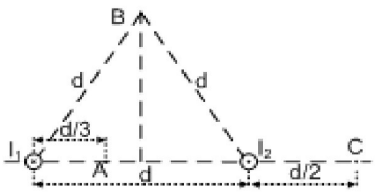
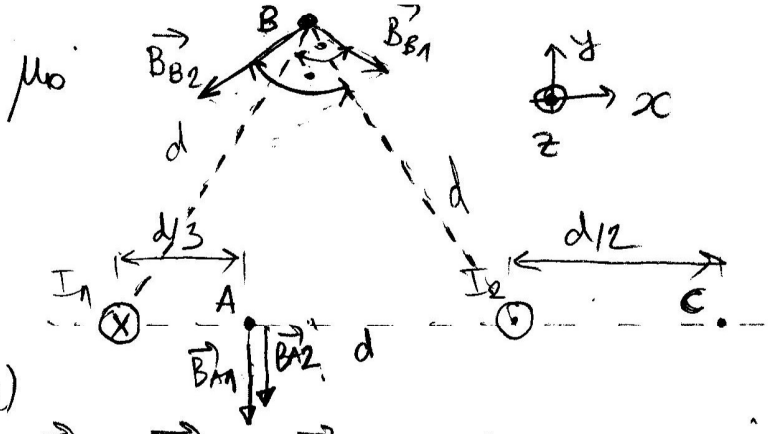
1. Na Slici 1 je prikazan poprečni presek dva paralelna veoma duga čka provodnika, kroz koje protiču struje intenziteta $I_1 = I$ i $I_2 = 2I$ u označenim smerovima. Provodnici se nalaze u vazduhu ($\mu_0 = 4\pi \cdot 10^{-7}$ H/m).

a) Odrediti i nacrtati rezultujući vektor magnetne indukcije u tačkama A, B i C.

b) Odrediti i nacrtati vektor podužne sile kojom provodnik sa strujom $I_2$ deluje na provodnik sa strujom $I_1$.




Slika 1




a)

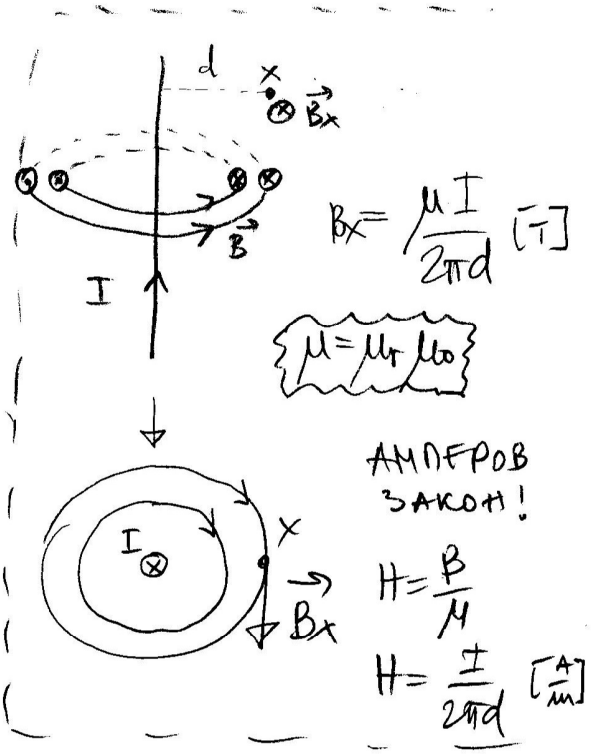
$\vec{B}_A = \vec{B}_{A1} + \vec{B}_{A2}$ – СУПЕРПОЗИЦИЈА!

$\vec{B}_{A1} = \frac{\mu_0 I_1}{2\pi \frac{d}{3}}(-\vec{j}) = \frac{3\mu_0 I}{2\pi d}(-\vec{j})$

$\vec{B}_{A2} = \frac{\mu_0 I_2}{2\pi \frac{2d}{3}}(-\vec{j}) = \frac{3\mu_0 2I}{4\pi d}(-\vec{j})$

$\vec{B}_A = \frac{12\mu_0 I}{4\pi d}(-\vec{j}) = -\frac{3\mu_0 I}{\pi d}\vec{j}$

