1. Na torusnom jezgru od magnetnog materijala relativne magnetne permeabilnosti μ_r, površine poprečnog preseka S i srednje linije l, ravnomerno je namotan namotaj sa N navojaka sa strujom I.

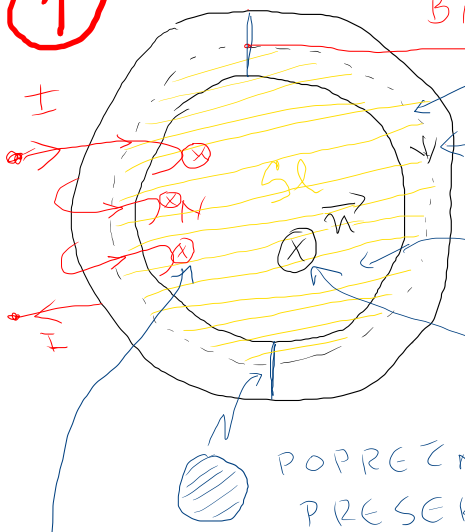
a) Odrediti fluks u jezgru.

b) Odrediti intezitet i nacrtati vektor magnetne indukcije i jačine magnetnog polja u jezgru.

c) Odrediti fluks kroz namotaj.

d) Odrediti sopstvenu induktivnost namotaja.

1



$\vec{B}, \vec{H}, \vec{dl}$ (SMER ZA $\Phi, \vec{B}, \vec{H}, \vec{dl}$ POKLAPAJU SE SA SMEROM ZA OBILAZAK ℓ)

ℓ (SREDNJA LINIJA)

VAŽI ZA "TANAK" TORUS !?

USVOJEN SMER OKILASKA ZA AMPEROV ZAKON

POVRŠINA NALEGLA NA KONTURU ℓ ($S\ell$)

SMER NORMALE PRIDRUŽENE POVRŠINI NALEGLOJ NA KONTURU ℓ ($S\ell$) (SMER za ℓ i $\vec{n}$ SU POVEZANI PRAVILOM DESNE RUKE)

POPREČNI PRESEK JEZGRA (S)

SMER STRUJE KROZ POVRŠINU NALEGLU NA ℓ ($S\ell$)

AMPEROV ZAKON:

$$\oint_\ell \vec{H}\,\vec{dl} = \sum_{S\ell} I$$

$$\oint_\ell \vec{H}\,\vec{dl} = \oint_\ell H\,dl = H\oint_\ell dl = H\ell$$

$$\sum_{S\ell} I = +NI \quad (+ \text{ SMER ZA } I \text{ SE POKLAPA SA SMEROM } \vec{n})$$

$$\Rightarrow \quad H\ell = NI, \quad H = NI/\ell$$

$$B = \mu_0\mu_r H = \frac{\mu_0\mu_r NI}{\ell}$$

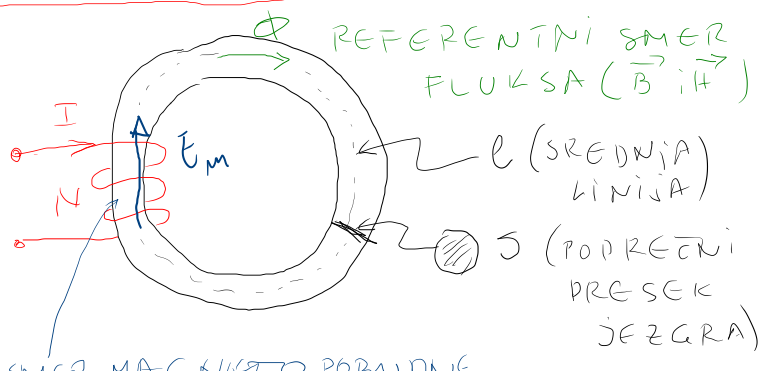
$$\Phi = \vec{B}\cdot\vec{S}$$

$$\Phi = B\cdot S = \frac{\mu_0\mu_r NIS}{\ell}$$

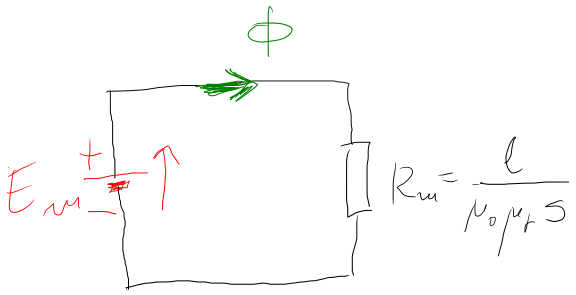
FLUKS KROZ NAMOTAJ: $\Psi = N\Phi = \frac{\mu_0 \mu_r N^2 I S}{\ell}$

