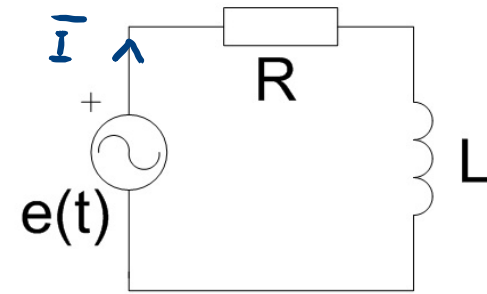


2. Na slici 2 prikazano je kolo naizmenične struje koje se napaja naizmeničnim naponom trenutne vrednosti  $e(t) = 20 \sin(\omega t + \pi/2) \text{V}$ , gde je  $\omega = 1000 \text{ rad/s}$ . Poznato je:  $R = 10 \Omega$  i  $L = 10 \text{ mH}$ . Odrediti:

- kompleksne vrednosti struje i napona svih elemenata u kolu,
- vremenski oblik struje u kolu i napona otpornika i kalema,
- nacrtati fazorski dijagram,
- snage svih elemenata u kolu.



Slika 2

a) bez kompleksnog računa

$$R = 10 \Omega$$

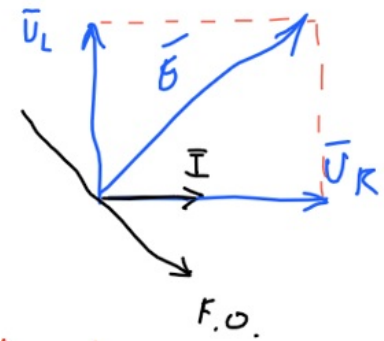
$$X_L = \omega L = 1000 \cdot 10 \cdot 10^{-3} = 10 \Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{2 \cdot 10^2} = 10\sqrt{2} \Omega$$

$$I = \frac{E}{Z} = \frac{10\sqrt{2}}{10\sqrt{2}} = 1 \text{ A}$$

$$U_R = Z_R I = R I = 10 \text{ V}$$

$$U_L = Z_L I = X_L I = 10 \text{ V}$$



kompleksni račun

$$\bar{E} = \frac{20}{\sqrt{2}} e^{j\pi/2} = 10\sqrt{2} e^{j\pi/2} = j10\sqrt{2} \text{ V}$$

$$\bar{I} = \frac{\bar{E}}{Z} = \frac{j10\sqrt{2}}{10 + j10} = \frac{10\sqrt{2} e^{j\pi/2}}{10\sqrt{2} e^{j\pi/4}} = 1 \cdot e^{j\pi/4} \text{ A}$$

$$\bar{I} = \frac{j\sqrt{2}}{1+j} \cdot \frac{1-j}{1-j} = \frac{\sqrt{2} + j\sqrt{2}}{1^2 + 1^2} = \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2} \text{ A}$$

$$\bar{U}_R = \bar{Z}_R \bar{I} = 10 e^{j\pi/4} \text{ V}$$

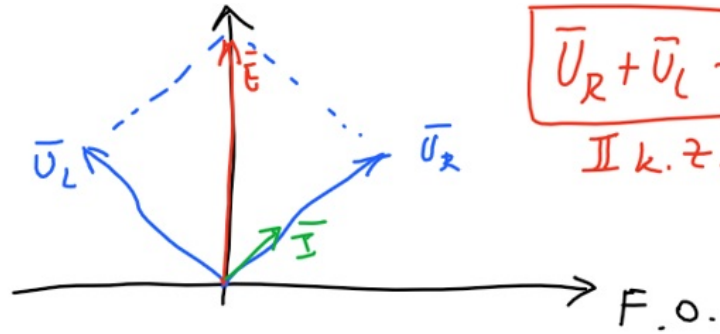
$$\bar{U}_L = \bar{Z}_L \bar{I} = j10 \cdot 1 \cdot e^{j\pi/4} = 10 e^{j\pi/2} \cdot 1 e^{j\pi/4} = 10 e^{j3\pi/4} \text{ V}$$

