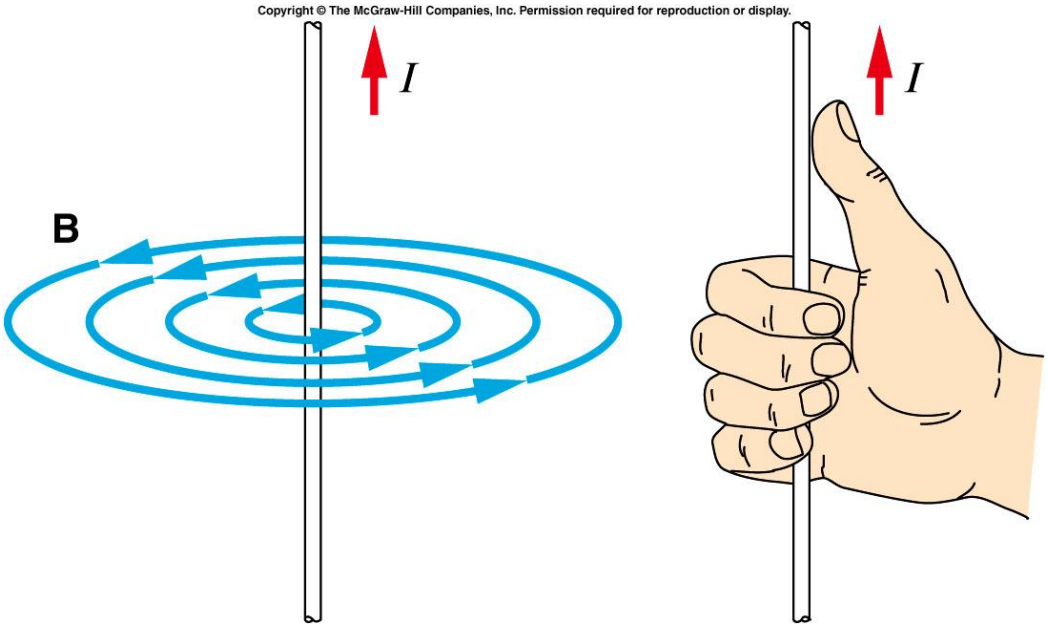
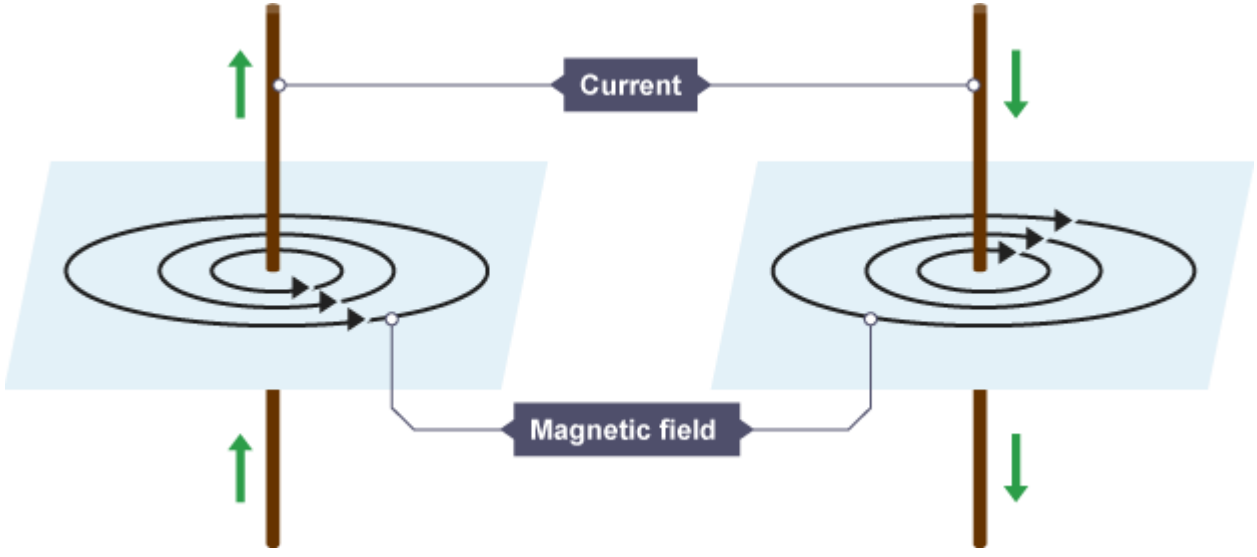
Magnetizam – 1 čas

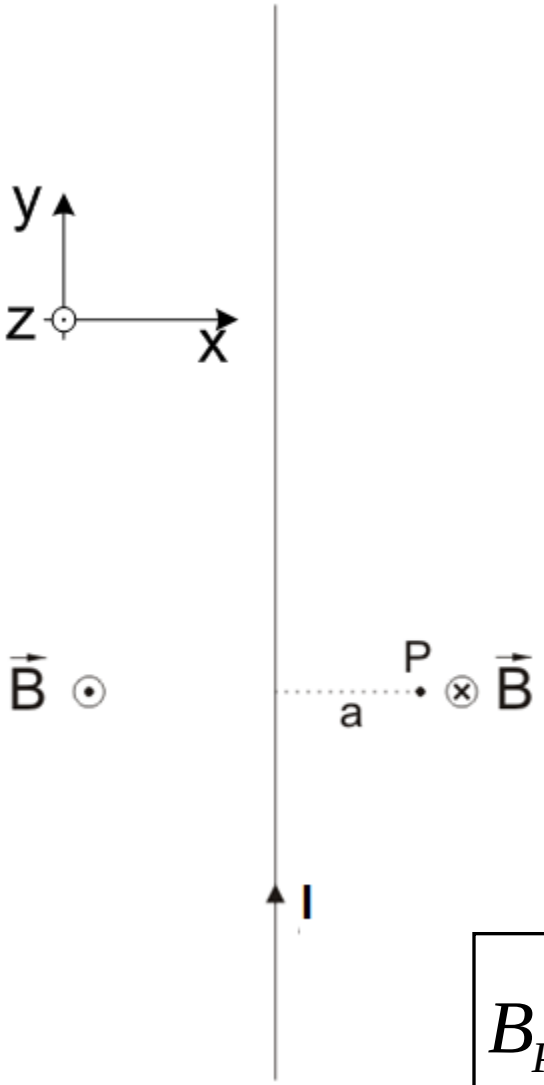
Provodnici

Magnetna indukcija u okolini besoknčnog provodnika





Magnetna indukcija u okolini besoknčnog provodnika



- **<u>Intenzitet</u>** vektora magnetne indukcije u okolini beskonačno dugačkog pravolinijskog provodnika je:

