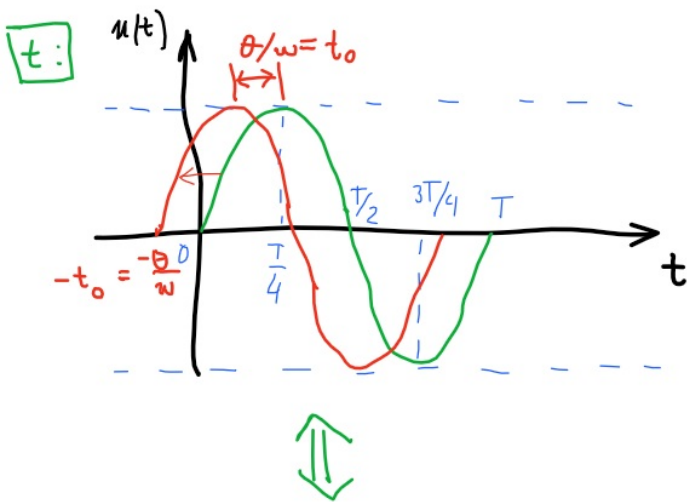


NAIŽMERNIČNE STRUJE



w: $\bar{U} = U e^{j\theta} = U \cos \theta + j U \sin \theta$

$U = \frac{U_m}{\sqrt{2}}$ - efektivna vrednost

θ - početna faza

ω - vektor rotiranja ∇

$$u(t) = \frac{U_m}{1) \quad 2) \quad 3)} \sin(\omega t + \theta)$$

1) U_m - amplituda [V]

$U = \frac{U_m}{\sqrt{2}}$ - efektivna vrednost

2) ω - kružna učestanost [rad/s]

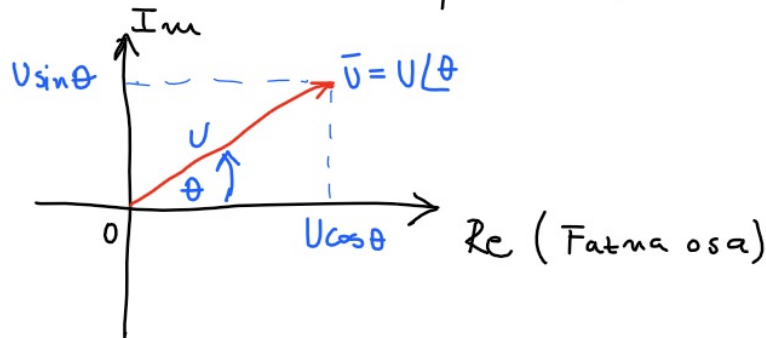
$f = \frac{\omega}{2\pi}$ - učestanost (frekvencija) [Hz = $\frac{1}{s}$]

$T = \frac{1}{f}$ - perioda [s]

3) $\alpha = \omega t + \theta$ - faza [rad]

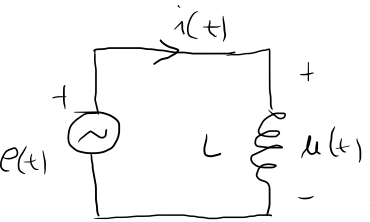
$t=0$: $\alpha = \theta$ - početna faza [rad]

$$\begin{pmatrix} \omega t_0 = \theta \\ t_0 = \frac{\theta}{\omega} \end{pmatrix}$$



1. Kolo naizmenične struje sastoji se od izvora naizmeničnog napona i kalema induktivnosti L . Vremenski oblik napona izvora je $e(t) = 200 \sin(100\pi t) \text{ V}$, dok efektivna vrednost struje u kolu iznosi 50A. Odrediti:

- vrednost induktivnosti kalema L ,
- kompleksne predstavnike napona i struje kalema,
- vremenski oblik struje kalema, i
- nacrtati fazorski dijagram.