$$\bar{Z}_R = R, \quad \bar{Z}_L = jX_L$$

$$\bar{Z} = \bar{Z}_R + \bar{Z}_L = R + jX_L = 10 + j10 \Omega$$

b)  $i(t) = \sqrt{2} \sin(1000t + \bar{u}/4) \text{ A}$   
 $u_R(t) = 10\sqrt{2} \sin(1000t + \bar{u}/4) \text{ V}$   
 $u_L(t) = 10\sqrt{2} \sin(1000t + 3\bar{u}/4) \text{ V}$

c)



$$\bar{U}_R + \bar{U}_L - \bar{E} = 0$$

II k.z.

d)  $\bar{S}_R = P_R + jQ_R = S_R e^{j\varphi_R}$

$P_R = R I^2 = 10 \text{ W}$  (AKTIVNA)

$Q_R = 0 \text{ VAR}$  (REAKTIVNA)

$S_R = \sqrt{P_R^2 + Q_R^2} = Z_R I^2 = 10 \text{ VA}$  (PRIVIONA)

$\bar{S}_R = 10 + j0 \text{ VA}$  (KOMPLEKSNIA PRIVIONA) =  $10 e^{j0} \text{ V}$

$\bar{S}_L = P_L + jQ_L = S_L e^{j\varphi_L}$

$P_L = 0 \text{ W}$

$Q_L = X_L I^2 = 10 \text{ VAR}$

$S_L = \sqrt{P_L^2 + Q_L^2} = Z_L I^2 = 10 \text{ VA}$

$\bar{S}_L = 0 + j10 \text{ VA} = 10 e^{j\bar{u}/2} \text{ VA}$

$$X_L > 0$$

$$Q_L > 0$$

!

ZAKON ODRŽANJA SNAGE

$\bar{S}_E = \bar{S}_R + \bar{S}_L = 10 + j10 \text{ VA}$

$P_E = P_R + P_L = P_R = 10 \text{ W}$

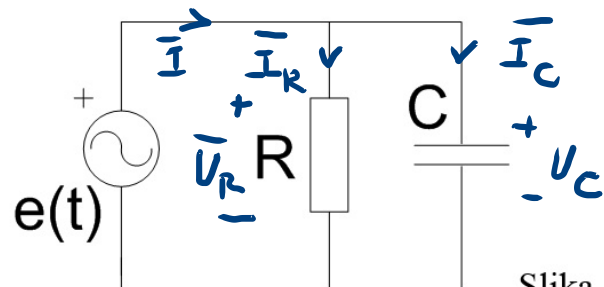
$Q_E = Q_R + Q_L = Q_L = 10 \text{ VAR}$

PAZI:  $S_E \neq S_R + S_L$  ! ! !

3. Na slici 3 prikazano je kolo naizmenične struje koje se napaja naizmeničnim naponom trenutne vrednosti  $e(t) = 20\sin(\omega t - \pi/4)$ , gde je  $\omega = 1000\text{rad/s}$ . Poznato je:  $R = 5\Omega$  i  $C = 200\mu\text{F}$ .

Odrediti:

- kompleksne vrednosti struja i napona svih elementima u kolu,
- vremenski oblik struja otpornika i kondenzatora,
- nacrtati fazorski dijagram,
- snage svih elemenata u kolu.



Slika 3

$$\bar{E} = 10\sqrt{2}e^{-j\pi/4} = 10\sqrt{2}\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right) = 10 - j10\text{V}$$

a)  $\bar{z}_R = R = 5\Omega$

$$X_C = \frac{-1}{\omega C} = \frac{-1}{1000 \cdot 200\mu} = -5\Omega$$

$$\bar{z}_C = +jX_C = -j5\Omega = 5e^{-j\pi/2}\Omega$$

$$\bar{U}_R = \bar{U}_C = \bar{E}$$

$$\bar{I}_R = \frac{\bar{U}_R}{\bar{z}_R} = \frac{10\sqrt{2}e^{-j\pi/4}}{5} = 2\sqrt{2}e^{-j\pi/4}\text{A} = 2 - j2\text{A}$$