$$B = \frac{\mu_0 \mu_r I}{2\pi a}$$

- I – intenzitet struje u provodniku,
- a – normalno rastojanje od tačke (u kojoj određujemo vektor magnetne indkucije) do provodnika,
- μ_r – relativna magnetna permeabilnost sredine u kojoj se provodnik nalazi,
- $\mu_0 = 4\pi \cdot 10^{-7}$ H/m –magnetna permeabilnost vakuma
- **<u>Intenziteti vektora</u>** magnetne indukcije i jačine magnetnog polja u tački P iznose:

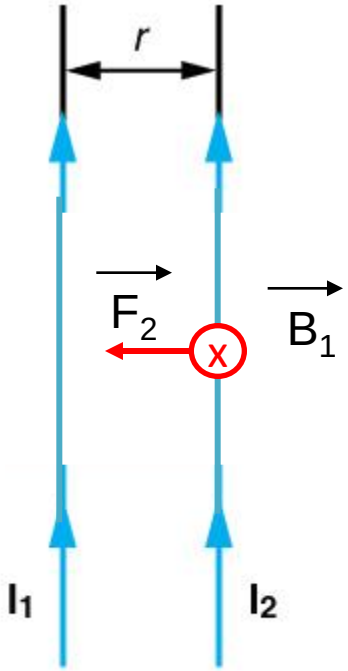
$$B_P = \frac{\mu_0 \mu_r I}{2\pi a}\,[\mathrm{T}]$$

$$H_P = \frac{B_P}{\mu_0 \mu_r} = \frac{I}{2\pi a}\left[\frac{\mathrm{A}}{\mathrm{m}}\right]$$

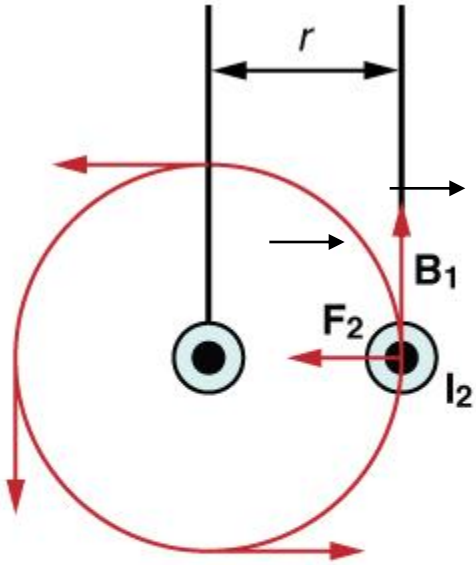
Magnetna sila

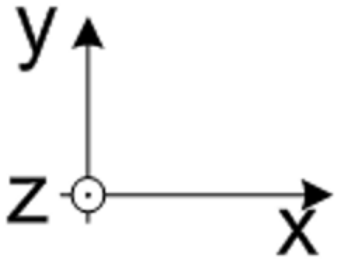
$$\overrightarrow{B_1} = \frac{\mu_0 \mu_r I_1}{2\pi r}\left(-\vec{k}\right) \quad \overrightarrow{l_{20}} = \vec{j}$$

$$\overrightarrow{B_1} = \frac{\mu_0 \mu_r I_1}{2\pi r}\,\vec{j} \quad \overrightarrow{l_{20}} = \vec{k}$$



(a)



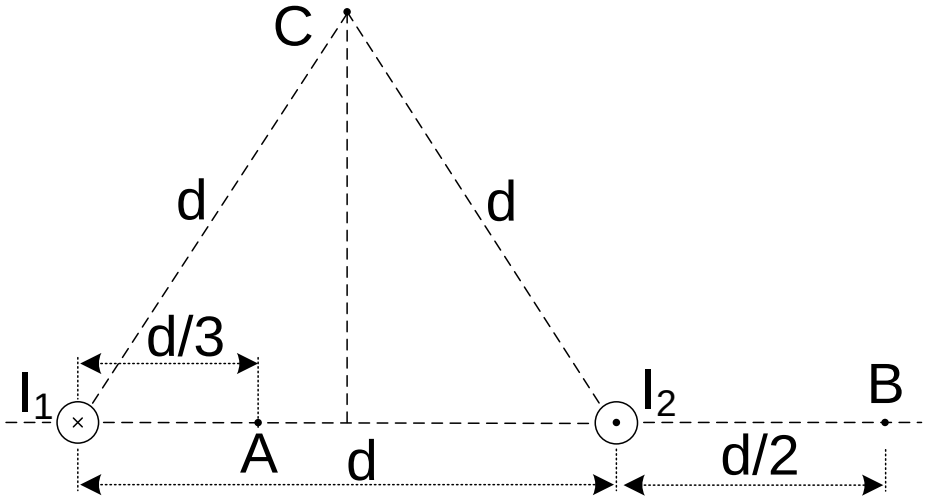


(b)

$$\overrightarrow{F_2'} = \frac{\overrightarrow{F_2}}{\left|\overrightarrow{l_2}\right|} = I_2 \frac{\overrightarrow{l_2}}{\left|\overrightarrow{l_2}\right|} \times \overrightarrow{B_1} = I_2 \overrightarrow{l_{20}} \times \overrightarrow{B_1} = \frac{\mu_0 \mu_r I_1 I_2}{2\pi r}\left(-\vec{i}\right)\left[\frac{\mathrm{N}}{\mathrm{m}}\right]$$

Zadatak 1

- Na slici je prikazan poprečni presek dva paralelna veoma dugačka provodnika, kroz koje protiču struje intenziteta $I_1 = I$ i $I_2 = 2I$ u označenim smerovima. Provodnici se nalaze u vazduhu ($\mu_0 = 4\pi \cdot 10^{-7}$ H/m).
- a) Odrediti i nacrtati rezultujući vektor jačine magnetnog polja u tačkama A, B i C.
- b) Odrediti i nacrtati vektor **podužne** sile kojom provodnik sa strujom I_2 deluje na provodnik sa strujom I_1.