$$e(t) = 200 \sin(100\pi t) \text{ V} = E_m \sin(\omega t + \theta)$$

$$E_m = 200 \text{ V}, \quad \omega = 100\pi \text{ rad/s}, \quad \theta = 0$$

$$E = \frac{E_m}{\sqrt{2}} = \frac{200}{\sqrt{2}} = 100\sqrt{2} \text{ V}$$

a) $u(t) = e(t) = U_m \sin(\omega t + \theta) \Rightarrow U_m = 200 \text{ V}, \omega = 100\pi \text{ rad/s}, \theta = 0$

$$i(t) = \frac{1}{L} \int u(t) dt = -\frac{U_m}{\omega L} \cos(\omega t) = \frac{U_m}{\omega L} \sin(\omega t - \pi/2) = I_m \sin(\omega t + \psi)$$

$$I_m = \frac{U_m}{\omega L}$$

$$I = \frac{U}{\omega L}$$

$$\psi = \theta - \pi/2$$

STRUJA KASNI
ZA $\pi/2$

\Rightarrow

$$\bar{Z}_L = \frac{\bar{U}}{\bar{I}} = \frac{U}{I} \angle \theta - \psi$$

$$\bar{Z}_L = \omega L \angle +\pi/2$$

$$Z_L = \frac{U}{I} = \frac{U_m/\sqrt{2}}{I} = \omega L$$

$$L = \frac{U_m}{\sqrt{2} I \omega} = \frac{200}{\sqrt{2} \cdot 50 \cdot 100\pi}$$

$$L \approx 9 \text{ mH}$$

$$b) \quad \bar{U} = U e^{j\theta} = U \cos\theta + jU \sin\theta$$

$$\bar{U} = \frac{100}{\sqrt{2}} e^{j0} = 100\sqrt{2} \text{ V}$$

$$\bar{U} = \frac{200}{\sqrt{2}} (\cancel{\cos 0} + j \cancel{\sin 0}) = \frac{200}{\sqrt{2}} = 100\sqrt{2} \text{ V}$$

$$\bar{Z} = Z e^{j\theta - \varphi}$$

$$\bar{Z} = \omega L \cdot e^{j\bar{u}/2} = \omega L (\cancel{\cos \frac{\bar{u}}{2}} + j \cancel{\sin \frac{\bar{u}}{2}})$$

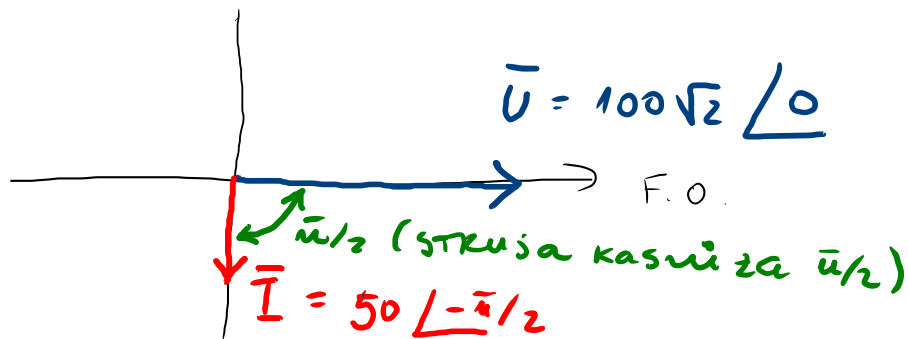
$$\bar{Z} = j\omega L = j \frac{200\sqrt{2}}{10} = j 2\sqrt{2} \Omega = 2\sqrt{2} e^{j\pi/2}$$

$$\bar{I} = \frac{\bar{U}}{\bar{Z}} = \frac{100\sqrt{2}}{2\sqrt{2} e^{j\pi/2}} = 50 e^{-j\pi/2}$$

$$\bar{I} = 50 (\cancel{\cos(-\pi/2)} + j \cancel{\sin(-\pi/2)}) = -j 50 \text{ A}$$