$\vec{H}_A = \frac{\vec{B}_A}{\mu_0} = -\frac{3I}{\pi d}\vec{j}$




$\vec{B}_B = \vec{B}_{B1} + \vec{B}_{B2}$

$\vec{B}_{B2} = \frac{\mu_0 I_2}{2\pi d} \cdot \vec{r}_{0B_{B2}} = \frac{\mu_0 I 2}{2\pi d}(\cos(-150°)\vec{i} + \sin(-150°)\vec{j})$ $\quad \measuredangle(\vec{B}_{B2}, 0x)$

$\vec{B}_{B2} = \frac{2\mu_0 I}{2\pi d}(-\frac{\sqrt{3}}{2}\vec{i} - \frac{1}{2}\vec{j})$

$\vec{B}_{B1} = \frac{\mu_0 I_1}{2\pi d}\vec{r}_{0B_{B1}} = \frac{\mu_0 I_1}{2\pi d}(\cos(-30°)\vec{i} + \sin(-30°)\vec{j})$ $\quad \measuredangle(B_{B1}, 0x)$

$\vec{B}_{B2} = \frac{\mu_0 I}{2\pi d}(\frac{\sqrt{3}}{2}\vec{i} - \frac{1}{2}\vec{j})$

$\measuredangle(\vec{B}_B, 0x) = -120°$

$\vec{B}_B = \frac{\mu_0 I}{\pi d}(-\frac{\sqrt{3}}{4}\vec{i} - \frac{3}{4}\vec{j}) = \frac{\sqrt{3}\mu_0 I}{2\pi d}(-\frac{1}{2}\vec{i} - \frac{\sqrt{3}}{2}\vec{j})$

$$\vec{B_C} = \vec{B_{C1}} + \vec{B_{C2}}$$

$$\vec{B_{C1}} = \frac{\mu_0 I_1}{2\pi \frac{3d}{2}} (-\vec{j}) = -\frac{\mu_0 I}{3\pi d} \vec{j}$$

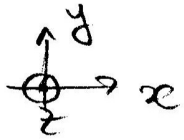
$$\vec{B_{C2}} = \frac{\mu_0 I_2}{2\pi \frac{d}{2}} \vec{j} = \frac{2\mu_0 I}{\pi d} \vec{j}$$

$$+$$

$$\vec{B_C} = \frac{5}{3} \frac{\mu_0 I}{\pi d} \vec{j}$$

---

б)





$\vec{F_{21}} = I_1 \vec{\ell} \times \vec{B}_{2\to1}$ – сила която действа на пров. $I_1$ дължине "$\ell$" = произволна

$\vec{F'_{21}} = \frac{\vec{F_{21}}}{\ell}$ – линейна сила

$\vec{B}_{2\to1}$ – вектор магн. инд. проводника $I_2$ на място $I_1$

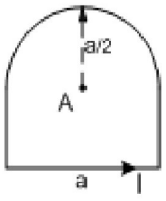
$$\vec{F_{21}} = I_1 \ell (-\vec{k}) \times \frac{\mu_0 I_2}{2\pi d} (-\vec{j})$$

$$\vec{F_{21}} = \frac{\mu_0 I_1 I_2 \ell}{2\pi d} \underbrace{(-\vec{k}) \times (-\vec{j})}_{\vec{k} \times \vec{j} = -\vec{i}\,!}$$

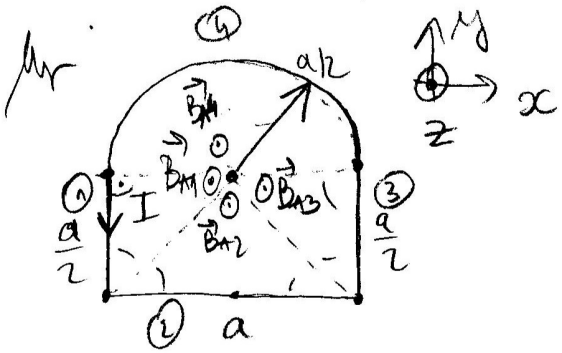
$$\vec{F_{21}} = \frac{\mu_0 I_1 I_2 \ell}{2\pi d} (-\vec{i})$$

$$\vec{F'_{21}} = \frac{\vec{F_{21}}}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi d} (-\vec{i}) = \frac{\mu_0 I^2}{\pi d} (-\vec{i})$$

2. Na Slici 3 je prikazana kontura koja se sastoji od polovine kružnice poluprečnika $a/2$, sa centrom u tački A, i donje polovine kvadrata dužine stranice $a$. Kroz konturu protiče struja intenziteta $I$. Odrediti i nacrtati vektor magnetne indukcije u tački A. Kontura se nalazi u prostoru relativne magnetne permeabilnosti $\mu_r$.