SOPSTVENA INDUKTIVNOST: $L = \frac{\Psi}{I} = \frac{\mu_0 \mu_r S}{\ell} N^2$

II NAČIN: EKVIVALENTNO ELEKTRIČNO KOLO



$\Leftrightarrow$



R_m – magnetna otpornost

(PAZI NA SMER Θ_m i Φ !)

$$\Phi = \frac{\Theta_m}{R_m} = \frac{NI}{\frac{\ell}{\mu_0 \mu_r S}} = \frac{\mu_0 \mu_r S N I}{\ell}$$

$$\Psi = N\Phi = \frac{\mu_0 \mu_r S I}{\ell} N^2$$

$$L = \frac{N^2}{R_m} = \frac{\Psi}{I} = \frac{\mu_0 \mu_r S}{\ell} N^2 > 0 \quad (\text{UVEK})$$

$\left(I \gtrless 0 \;?\; \Rightarrow B, \Phi, H \gtrless 0\right)$ $B = \frac{\Phi}{S} = \frac{\mu_0 \mu_r N I}{\ell}$, $H = \frac{B}{\mu_0 \mu_r} = \frac{NI}{\ell}$

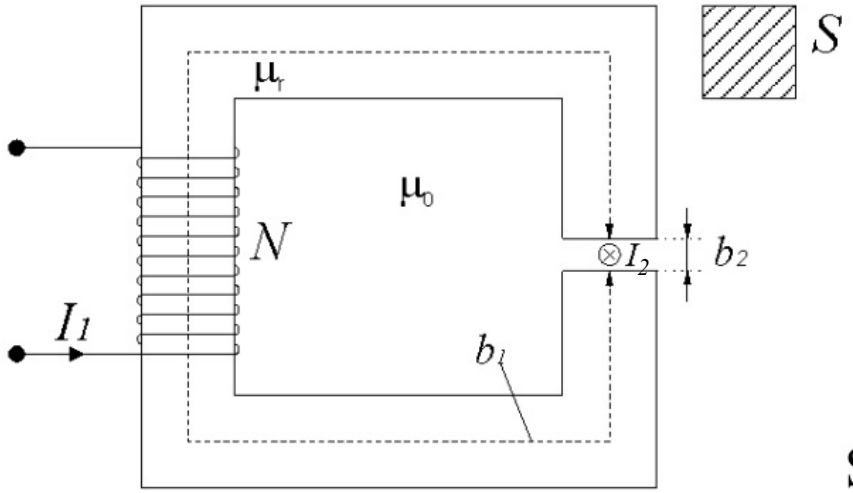
↳ INTENZITET: $|\Phi| = \ldots$, $|B| = \ldots$, $|H| = \ldots$

2. Na pravougaonom jezgru od feromagnetnog materijala relativne magnetne permeabilnosti $\mu_r = 1000$, sa vazdušnim procepom širine $b_2 = 2\,\mathrm{mm}$, ravnomerno je namotano $N = 2000$ navojaka tako da nema rasipanja, kao što je prikazano na Slici 2. Dužina srednje linije magnetnog jezgra je $b_1 = 20\,\mathrm{cm}$, a površina kvadratnog poprečnog preseka jezgra iznosi $S = 9\,\mathrm{cm}^2$. Kroz namotaj sa *N* navojaka protiče struja intenziteta $I_1 = 100\,\mathrm{mA}$.

a) Odrediti intenzitet vektora jačine magnetnog polja i vektora magnente indukcije u vazdušnom procepu.