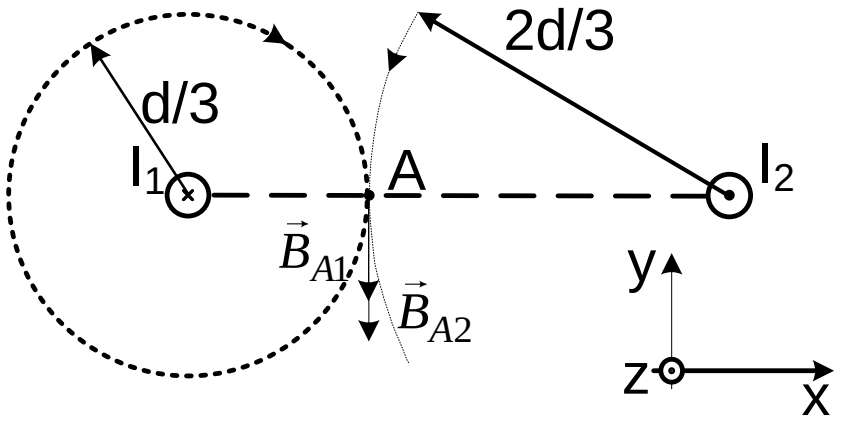


Zadatak 1 a) – polje u tački A

$$\vec{B}_A = \vec{B}_{A1} + \vec{B}_{A2}$$

$$\vec{B}_{A1} = \frac{\mu_0 I_1}{2\pi \frac{d}{3}}(-\vec{j}) = -\frac{3\mu_0 I_1}{2\pi d}\vec{j}$$

$$\vec{B}_{A2} = \frac{\mu_0 I_2}{2\pi \cdot \frac{2d}{3}}(-\vec{j}) = -\frac{3\mu_0 I_2}{4\pi d}\vec{j}$$

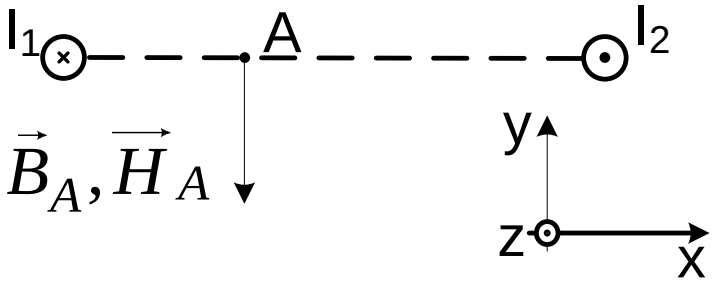


$$\vec{B}_A = -\frac{3\mu_0}{4\pi d}(2I_1 + I_2)\vec{j}$$

$$\vec{B}_A = -\frac{3\mu_0}{4\pi d}(2\cdot I + 2I)\vec{j}$$

$$\boxed{\vec{B}_A = -\frac{3\mu_0 I}{\pi d}\vec{j}\ [T]}$$

$$\boxed{\vec{H}_A = \frac{\vec{B}_A}{\mu_0} = -\frac{3I}{\pi d}\vec{j}\left[\frac{A}{m}\right]}$$



Zadatak 1 a) – polje u tački B

$$\vec{B}_B = \vec{B}_{B_1} + \vec{B}_{B_2}$$

$$\vec{B}_{B_1} = \frac{\mu_0 I_1}{2\pi \frac{3d}{2}} \left(-\vec{j}\right) = -\frac{\mu_0 I_1}{3\pi d}\vec{j}$$

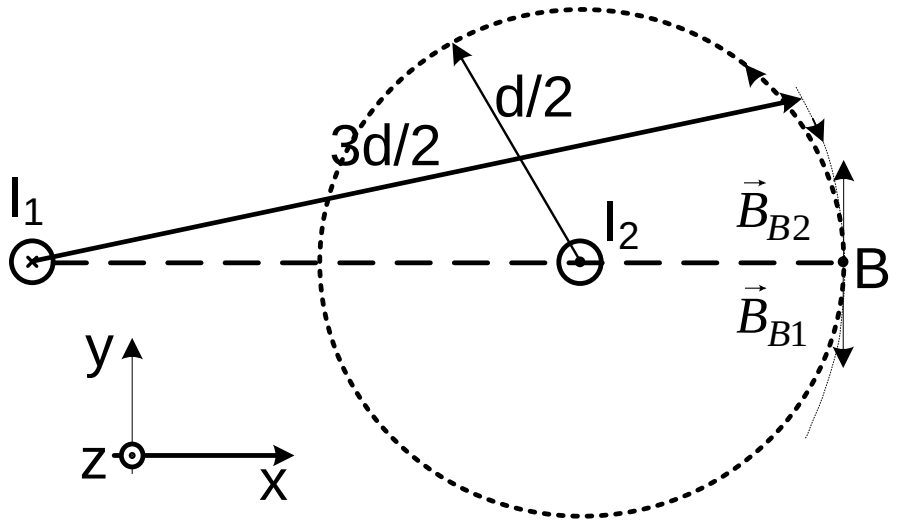
$$\vec{B}_{B_2} = \frac{\mu_0 I_2}{2\pi \cdot \frac{d}{2}}\vec{j} = \frac{\mu_0 I_2}{\pi d}\vec{j}$$