$$\bar{I}_C = \frac{\bar{U}_C}{\bar{z}_C} = \frac{10\sqrt{2}e^{-j\pi/4}}{5e^{-j\pi/2}} = 2\sqrt{2}e^{j\pi/4}\text{A} = 2 + j2\text{A}$$

$$\bar{I} = \bar{I}_R + \bar{I}_C \quad \text{I.R.Z.}$$

PAZI!

$$\bar{I}_R + \bar{I}_C = 2\sqrt{2} + 2\sqrt{2} = 4\sqrt{2} \neq \bar{I}$$

$$\bar{I} \neq \bar{I}_R + \bar{I}_C$$

$$\bar{z}_e = \bar{z}_R \parallel \bar{z}_C = \frac{\bar{z}_R \bar{z}_C}{\bar{z}_R + \bar{z}_C}$$

$$\bar{z}_e = \frac{5(-j5)}{5 - j5} = \frac{-j5}{1 - j} \cdot \frac{1 + j}{1 + j}$$

$$\bar{z}_e = \frac{5 - j5}{1^2 + 1^2} = 2,5 - j2,5\Omega$$

$$\bar{I} = \frac{\bar{E}}{\bar{z}_e} = \frac{10 - j10}{2,5 - j2,5} = 4\text{A}$$

$$\bar{I} = \bar{I}_R + \bar{I}_C = 4\text{A}$$

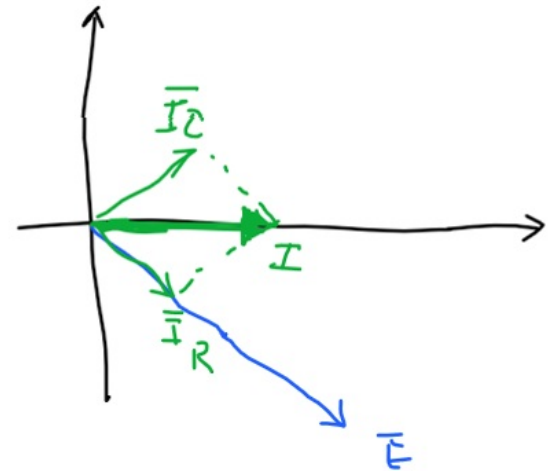
ALTERNATIVNO

$$b) i_R(t) = 4 \sin(1000t - \pi/4) \text{ A}$$

$$i_C(t) = 4 \sin(1000t + \pi/4) \text{ A}$$

$$i(t) = i_R(t) + i_C(t) = 4\sqrt{2} \sin(1000t) \text{ A}$$

c)



$$d) P_R = R I_R^2 = 5 \left( \sqrt{2^2 + (-2)^2} \right)^2 = 5 \cdot 8 = 40 \text{ W}$$

$$Q_R = 0 \text{ VAR}$$

$$\bar{S}_R = 40 + j0 \text{ VA}$$

$$P_C = 0 \text{ W}$$

$$Q_C = X_C I_C^2 = -5 \cdot \left( \sqrt{2^2 + 2^2} \right)^2 = -5 \cdot 8 = -40 \text{ VAR}$$

$$\bar{S}_C = 0 - j40 \text{ VA}$$

$$\bar{S}_E = \bar{S}_R + \bar{S}_C = 40 - j40 \text{ VA}$$

$$P_E = P_R + P_C = P_R = 40 \text{ W}$$

$$Q_E = Q_R + Q_C = Q_C = -40 \text{ VAR}$$

$$X_C < 0$$

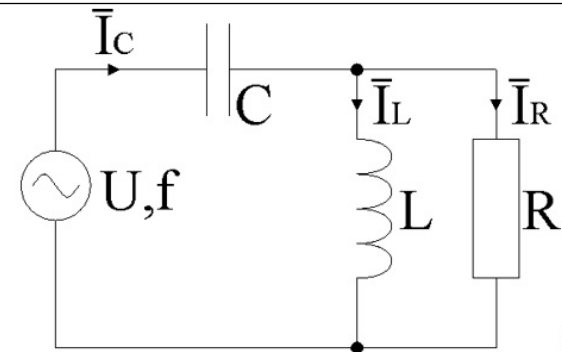
$$Q_C < 0$$

!