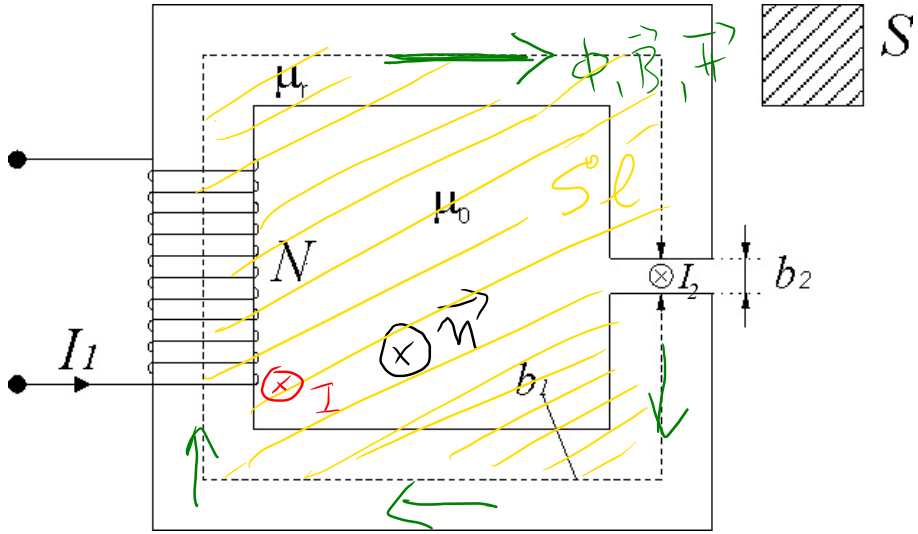
b) Odrediti induktivnost namotaja.

c) Ako kroz tanak pravolinijski provodnik, postavljen paralelno gornoj i donjoj ivici vazdušnog procepa, protiče struja inteziteta $I_2 = 1\,\mathrm{A}$, odrediti i nacrtati vektor mehaničke sile koja deluje na ovaj provodnik.



Slika 2.

2.



$\mu_r = 1000$

$b_1 = 20\,cm = 0{,}2\,m$

$b_2 = 2\,mm = 2 \cdot 10^{-3}\,m$

$N = 2000$

$S = 9\,cm^2 = 9 \cdot (10^{-2})^2\,m^2 = 9 \cdot 10^{-4}\,m^2$

$I = 100\,mA$, $I_2 = 1\,A$

---

a) AMPEROV ZAKON: $\oint_e \vec{H}\,d\vec{l} = \sum_{Se} I$

$H_1 b_1 + H_2 b_2 = (+) N I_1$

$\frac{B_1 b_1}{\mu_0 \mu_r} + \frac{B_2 b_2}{\mu_0} = N I_1$

PRORET TANAK: $S = S_1 \approx S_2$, $B_1 \cong B_2$ (b_2 malo)

$B_2 \left(\frac{b_1}{\mu_0 \mu_r} + \frac{b_2}{\mu_0} \right) = N I_1$

$B_2 = \frac{N I_1}{\frac{b_1}{\mu_0 \mu_r} + \frac{b_2}{\mu_0}}$, $H_2 = \frac{B_2}{\mu_0} = \frac{N I_1}{b_1/\mu_r + b_2}$

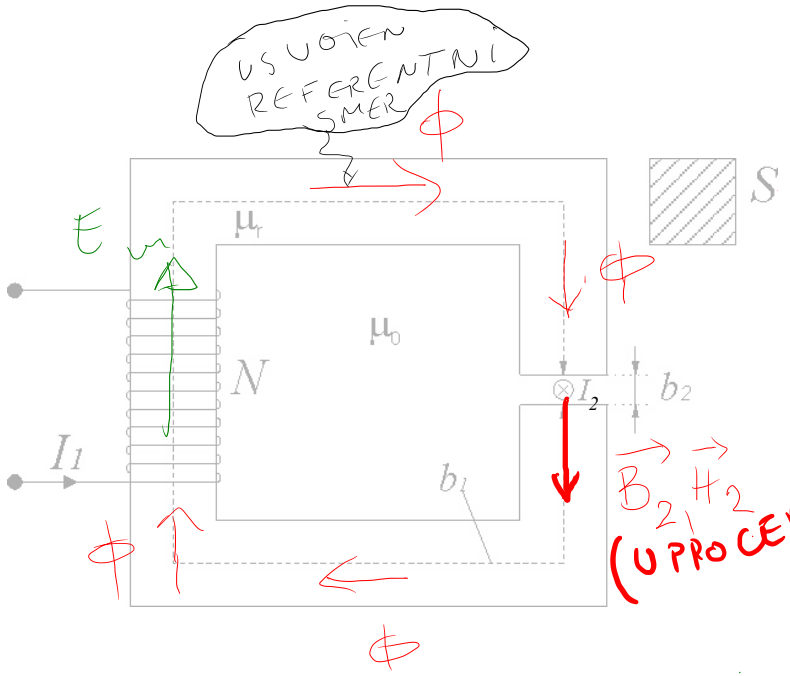
podatak:

$B_1 = B_2 = \ldots$

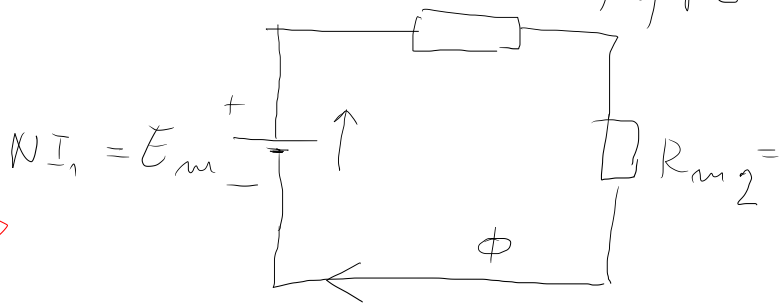
$H_1 = \frac{B_1}{\mu_0 \mu_r} = \ldots$

$\Phi = B_2 S = \ldots$

II NAČIN: EKVIVALENTNO ELEKTRIČNO KOLO



$\Leftrightarrow$



$$\Phi = \frac{E_m}{R_{m1} + R_{m2}} = \frac{NI_1}{\frac{b_1}{\mu_0 \mu_r S} + \frac{b_2}{\mu_0 S}}$$

$$B_1 = B_2 = \frac{\Phi}{S} = \frac{NI_1}{\frac{b_1}{\mu_0 \mu_r} + \frac{b_2}{\mu_0}} = 0{,}114 \text{ T}$$

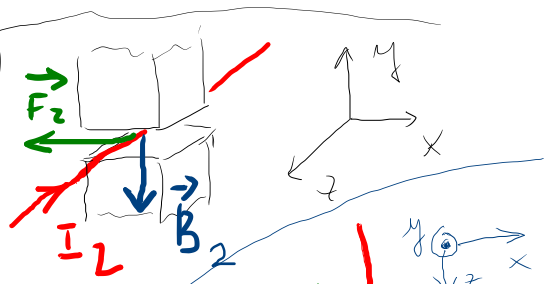
$$H_2 = \frac{B_2}{\mu_0} = \frac{NI_1}{\frac{b_1}{\mu_r} + b_2} \approx 90 \text{ kA/m}$$

$$\left(H_1 = \frac{B_1}{\mu_0 \mu_r} \approx 90 \text{ A/m} \right)$$

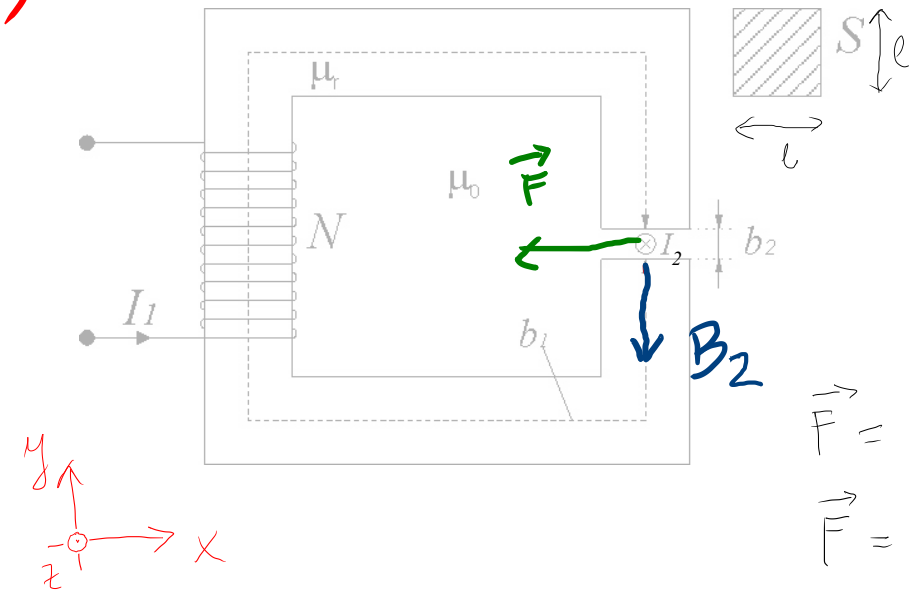
b)