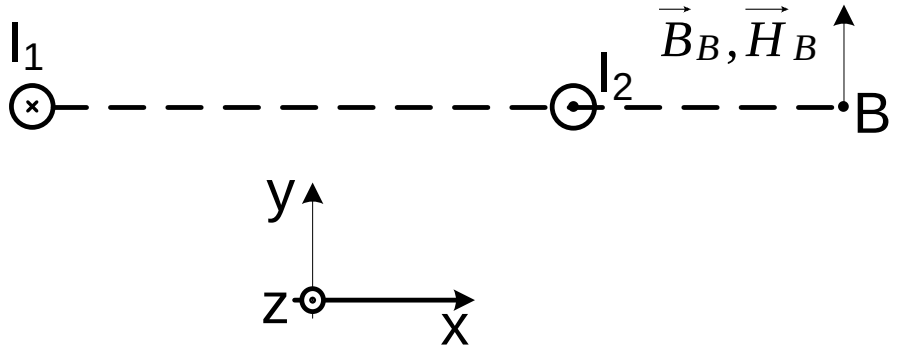
$$\vec{B}_B = \frac{\mu_0}{3\pi d}\left(-I_1 + 3I_2\right)\vec{j}$$

$$\vec{B}_B = \frac{\mu_0}{3\pi d}\left(-I + 6I\right)\vec{j}$$

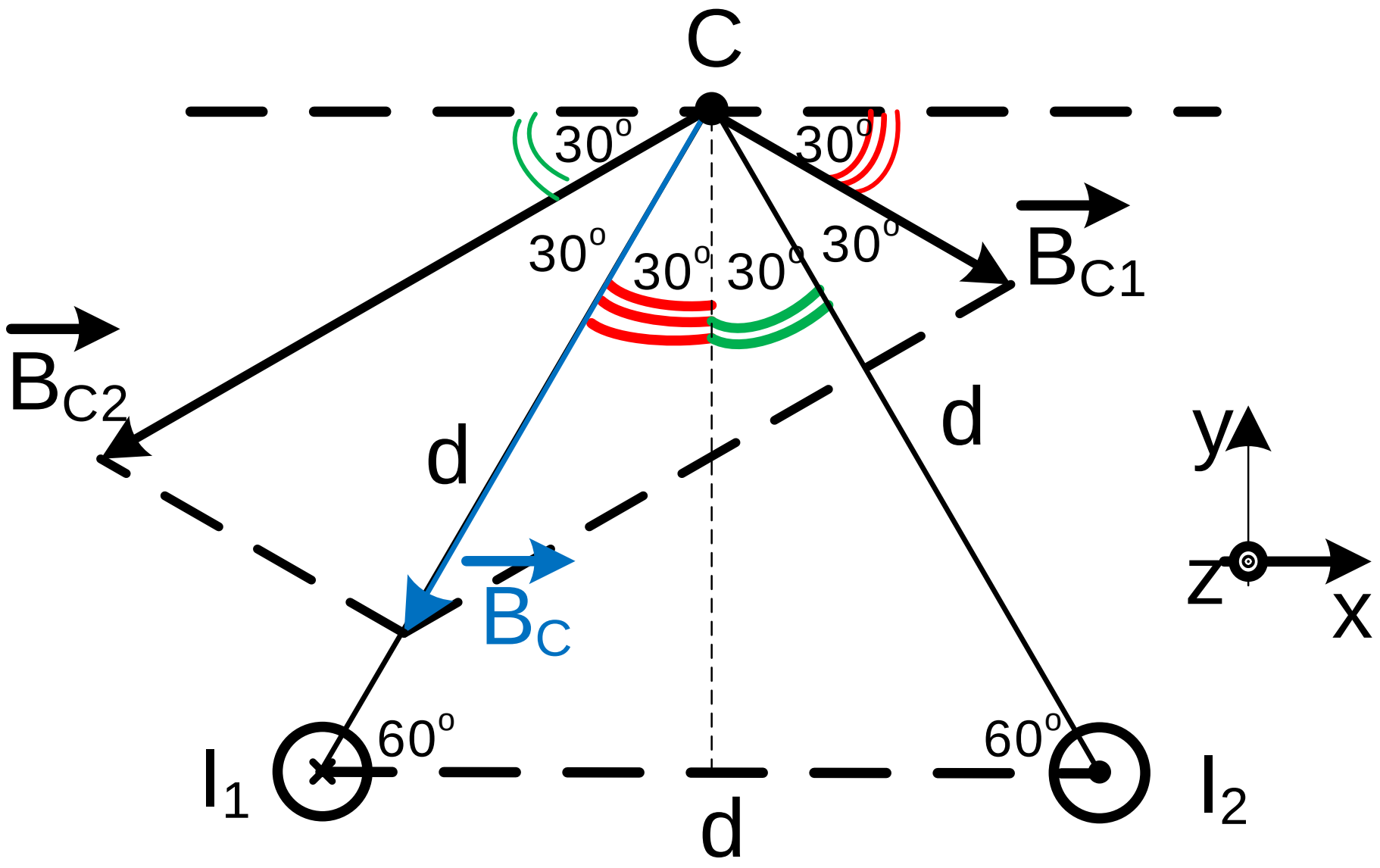
$$\vec{B}_B = \frac{5\mu_0 I}{3\pi d}\vec{j} \quad [T]$$

$$\vec{H}_B = \frac{\vec{B}_B}{\mu_0} = \frac{5I}{3\pi d}\vec{j} \quad \left[\frac{A}{m}\right]$$