$$\left. \begin{aligned} \bar{S}_E &= \bar{Z}_e I^2 = (2.5 - j2.5) \cdot 4^2 \\ \bar{S}_E &= 40 - j40 \text{ VA} \end{aligned} \right\} \text{ ili}$$

4. Na slici 4 je prikazano kolo naizmenične struje koje se napaja naponom trenutne vrednosti:  $u(t) = 100\sqrt{2}\sin(\omega t)$ , gde je  $\omega = 10^3 \text{ rad/s}$ . Poznate su sledeće vrednosti elemenata u kolu:  $R = 50\Omega$ ,  $C = 20\mu\text{F}$ ,  $L = 50\text{mH}$ .

- Odrediti kompleksne izraze za označene struje;
- Odrediti aktivnu, reaktivnu i prividnu snagu celokupnog potrošača;
- Odrediti trenutnu vrednost struje kalemata.



Slika 4

$$\bar{U} = 100\text{V}$$

$$\bar{Z}_R = R = 50\Omega$$

$$X_L = \omega L = 10^3 \cdot 50 \cdot 10^{-3} = 50\Omega$$

$$\bar{Z}_L = jX_L = j50\Omega$$

$$X_C = \frac{-1}{\omega C} = \frac{-1}{10^3 \cdot 20 \cdot 10^{-6}} = -50\Omega$$

$$\bar{Z}_C = +jX_C = -j50\Omega$$

$$\bar{I}_L = \frac{\bar{U}_L}{\bar{Z}_L} = \frac{j100}{j50} = 2\text{A}$$

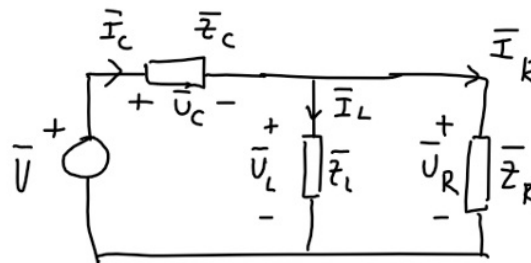
$$\bar{I}_R = \frac{\bar{U}_R}{\bar{Z}_R} = \frac{j100}{50} = j2\text{A} = 2e^{j\pi/2}\text{A}$$

$$\bar{U}_C = \bar{Z}_C \bar{I}_C = \bar{U} - \bar{U}_{RL} = 100 - j100\text{V}$$

$$\bar{U}_C = 100\sqrt{2} e^{-j\pi/4}\text{V}$$

$$U_C + U_{RL} = 100\sqrt{2} + 100 = 100(1 + \sqrt{2}) \neq U = 100$$

**PAZI!  $U_C + U_{RL} \neq U$ !**

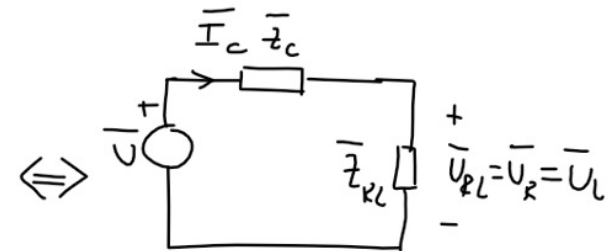


$$\bar{Z}_{RL} = \bar{Z}_R \parallel \bar{Z}_L = \frac{\bar{Z}_R \cdot \bar{Z}_L}{\bar{Z}_R + \bar{Z}_L}$$