$$L = \frac{\Psi}{I_1} = \frac{N\Phi}{I_1} = \frac{N^2}{R_{m1} + R_{m2}} = \frac{N^2}{\frac{b_1}{\mu_0 \mu_r S} + \frac{b_2}{\mu_0 S}}$$

$$L = 2{,}1\ \text{H}$$



c)



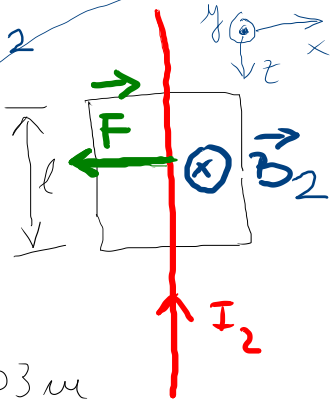
$$\vec{F} = I_2 \vec{\ell} \times \vec{B}_2$$

$I_2 = 1\ \text{A}$, $B_2 = 0{,}114\ \text{T}$

$$\ell = |\vec{\ell}| = \sqrt{S} = 3\ \text{cm} = 0{,}03\ \text{m}$$

$(S = \ell^2)$

(SILA DELUJE SAMO NA ONAJ DEO PROVODNIKA KOJI JE U MAG. POLJU B_2)



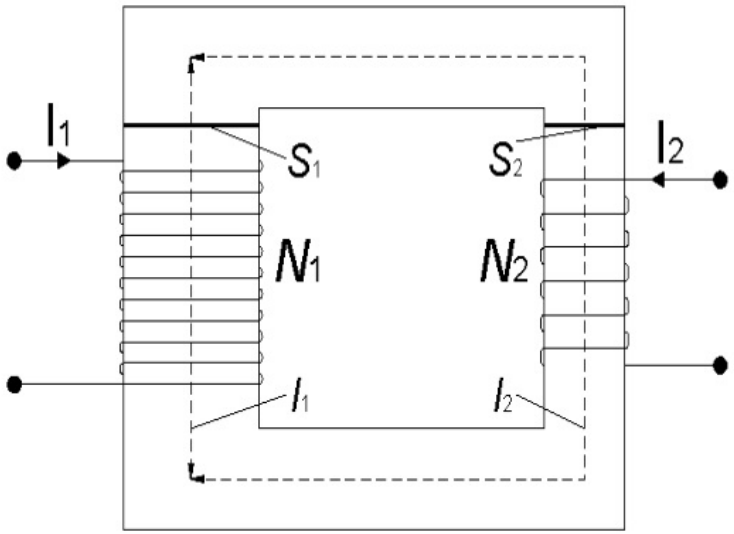
$$\vec{F} = I_2 \ell B_2 (-\vec{k}) \times (-\vec{j})$$

$$\vec{F} = I_2 \ell B_2 \vec{k} \times \vec{j} = I_2 \ell B_2 (-\vec{i})$$

$$F = |\vec{F}| \approx 3{,}4\ \text{mN}$$

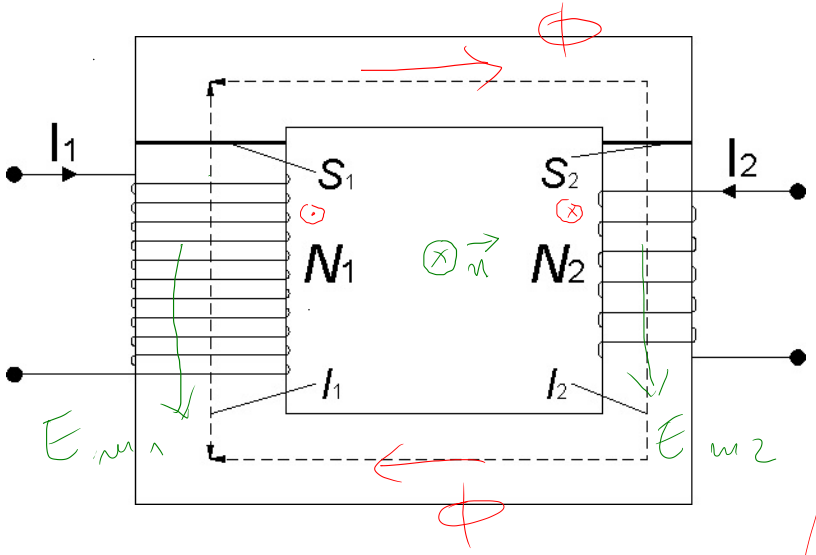
3. Na Slici 3 prikazano je magnetno kolo, koje je sačinjeno od jezgra relativnog magnetnog permeabiliteta μ_r. Jezgro se sastoji iz dva dela, koji imaju dužine srednjih linija l_1 i l_2 i površine poprečnih preseka S_1 i S_2. Na jezgro su na motana dva namotaja sa N_1 i N_2 navojaka, kroz koje protiču struje intenziteta I_1 i I_2.