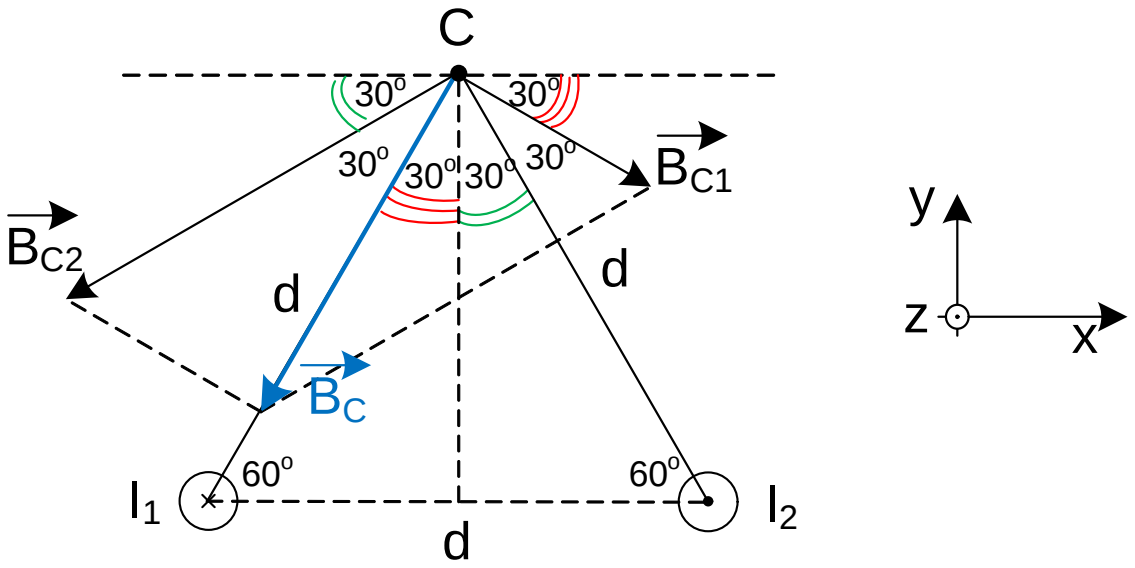
Zadatak 1 a) – polje u C



Zadatak 1 a) – polje u C

$$B_{C1} = \frac{\mu_0 I_1}{2\pi d} = \frac{\mu_0 I}{2\pi d}$$

$$B_{C2} = \frac{\mu_0 I_2}{2\pi d} = \frac{\mu_0 I}{\pi d}$$



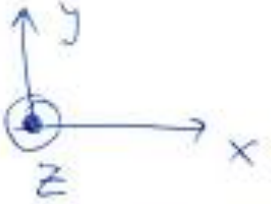
$$\vec{B}_{C1} = B_{C1}\cos 30^\circ \vec{i} - B_{C1}\sin 30^\circ \vec{j} = \frac{\sqrt{3}}{2} B_{C1}\vec{i} - \frac{1}{2} B_{C1}\vec{j}$$

$$\vec{B}_{C2} = -B_{C2}\cos 30^\circ \vec{i} - B_{C2}\sin 30^\circ \vec{j} = -\frac{\sqrt{3}}{2} B_{C2}\vec{i} - \frac{1}{2} B_{C2}\vec{j}$$

$$\vec{B}_C = \vec{B}_{C1} + \vec{B}_{C2} = \frac{\mu_0\sqrt{3}}{4\pi d}(I_1 - I_2)\vec{i} - \frac{\mu_0}{4\pi d}(I_1 + I_2)\vec{j} = -\frac{\mu_0 I\sqrt{3}}{4\pi d}\vec{i} - \frac{\mu_0 3I}{4\pi d}\vec{j}$$

$$\vec{B}_C = \frac{\mu_0 I\sqrt{3}}{2\pi d}\left(-\frac{1}{2}\vec{i} - \frac{\sqrt{3}}{2}\vec{j}\right) = -B_C\cos 60^\circ \vec{i} - B_C\sin 60^\circ \vec{j},\ B_C = \frac{\mu_0 I\sqrt{3}}{2\pi d}$$

Zadatak 1 b) – podužna sila



$\vec{i} \times \vec{j} = \vec{k}$

$\vec{j} \times \vec{k} = \vec{i}$

$\vec{k} \times \vec{i} = \vec{j}$

$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

$$\vec{F_{21}'} = I_1 \vec{l_{o_1}} \times \vec{B_2}$$

$$\vec{B_2} = \frac{\mu_0 I_2}{2\pi d}(-\vec{j})$$

$$\vec{l_{o_1}} = -\vec{k}$$

$$\vec{F_{21}'} = I_1(-\vec{k}) \times \frac{\mu_0 I_2}{2\pi d}(-\vec{j})$$

$$= \frac{\mu_0 I_1 I_2}{2\pi d} \vec{k} \times \vec{j}$$

$$\vec{F_{21}'} = -\frac{\mu_0 I_1 I_2}{2\pi d}\vec{i} \quad \left[\frac{N}{m}\right]$$

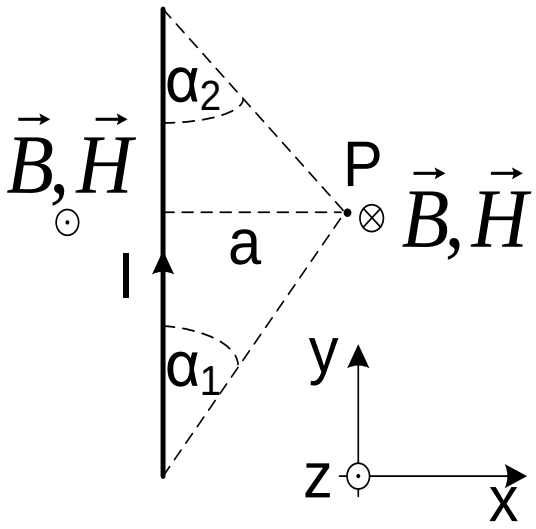
Magnetna indukcija u okolini provodnika konačne dužine

- Magnetna indukcija u okolini pravolinijskog provodnika konačne dužine u tački P, na rastojanju *a* od provodnika je:

$$B_P = \frac{\mu_0 \mu_r I}{4\pi a}\left(\cos\alpha_1 + \cos\alpha_2\right)$$

- Jačina magnetnog polja u tački P iznosi:

$$H_P = \frac{B_P}{\mu_0 \mu_r} = \frac{I}{4\pi a}\left(\cos\alpha_1 + \cos\alpha_2\right)$$



- Ukoliko provodnik konačne dužine "razvučemo" u beskonačni provodnik, uglovi α_1 i α_2 smanjuju se i teže nuli, pa se formula za provodnik konačne dužine svodi na formulu za beskonačni provodnik!

