frac{1}{\frac{1}{R_4} + \frac{1}{R_5 + R_6}}}}$$

$$= R + \frac{5R}{8} = \frac{13}{8}R$$

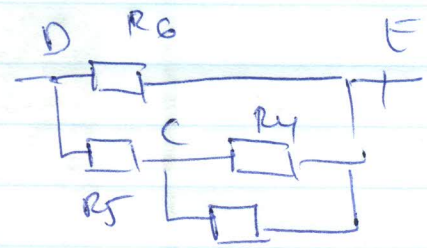


$R_{AE} = 26 \Omega$

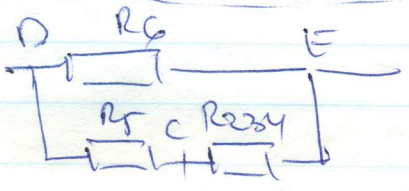
R_{DE}



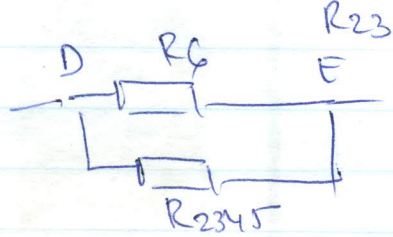
$$R_{DE} = \frac{1}{\frac{1}{R_6} + \frac{1}{R_5 + \frac{1}{\frac{1}{R_4} + \frac{1}{R_3 + R_2}}}}$$



$$R_{23} = R_2 + R_3 = 2R$$



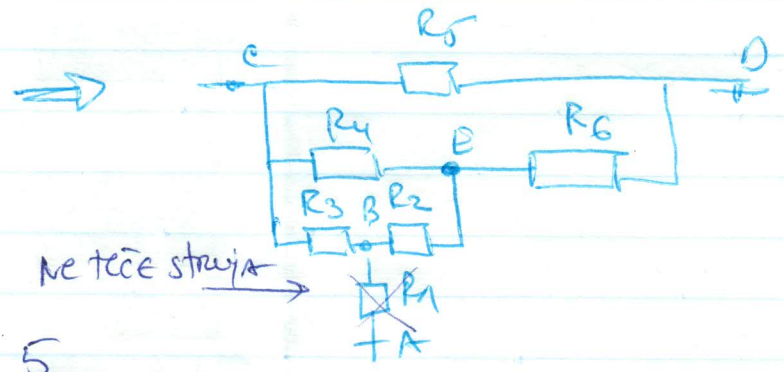
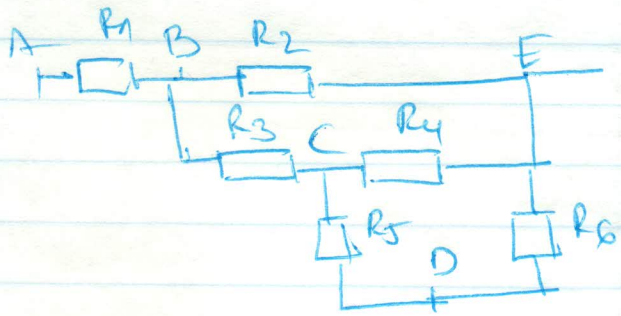
$$R_{234} = \frac{1}{\frac{1}{R_4} + \frac{1}{R_{23}}} = \frac{1}{\frac{1}{R} + \frac{1}{2R}} = \frac{2R}{3}$$



$$R_{2345} = R_5 + R_{234} = R + \frac{2R}{3} = \frac{5R}{3}$$

$$R_{DE} = \frac{1}{\frac{1}{R_6} + \frac{1}{R_{2345}}} = \frac{1}{\frac{1}{R} + \frac{3}{5R}} = \frac{5}{8}R$$

$R_{DE} = 10\Omega$



$$R_{CD} = \frac{1}{\frac{1}{R_5} + \frac{1}{R_6 + \frac{1}{\frac{1}{R_4} + \frac{1}{R_2 + R_3}}}} = \frac{5}{8}R = 10\Omega$$