a) Odrediti magnetni fluks u jezgru.
b) Odrediti intezitete vektora magnetne indukcije i jačine magnetnog polja u oba dela jezgra.
c) Odrediti sopstevene i međusobne induktivnosti namoraja.
d) Odrediti gustinu energije magnetnog polja i energiju magnetnog polja u delu jezgra dužine l_1.



Slika 3.

3



AMPEROV ZAKON

a)

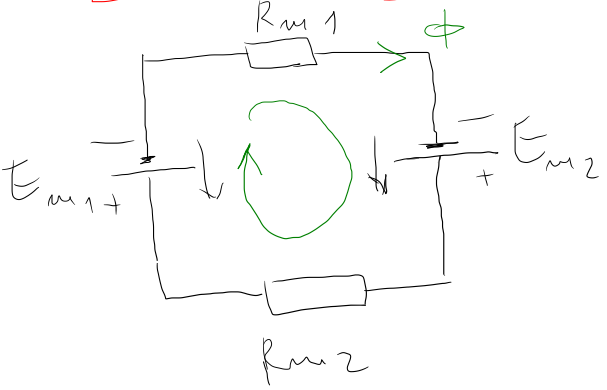
$$H_1 l_1 + H_2 l_2 = -N_1 I_1 + N_2 I_2$$

$$\frac{B_1}{\mu_0 \mu_r} l_1 + \frac{B_2}{\mu_0 \mu_r} l_2 = N_2 I_2 - N_1 I_1$$

$$\frac{\Phi l_1}{\mu_0 \mu_r S_1} + \frac{\Phi l_2}{\mu_0 \mu_r S_2} = N_2 I_2 - N_1 I_1$$

$$\Phi = \frac{N_2 I_2 - N_1 I_1}{\frac{l_1}{\mu_0 \mu_r S_1} + \frac{l_2}{\mu_0 \mu_r S_2}} \geq 0 \; ?$$

EKV. EL. VOLO



$$E_{m1} = N_1 I_1$$

$$E_{m2} = N_2 I_2$$

$$R_{m1} = \frac{l_1}{\mu_0 \mu_r S_1}$$

$$R_{m2} = \frac{l_2}{\mu_0 \mu_r S_2}$$

$$-E_{m1} - R_{m1}\Phi + E_{m2} - R_{m2}\Phi = 0$$

$$\Phi = \frac{E_{m2} - E_{m1}}{R_{m1} + R_{m2}} \geq 0 \; ?$$

b)

$$B_1 = \Phi / S_1 = \frac{N_2 I_2 - N_1 I_1}{\frac{\ell_1}{\mu_0 \mu_r} + \frac{\ell_2 S_1}{\mu_0 \mu_r S_2}} \gtrless 0 ?$$

$$B_2 = \Phi / S_2 = \frac{N_2 I_2 - N_1 I_1}{\frac{\ell_1 S_2}{\mu_0 \mu_r S_1} + \frac{\ell_2}{\mu_0 \mu_r}} \gtrless 0 ?$$

$$H_1 = \frac{B_1}{\mu_0 \mu_r} = \frac{N_2 I_2 - N_1 I_1}{\ell_1 + \ell_2 S_1 / S_2} \gtrless 0 ?$$

$$H_2 = \frac{B_2}{\mu_0 \mu_r} = \frac{N_2 I_2 - N_1 I_1}{\ell_1 S_2 / S_1 + \ell_2} \gtrless 0 ?$$

c)

$$L_{11} = \left.\frac{\Psi_{11}}{I_1}\right|_{I_2=0} = \left.\frac{N_1 \Phi_{11}}{I_1}\right|_{I_2=0} = \frac{N_1^2}{R_{m1} + R_{m2}} = \frac{N_1^2}{\frac{\ell_1}{\mu_0 \mu_r S_1} + \frac{\ell_2}{\mu_0 \mu_r S_2}} > 0$$