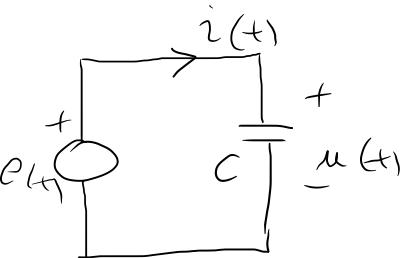
$$c) \quad i(t) = I_m \sin(\omega t + \varphi) = I \sqrt{2} \sin(\omega t + \varphi) = 50\sqrt{2} \sin(100\bar{u}t - \bar{u}/2) \text{ A}$$

d)



2. Kolo naizmenične struje sastoji se od izvora naizmeničnog napona i kondenzatora kapacitivnosti $C = 5\text{mF}$. Vremenski oblik struje kondenzatora je $i(t) = 20\sin(100t)\text{A}$. Odrediti:

- kompleksne predstavnike napona i struje kondenzatora,
- vremenski oblik napona na kondenzatoru, i
- nacrtati fazorski dijagram.



$$i(t) = 20 \sin(100t) \text{ A} = I_m \sin(\omega t + \psi)$$

$$I_m = 20 \text{ A} \quad \omega = 100 \text{ rad/s}$$

$$I = I_m / \sqrt{2} = 10\sqrt{2} \text{ A} \quad \psi = 0 \text{ rad}$$

$$\left. \begin{array}{l} \\ \\ \end{array} \right\} \Rightarrow \bar{I} = I \angle 0$$

$$\bar{I} = 10\sqrt{2} \angle 0 \text{ A}$$

a) $Z_C = \frac{1}{\omega C} = \frac{1}{100 \cdot 5 \cdot 10^{-3}} = 2 \Omega$

$$U = Z_C I = \frac{I}{\omega C} = 2 \cdot 10\sqrt{2} = 20\sqrt{2} \text{ V}$$

$$\Rightarrow \bar{U} = 20\sqrt{2} e^{-j\pi/2}$$

$$\bar{Z}_C = \frac{1}{\omega C} \angle -\pi/2$$

$$U_m = Z_C I_m = 2 \cdot 20 = 40 \text{ V} \quad (U_m = U\sqrt{2})$$

$$\theta = \psi - \pi/2 = 0 - \pi/2 = -\pi/2 \text{ rad} \quad (\text{napon ka sinu za } \pi/2)$$

$$\bar{Z}_C = \frac{1}{\omega C} e^{-j\pi/2} = -j \frac{1}{\omega C}$$

$$\bar{U} = \bar{Z}_C \bar{I} = (-j2) \cdot (10\sqrt{2}) = -j20\sqrt{2} = 20\sqrt{2} e^{-j\pi/2} \text{ V}$$