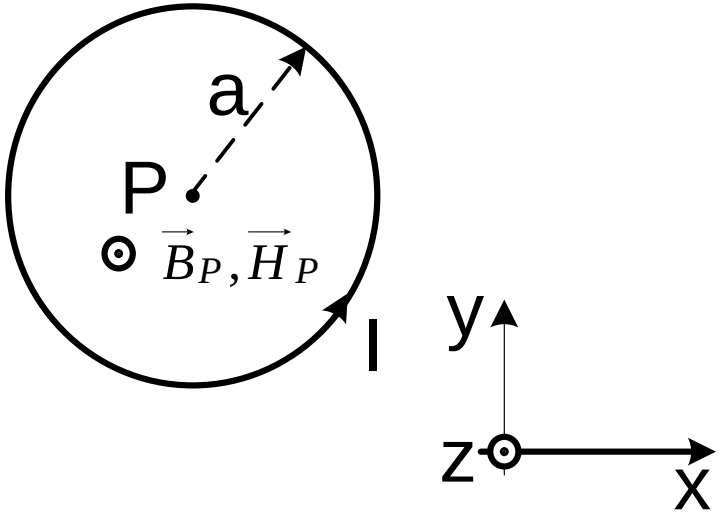
$$B_{P\_bekonacan} = \frac{\mu_0 \mu_r I}{4\pi a}\left(\cos 0^\circ + \cos 0^\circ\right) = \frac{\mu_0 \mu_r I}{4\pi a}(1+1) = \frac{\mu_0 \mu_r I}{2\pi a}$$

Magnetna indukcija u okolini kružnog provodnika

- Magnetne indukcija u centru kružnog provodnika poluprečnika *a*, je:

$$B_P = \frac{\mu_0 \mu_r I}{2a}$$

$\vec{B}, \vec{H}$ ⊗



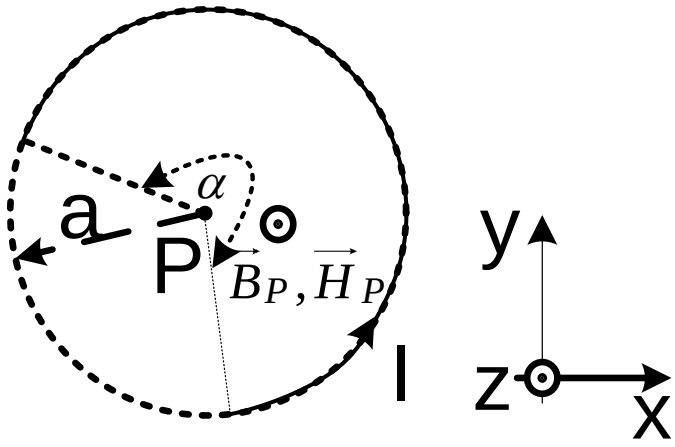
- Jačina magnetnog polja u tački P iznosi:

$$H_P = \frac{B_P}{\mu_0 \mu_r} = \frac{I}{2a}$$

- Ukoliko se radi o kružnom isečku:

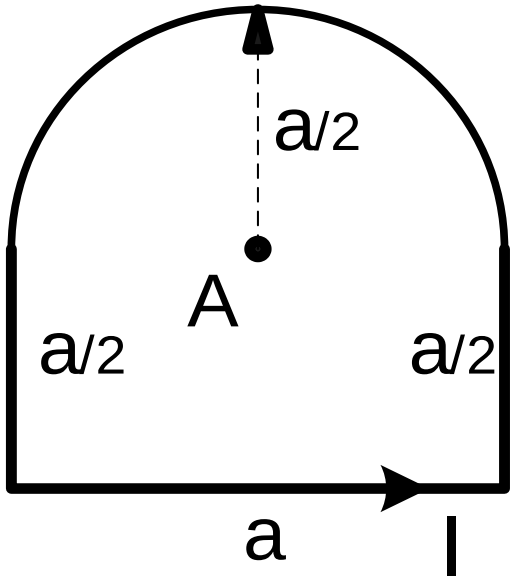
$$B_P = \frac{\mu_0 \mu_r I}{2a} \cdot \frac{\alpha}{2\pi} = \frac{\mu_0 \mu_r I}{4\pi a} \alpha$$

$$H_P = \frac{B_P}{\mu_0 \mu_r} = \frac{I}{2a} \cdot \frac{\alpha}{2\pi} = \frac{I}{4\pi a} \alpha$$



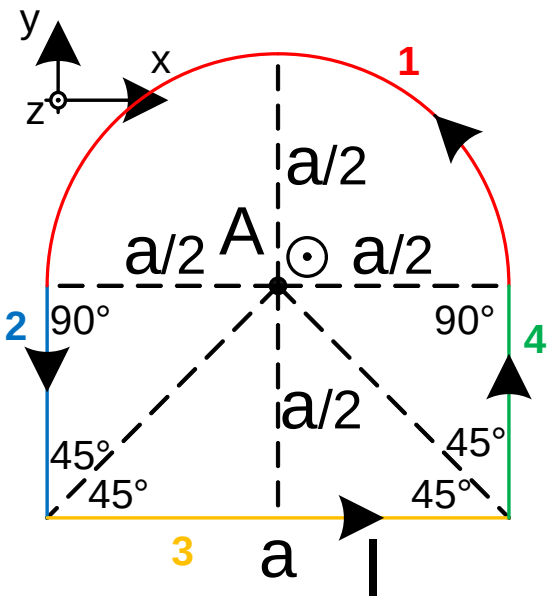
Zadatak 2

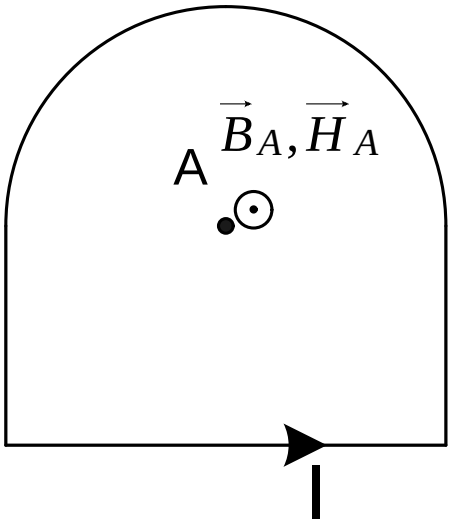
- Na slici je prikazana kontura koja se sastoji od polovine kružnice poluprečnika *a*/2, sa centrom u tački A, i donje polovine kvadrata dužine stranice *a*. Kroz konturu protiče struja intenziteta *I*. Kontura se nalazi u vazduhu ($\mu_0 = 4\ \pi \cdot 10^{-7}$ H/m). Odrediti i nacrtati vektor magnetne indukcije i vektor magnetnog polja u tački A.