$\Phi = \frac{+N_1 I_1}{R_{m1}+R_{m2}}$

$$L_{22} = \left.\frac{\Psi_{22}}{I_2}\right|_{I_1=0} = \left.\frac{N_2 \Phi_{22}}{I_2}\right|_{I_1=0} = \frac{N_2^2}{R_{m1}+R_{m2}} = \frac{N_2^2}{\frac{\ell_1}{\mu_0 \mu_r S_1} + \frac{\ell_2}{\mu_0 \mu_r S_2}} > 0$$

$\Phi = \frac{N_2 I_2}{R_{m1}+R_{m2}}$

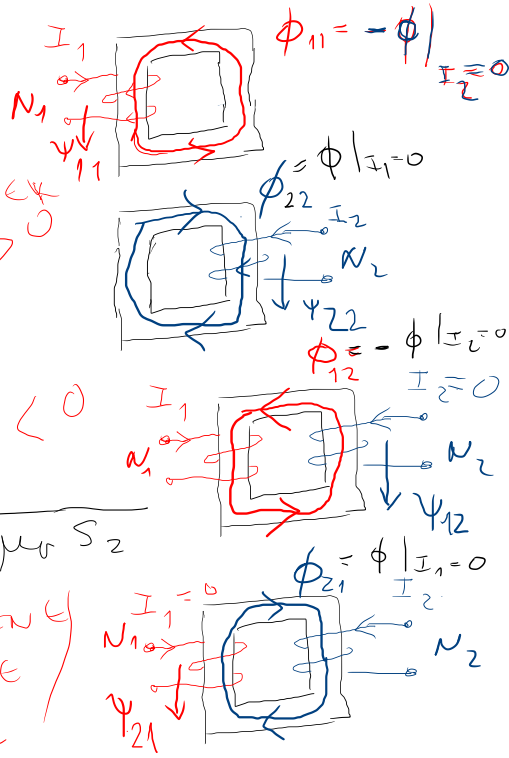
$$L_{12} = \left.\frac{\Psi_{12}}{I_1}\right|_{I_2=0} = -\left.\frac{N_2 \Phi_{12}}{I_1}\right|_{I_2=0} = -\frac{N_1 N_2}{R_{m1}+R_{m2}} = \frac{-N_1 N_2}{\frac{\ell_1}{\mu_0 \mu_r S_1} + \frac{\ell_2}{\mu_0 \mu_r S_2}} < 0$$

$-\Phi = \frac{+N_1 I_1}{R_{m1}+R_{m2}}$

$$L_{21} = \left.\frac{\Psi_{21}}{I_2}\right|_{I_1=0} = -\left.\frac{N_1 \Phi_{21}}{I_2}\right|_{I_1=0} = L_{12} < 0$$

$\Phi = \frac{N_2 I_2}{R_{m1}+R_{m2}}$

(za usvojene smerove I_1 i I_2)



d)

$$w_1 = \frac{1}{2} B_1 H_1 = \frac{1}{2} \mu_0 \mu_r H_1^2 = \frac{1}{2} \mu_0 \mu_r \left(\frac{N_2 I_2 - N_1 I_1}{l_1 + l_2 S_1/S_2} \right)^2$$