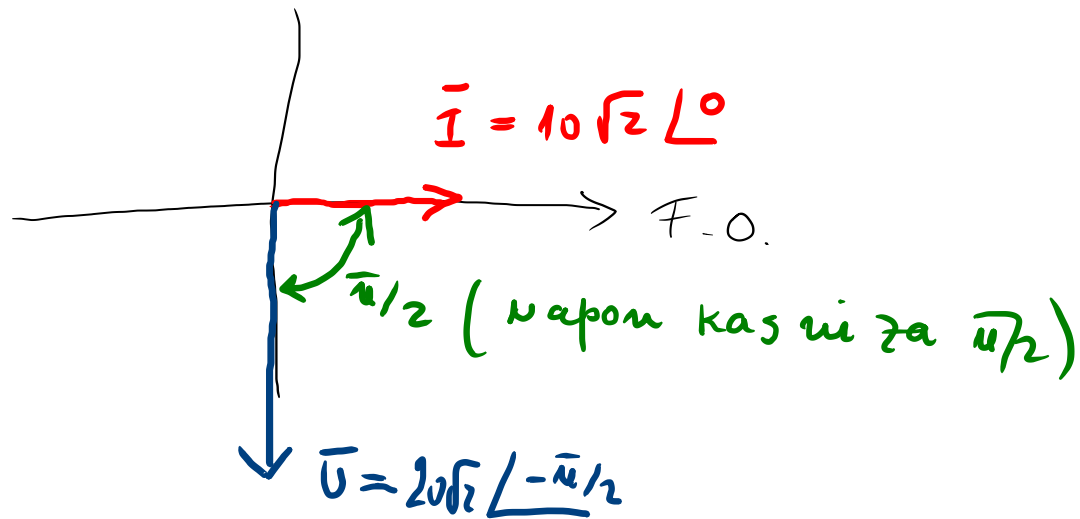
$$Z_C = -j2 \Omega$$

$$U = 20\sqrt{2} \text{ V}, \theta = -\pi/2 \text{ rad}$$

b) $u(t) = U_m \sin(\omega t + \theta)$

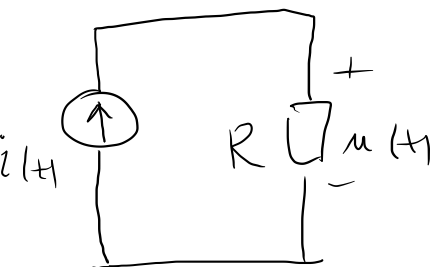
$u(t) = 40 \sin(100t - \pi/2) \text{ V}$

c)



3. Kolo naizmenične struje sastoji se od izvora naizmenične struje i otpornika otpornosti R . Vremenski oblik struje izvora je $i(t) = 10 \sin(314t) \text{ A}$, dok efektivna vrednost napona na otporniku iznosi 5V. Odrediti:

- vrednost otpornosti R ,
- kompleksne predstavnik napona i struje otpornika,
- vremenski oblik napona na otporniku, u
- nacrtati fazorski dijagram.



$$i(t) = 10 \sin(314t) \text{ A} = I_m \sin(\omega t + \psi)$$

$$I_m = 10 \text{ A}$$

$$\omega = 314 \text{ rad/s}$$

$$I = \frac{I_m}{\sqrt{2}} = 5\sqrt{2} \text{ A}$$

$$\psi = 0 \text{ rad}$$

a) $U = 5 \text{ V}$
 $\bar{Z}_R = R \angle 0$
 NAPONI I STRUJA
 SU U FAZI

$$\bar{Z}_R = R = \frac{U}{I} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \Omega$$

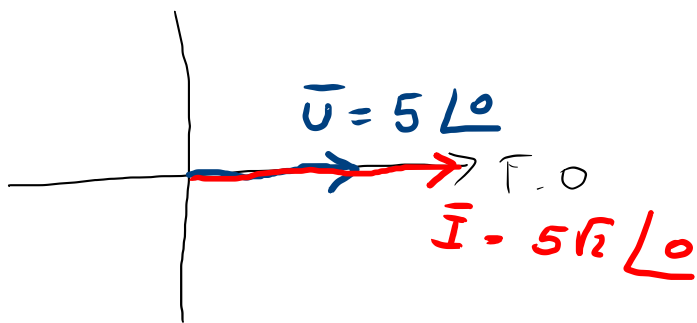
$$b) \quad \bar{I} = I e^{j\psi} = 5\sqrt{2} e^{j0} = 5\sqrt{2} \text{ A} \quad (5\sqrt{2} \angle 0)$$

$$\bar{U} = U e^{j\theta} = \bar{Z} \cdot \bar{I} = \frac{1}{\sqrt{2}} \cdot 5\sqrt{2} e^{j0} = 5 e^{j0} = 5 \text{ V} \quad (5 \angle 0)$$

$\theta = \psi = 0$ (napon i struja su u fazi)

$$c) \quad u(t) = U_m \sin(\omega t + \theta) = U\sqrt{2} \sin(\omega t + \theta) = 5\sqrt{2} \sin(314t) \text{ V}$$

d)



Napon i struja su u fazi!