$$\bar{Z}_{RL} = \frac{50 \cdot j50}{50 + j50} = \frac{j50}{1 + j} \cdot \frac{1 - j}{1 - j}$$

$$\bar{Z}_{RL} = \frac{50 + j50}{2} = 25 + j25\Omega$$

$$\bar{Z}_{RL} = 25\sqrt{2} e^{j\pi/4}\Omega$$



$$\bar{I}_C = \frac{\bar{U}}{\bar{Z}_C + \bar{Z}_{RL}} = \frac{100}{-j50 + 25 + j25}$$

$$\bar{I}_C = \frac{100}{25 - j25} = \frac{4}{1 - j} \cdot \frac{1 + j}{1 + j}$$

$$\bar{I}_C = 2 + j2\text{A}$$

$$\bar{I}_C = \frac{100}{25\sqrt{2} e^{-j\pi/4}} = 2\sqrt{2} e^{j\pi/4}\text{A}$$

$$\bar{U}_{RL} = \bar{U}_R = \bar{U}_L = \bar{Z}_{RL} \bar{I}_C$$

$$\bar{U}_{RL} = 25\sqrt{2} e^{j\pi/4} \cdot 2\sqrt{2} e^{j\pi/4} = 100 e^{j\pi/2}\text{V}$$

$$\bar{U}_{RL} = j100\text{V}$$

b)

$$\text{I} \quad \bar{S}_z = \sum_{i=1} \bar{S}_i = \bar{S}_R + \bar{S}_L + \bar{S}_C = R I_R^2 + j X_L I_L^2 + j X_C I_C^2 = 50 \cdot 2^2 + j 50 \cdot 2^2 - j 50 \cdot (2\sqrt{2})^2$$

$$\bar{S}_z = 200 + j 200 - j 400 = 200 - j 200 \text{ VA}$$

$$\text{II} \quad \bar{Z} = \bar{Z}_C + \bar{Z}_{RL} = -j 50 + 25 + j 25 = 25 - j 25 \Omega = 25\sqrt{2} e^{-j\pi/4} \Omega$$

$$\bar{S}_z = \bar{Z} I_C^2 = (25 - j 25) \cdot (2\sqrt{2})^2 = 200 - j 200 \text{ VA}$$

$$c) \quad i_L(t) = 2\sqrt{2} \sin(1000t) \text{ A}$$

$$i_R(t) = 2\sqrt{2} \sin(1000t + \bar{u}/2) \text{ A}$$

$$i_C(t) = 4 \sin(1000t + \bar{u}/4) \text{ A}$$

$$i_C(t) = i_R(t) + i_L(t)$$

$$\bar{I}_C = \bar{I}_R + \bar{I}_L$$

$$u_L(t) = 100\sqrt{2} \sin(1000t + \bar{u}/2) \text{ V}$$

$$u_R(t) = u_L(t)$$

$$u_C(t) = 200 \sin(1000t - \bar{u}/4) \text{ V}$$

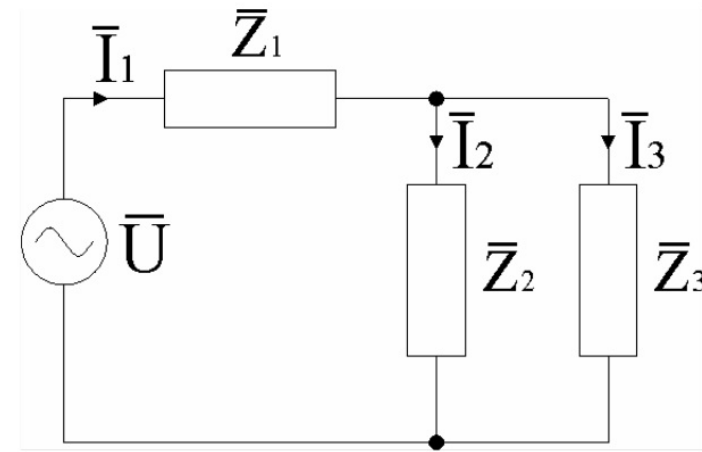
$$u(t) = u_C(t) + u_{RL}(t)$$

$$\bar{U} = \bar{U}_C + \bar{U}_{RL}$$

6. Na Slici 3 je prikazano kolo naizmenične struje koje čine naponski generator efektivne vrednosti napona  $U = 50V$  i tri potrošača impedansi  $Z_1 = (1 - 2j)\Omega$ ,  $Z_2 = (1 - j)\Omega$  i