Slika 3




4 СЕГМЕНТА КОНТУРА ...

$$\vec{B}_A = \vec{B}_{A1} + \vec{B}_{A2} + \vec{B}_{A3} + \vec{B}_{A4}$$

$$\vec{B}_{A1} = \frac{\mu_0\mu_r I}{4\pi \frac{a}{2}} \left(\cos\frac{\pi}{4} + \cos\frac{\pi}{2}\right)\cdot \vec{k} = \frac{\sqrt{2}\,\mu_0\mu_r I}{4\pi a}\vec{k}$$

$$\vec{B}_{A2} = \frac{\mu_0\mu_r I}{4\pi \frac{a}{2}} \left(\cos\frac{\pi}{4} + \cos\frac{\pi}{4}\right)\vec{k} = \frac{\sqrt{2}\,\mu_0\mu_r I}{2\pi a}\vec{k}$$

$$\vec{B}_{A3} = \ldots = \vec{B}_{A1} = \frac{\sqrt{2}\,\mu_0\mu_r I}{4\pi a}\vec{k}$$

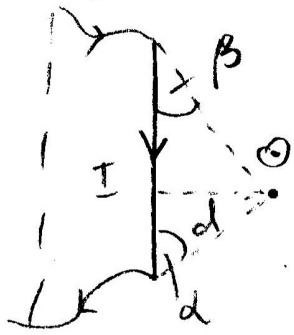
$$\vec{B}_{A4} = \frac{1}{2}\,\frac{\mu_0 I}{2\frac{a}{2}}\vec{k} \quad \left(\text{или} = \frac{\mu_0 I}{2\frac{a}{2}}\cdot\frac{\pi}{2\pi}\right)$$

$$\vec{B}_{A4} = \frac{\mu_0 I}{2a}\vec{k}$$

$$+$$

$$\vec{B}_A = \underbrace{\frac{\sqrt{2}\,\mu_0\mu_r I}{\pi a}\vec{k}}_{\vec{B}_{\sqcup}} + \underbrace{\frac{\mu_0\mu_r I}{2a}\vec{k}}_{\vec{B}_{\wedge}}$$

$$\vec{B}_A = \frac{\mu_0\mu_r I}{a}\left(\frac{\sqrt{2}}{\pi} + \frac{1}{2}\right)\vec{k}$$

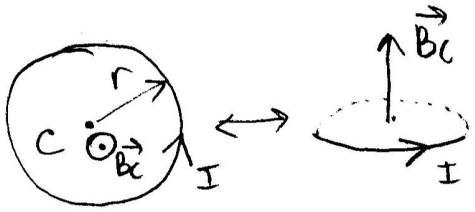



$$B_Y = \frac{\mu I}{4\pi d}\cdot(\cos\alpha + \cos\beta)$$

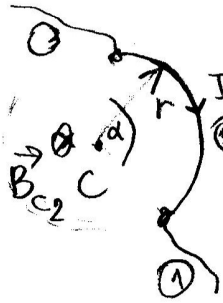
ВЕОМА ДУГАЧАК ПРОВОДНИК

$$\Rightarrow \alpha \to 0 \Rightarrow \alpha, \beta \to 0$$

$$\Rightarrow B_Y \to \frac{\mu I}{2\pi d}\ !$$

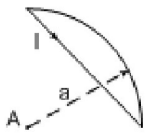



$$|\vec{B}_C| = \frac{\mu I}{2r}$$

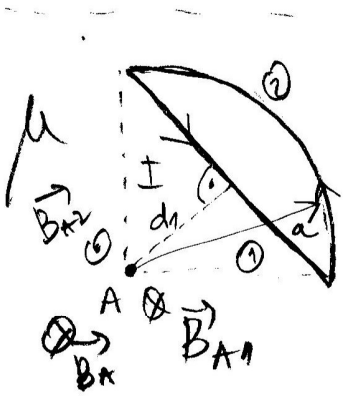


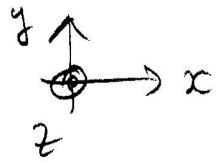

$$|\vec{B}_2| = \frac{\mu I}{2r}\cdot\frac{\alpha}{2\pi}$$

3. Na Slici 4 je prikazana kontura koja se sastoji od četvrtine kružnice poluprečnika $a$, sa centrom u tački A, i pravolinijskog segmenta. Kroz konturu protiče struja intenziteta $I$. Odrediti i nacrtati vektor magnetne indukcije u tački A. Kontura se nalazi u prostoru magnetne permeabilnosti $\mu$.