← ZAPREMINSKA GUSTINA ENERGIJE

$$W_1 = V_1 \cdot w_1$$

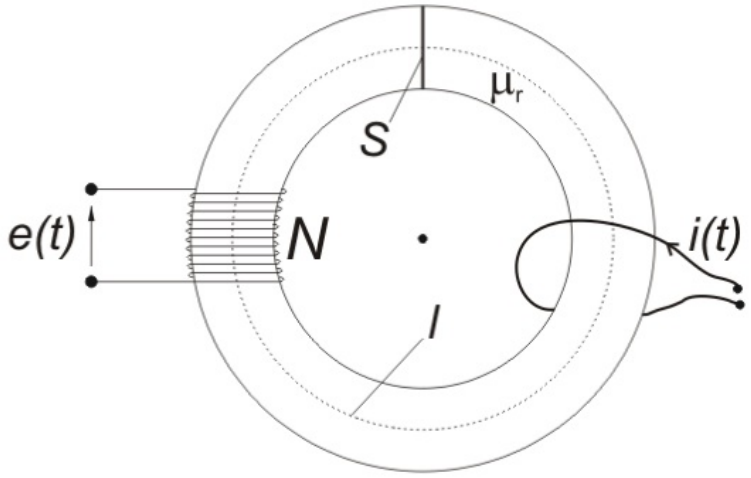
← ENERGIJA MAGNETNOG POLJA

$$V_1 = S_1 \cdot l_1$$

← ZAPREMINA

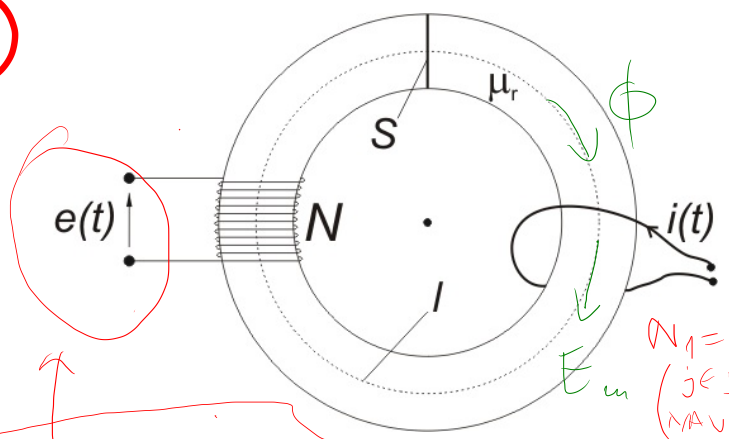
$$W_1 = \frac{1}{2} \mu_0 \mu_r S_1 l_1 \left(\frac{N_2 I_2 - N_1 I_1}{l_1 + l_2 S_1/S_2} \right)^2$$

4. Na tanak torus od feromagnetnog materijala relativne magnetne permeabilnosti $\mu_r = 1000$, poprečnog preseka $S = 5cm^2$ i srednje dužine $l = 0.8m$, ravnomerno je namotano $N = 2000$ navojaka žice. U provodniku proizvoljnog oblika koji obuhvata torus (Slika 4), postoji električna struja trenutne vrednosti $i(t) = I_m \cos \omega t$, $I_m = 50A$, $\omega = 314 rad/s$. Odrediti indukovanu elektromotornu silu na krajevima torusnog namotaja.



Slika 4.

4



OTVORENI KRAJEVI
=> NEMA STRUJE!

$N_1 = 1$ (JEDAN NAVOJAK)

E_m

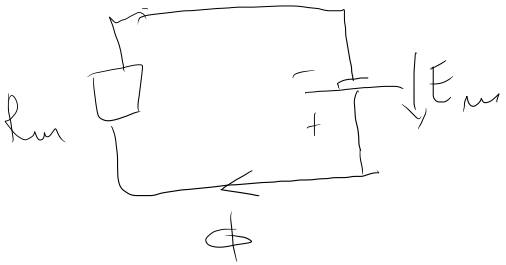
$$\Phi = \frac{E_m}{R_m} = \frac{N_1 \hat{i}(t)}{\frac{\ell}{\mu_0 \mu_r S}} = \frac{\mu_0 \mu_r S}{\ell} i(t)$$

$$\Phi(t) = \frac{\mu_0 \mu_r S I_m}{\ell} \cos(\omega t)$$

$$e(t) = -\frac{d\Psi}{dt} = -\frac{d}{dt}\left(N \Phi(t)\right)$$

$$e(t) = -\frac{\mu_0 \mu_r S I_m N}{\ell} \frac{d}{dt}\left[\cos(\omega t)\right]$$

$$e(t) = \frac{\mu_0 \mu_r S I_m N \omega}{\ell} \sin(\omega t) = E_{max} \sin(\omega t)$$



$$E_{max} = (\mu_0 \mu_r S I_m N \omega)/\ell = 24{,}66\ V$$