$Z_3 = (1 + 3j)\Omega$ . Odrediti:

- kompleksne izraze za struje u svim granama kola;
- efektivnu vrednost napona na potrošaču  $Z_1$ ;
- aktivnu snagu potrošača  $Z_2$  i kompleksnu prividnu snagu potrošača  $Z_3$ .



Slika 3

$$a) \bar{U} = U e^{j0} = 50 e^{j0} = 50 \text{ V}$$

$$\bar{Z}_{23} = \bar{Z}_2 \parallel \bar{Z}_3 = \frac{\bar{Z}_2 \bar{Z}_3}{\bar{Z}_2 + \bar{Z}_3} = \frac{(1-j)(1+j3)}{1-j+1+j3} = \frac{1+3-j+j^3}{2-j^2} = \frac{4-j^2}{2-j^2} = \frac{2-j}{1-j} \cdot \frac{1+j}{1+j}$$

$$\bar{Z}_{23} = \frac{2+1-j+j^2}{2} = 1,5 - j0,5 \Omega$$

$$\bar{Z}_{123} = \bar{Z}_1 + \bar{Z}_{23} = 1-j^2 + 1,5-j0,5 = 2,5 - j2,5 \Omega$$

$$\bar{I}_1 = \frac{\bar{U}}{\bar{Z}_{123}} = \frac{50}{2,5-j2,5} = \frac{500}{25-j25} = \frac{20}{1-j} \cdot \frac{1+j}{1+j} = \frac{20(1+j)}{2} = 10 + j10 \text{ A}$$

$$\bar{U}_1 = \bar{Z}_1 \bar{I}_1 = (1-j^2)(10+j10) = 10+20+j10-j^2 20 = 30-j10 \text{ V}$$

$$\bar{U}_2 = \bar{U}_3 = \bar{U} - \bar{U}_1 = \bar{Z}_{23} \bar{I}_1 = 50 - (30-j10) = 20 + j10 \text{ V}$$

$$\bar{I}_2 = \frac{\bar{U}_2}{\bar{Z}_2} = \frac{20 + j^{10}}{1 - j} \cdot \frac{1 + j}{1 + j} = \frac{20 - 10 + j^{10} + j^{20}}{2} = 5 + j^{15} \text{ A}$$

$$\bar{I}_3 = \frac{\bar{U}_3}{\bar{Z}_3} = \frac{20 + j^{10}}{1 + j^3} \cdot \frac{1 - j^3}{1 - j^3} = \frac{20 + 30 + j^{10} - j^{60}}{1 + 9} = 5 - j^5 \text{ A}$$

$$b) \quad \bar{U}_1 = 30 - j^{10} \text{ V} \Rightarrow U_1 = \sqrt{30^2 + (-10)^2} = \sqrt{10^2 (3^2 + 1^2)} = 10 \sqrt{10} \text{ V}$$

$$c) \quad \bar{S}_2 = \bar{Z}_2 \bar{I}_2^2 = (1 - j) (5^2 + 15^2) = 250 - j^{250} \text{ VA} \Rightarrow P_2 = 250 \text{ W}$$

$$Q_2 = -250 \text{ VAR}$$

$$\bar{S}_3 = \bar{Z}_3 \bar{I}_3^2 = (1 + 3j) (5^2 + (-5)^2) = (1 + 3j) \cdot 50 = 50 + j^{150} \text{ VA}$$

$$P_3 = 50 \text{ W}$$

$$Q_3 = 150 \text{ VAR}$$