Slika 4




$$\vec{B}_A = \vec{B}_{A1} + \vec{B}_{A2}$$

$$\vec{B}_{A1} = \frac{\mu I}{4\pi d_1}\left(\cos\frac{\pi}{4} + \cos\frac{\pi}{4}\right)(-\vec{k})$$

$$\vec{B}_{A1} = \frac{\mu I}{4\pi \frac{a\sqrt{2}}{2}} \cdot \sqrt{2}\,(-\vec{i}) = -\frac{\mu I}{2\pi a}\vec{k}$$

$$\vec{B}_{A2} = \frac{\mu I}{2a}\cdot\frac{1}{4}\vec{k} = \frac{\mu I}{8a}\vec{k}$$

$\frac{\pi}{2} / 2\pi$

$$\vec{B}_A = \ldots = \frac{\mu I}{2a}\left(-\frac{1}{\pi} + \frac{1}{4}\right)\vec{k}$$

$$\vec{B}_A = -\frac{4-\pi}{8\pi}\frac{\mu I}{a}\vec{k} \ldots$$

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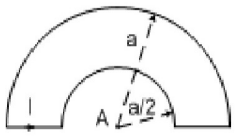
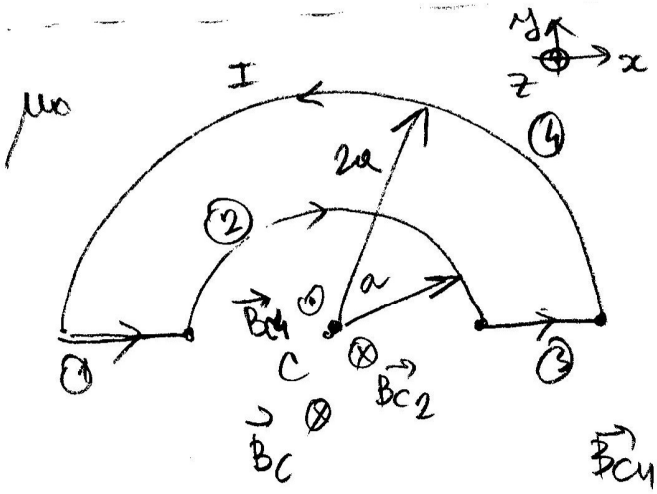
4. Na Slici 5 je prikazana kontura koja se sastoji od dve polovine kružnice poluprečnika $a$ i $2a$, sa centrom u tački A, koje su povezane odgovarajućim pravolinijskim segmentima. Kroz konturu protiče struja intenziteta $I$. Odrediti i nacrtati vektor magnetne indukcije u tački A. Kontura se nalazi u vazduhu ($\mu_0 = 4\pi \cdot 10^{-7}$ H/m).




Slika 5







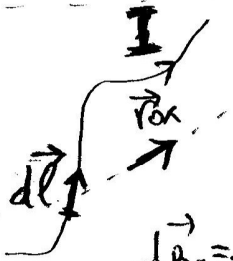
$$\vec{B}_C = \vec{B}_{C1} + \vec{B}_{C2} + \vec{B}_{C3} + \vec{B}_{C4}$$

$$\vec{B}_{C1} = \vec{B}_{C3} = 0$$

$$\vec{B}_{C2} = \frac{1}{2}\frac{\mu_0 I}{2a}(-\vec{i})$$

$$\vec{B}_{C4} = \frac{1}{2}\frac{\mu_0 I}{2\cdot 2a}(\vec{k})$$

$$\vec{B}_C = \ldots = -\frac{\mu_0 I}{8a}\vec{k}$$




$$d\vec{B}_x = \frac{\mu}{4\pi}\frac{I\,d\vec{l}\times\vec{r}_{0x}}{r_x^2}$$

$\sphericalangle(d\vec{l}, \vec{r}_{0x}) = 0$ или $\pi$

$$\Downarrow$$

$$d\vec{B}_x = 0!$$

$(\sin 0 = \sin\pi = 0)$