Zadatak 2

- Kontura se sastoji iz 4 provodnika, 3 pravolinijska i jednog koji je pola kruga:





$$\vec{B}_A = \vec{B}_{A1} + \vec{B}_{A2} + \vec{B}_{A3} + \vec{B}_{A4}$$

$$\vec{B}_{A1} = \frac{\mu_0 I}{4\pi a/2} \cdot \pi \vec{k} = \frac{\mu_0 I}{2a} \vec{k}$$

$$\vec{B}_{A2} = \frac{\mu_0 I}{4\pi \frac{a}{2}} (\cos 90^\circ + \cos 45^\circ) \vec{k}$$

$$= \frac{\mu_0 I}{2\pi a} \cdot \frac{\sqrt{2}}{2} \vec{k} = \frac{\mu_0 I \sqrt{2}}{4\pi a} \vec{k}$$

$$\vec{B}_{A3} = \frac{\mu_0 I}{4\pi \frac{a}{2}} (\cos 45^\circ + \cos 45^\circ) \vec{k}$$

$$= \frac{\mu_0 I}{2\pi a} \cdot \sqrt{2} \vec{k} = \frac{\mu_0 I \sqrt{2}}{2\pi a} \vec{k}$$

$$\vec{B}_{A4} = \frac{\mu_0 I}{4\pi \frac{a}{2}} (\cos 45^\circ + \cos 90^\circ) \vec{k}$$

$$= \frac{\mu_0 I}{2\pi a} \cdot \frac{\sqrt{2}}{2} \vec{k} = \frac{\mu_0 I \sqrt{2}}{4\pi a} \vec{k}$$

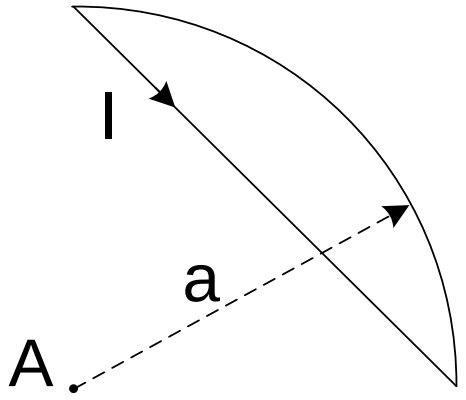
$$\vec{B}_A = \left(\frac{\mu_0 I}{2a} + \frac{\mu_0 I \sqrt{2}}{\pi a} \right) \vec{k} = \frac{\mu_0 I}{\pi a} \left(\sqrt{2} + \frac{\pi}{2} \right) \vec{k}$$

$$\boxed{\vec{B}_A \approx \frac{3\mu_0 I}{\pi a} \vec{k}}$$

$$\boxed{\vec{H}_A = \frac{\vec{B}_A}{\mu_0} = \frac{3I}{\pi a} \vec{k}}$$

Zadatak 3

- Na slici je prikazana kontura koja se sastoji od četvrtine kružnice poluprečnika *a*, sa centrom u tački A, i pravolinijskog segmenta. Kroz konturu protiče struja intenziteta *I*. Odrediti i nacrtati vektor magnetne indukcije u tački A. Kontura se nalazi u vazduhu ($\mu_0 = 4\pi \cdot 10^{-7}$ H/m).



Zadatak 3

- Kontura se sastoji iz 2 provodnika, 1 pravolinijskog i jednog koji je četvrtina kruga:

$$\vec{B}_A = \vec{B}_{A1} + \vec{B}_{A2}$$

$$\vec{B}_{A1} = \frac{\mu_0 I}{4\pi a} \cdot \frac{\pi}{2} \vec{k} = \frac{\mu_0 I}{8a} \vec{k}$$

$$\vec{B}_{A2} = \frac{\mu_0 I}{4\pi \frac{a\sqrt{2}}{2}} (\cos 45^\circ + \cos 45^\circ)(-\vec{k}) \qquad \cos 45^\circ = \frac{\sqrt{2}}{2}$$

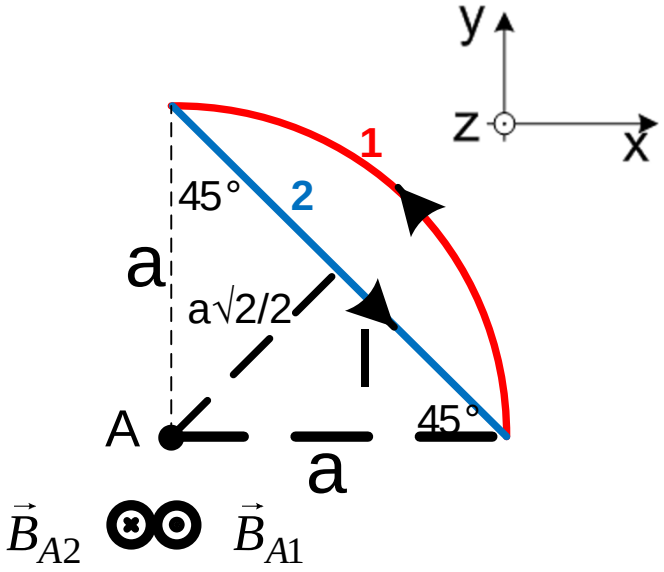
$$= \frac{\mu_0 I}{2\pi a \sqrt{2}} \cdot \sqrt{2} (-\vec{k}) = -\frac{\mu_0 I}{2\pi a} \vec{k}$$

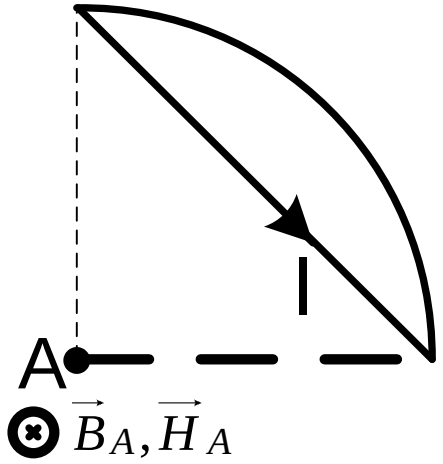
$$\vec{B}_A = \frac{\mu_0 I}{8a} \vec{k} - \frac{\mu_0 I}{2\pi a} \vec{k}$$

$$= \frac{\mu_0 I}{8a} \left(1 - \frac{4}{\pi}\right) \vec{k}$$

$$\vec{B}_A \simeq -\frac{0{,}27\,\mu_0 I}{8a} \vec{k}$$

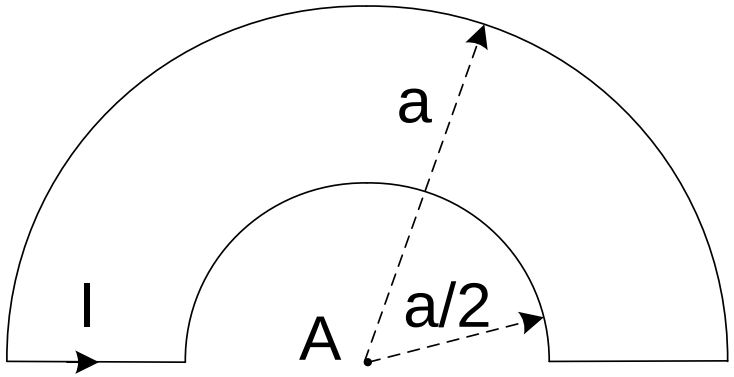
$$\vec{H}_A = \frac{\vec{B}_A}{\mu_0} = -\frac{0{,}27\,I}{8a} \vec{k}$$





Zadatak 4

- Na slici je prikazana kontura koja se sastoji od dve polovine kružnice poluprečnika $a/2$ i a, sa centrom u tački A, koje su povezane odgovarajućim pravolinijskim segmentima. Kroz konturu protiče struja intenziteta I. Odrediti i nacrtati vektor magnetne indukcije u tački A. Kontura se nalazi u vazduhu ($\mu_0 = 4\pi \cdot 10^{-7}$ H/m).



Zadatak 4

- Kontura se sastoji iz 4 provodnika, 2 pravolinijska i 2 polukruga:

$$\vec{B}_A = \vec{B}_{A1} + \vec{B}_{A2} + \vec{B}_{A3} + \vec{B}_{A4}$$

$$\vec{B}_{A_2} = \vec{B}_{A_4} = \vec{0}$$

$$\vec{B}_{A1} = \frac{\mu_0 I}{4\pi a} \cdot \pi \cdot (\vec{k})$$

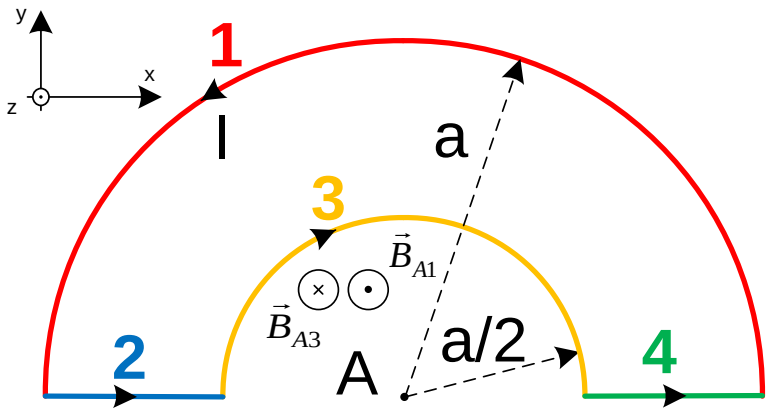
$$\vec{B}_{A_2} = \frac{\mu_0 I}{4\pi \frac{a}{2}} \cdot \pi \, (-\vec{k})$$

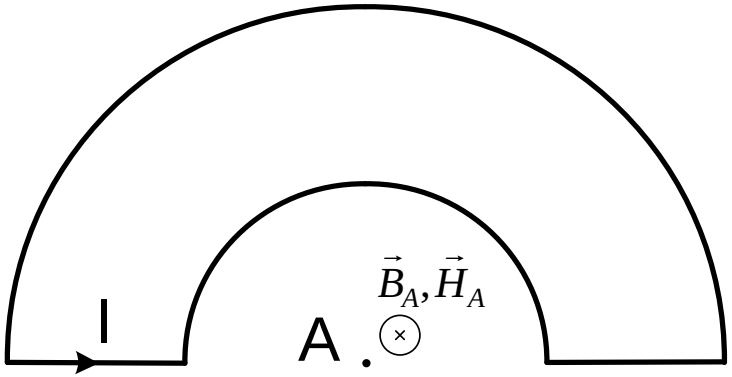
$$\vec{B}_A = \frac{\mu_0 I}{4a} \vec{k} - \frac{\mu_0 I}{4\frac{a}{2}} \vec{k}$$

$$= \frac{\mu_0 I}{4a} \vec{k} - \frac{\mu_0 I}{2a} \vec{k}$$

$$\boxed{\vec{B}_A = -\frac{\mu_0 I}{4a} \vec{k}}$$

$$\boxed{\vec{H}_A = \frac{\vec{B}_A}{\mu_0} = -\frac{I}{4a} \vec{k}}$$





Napomena!!!

- U prethodnim zadacima, ukoliko se provodnici nalaze u nekoj sredini koja nije vazduh, relativne magnetne permeabilnost μ_r u svim jednačinama, osim μ_0 figuriše i μ_r. Na primer: $$B = \frac{\mu_0 \mu_r I}{4\pi d}$$

- Obratiti pažnju da li je dato μ_r (RELATIVNA magnetna permeabilnost) ili μ (magnetna permeabilnost)!!!

$$\mu = \mu_0 \mu_r$$