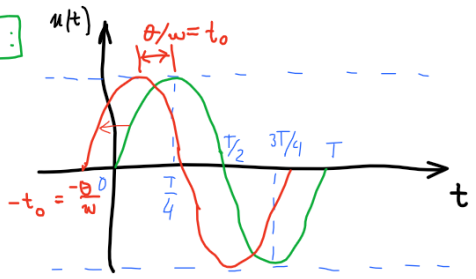
NAIZMENIČNE STRUJE

t:



$$u(t) = \underset{1)}{U_m} \sin(\underset{2)}{\omega t} + \underset{3)}{\theta})$$

1) U_m - amplituda [V]

$U = \frac{U_m}{\sqrt{2}}$ - efektivna vrednost

2) ω - krožna učestanost [rad/s]

$f = \frac{\omega}{2\pi}$ - učestanost (frekvencija) [Hz = $\frac{1}{s}$]

$T = \frac{1}{f}$ - perioda [s]

3) $\alpha = \omega t + \theta$ - faza [rad]

$t = 0$: $\alpha = \theta$ - početna faza [rad]

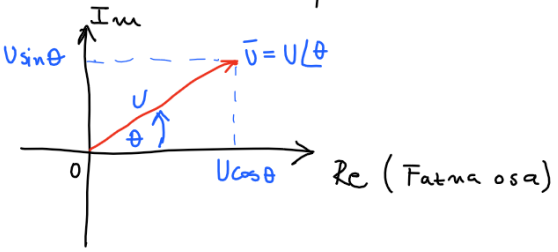
$(\omega t_0 = \theta,\ t_0 = \frac{\theta}{\omega})$

w: $\bar{U} = U e^{j\theta} = U\cos\theta + jU\sin\theta$

$U = \frac{U_m}{\sqrt{2}}$ - efektivna vrednost

θ - početna faza

ω - vektor rotira ↻



PRIMER 1:

$u(t) = 10\sin(314t + \frac{\pi}{4})$ [V]

$U_m = 10$ V

$U = \frac{10}{\sqrt{2}} = 5\sqrt{2}$ V

$\omega = 314 \frac{rad}{s} \Rightarrow f = \frac{314}{2\cdot 3{,}14} = 50$ Hz

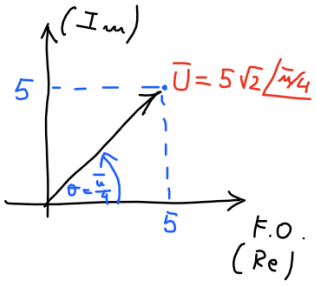
$T = \frac{1}{T} = 20$ ms

$\theta = \frac{\pi}{4}$ rad

$\bar{U} = Ue^{j\theta} = 5\sqrt{2}\, e^{j\pi/4}$

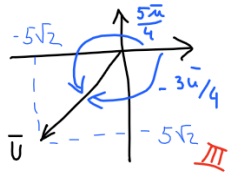
$\bar{U} = 5\sqrt{2}\left(\cos(\frac{\pi}{4}) + j\sin(\frac{\pi}{4})\right)$

$\bar{U} = 5 + j5$ [V]



PRIMER 2:

$\bar{U} = -5\sqrt{2} - j5\sqrt{2}$, $f = 100$ Hz



$U = |\bar{U}| = \sqrt{(-5\sqrt{2})^2 + (-5\sqrt{2})^2} = 10$ V

$\theta = \arg\{\bar{U}\} = \operatorname{arctg}\frac{-5\sqrt{2}}{-5\sqrt{2}} + k\pi$

$\theta = \operatorname{arctg}(1) + k\pi = \frac{\pi}{4} + k\pi$

III: $\theta = \pi/4 - \pi = -3\pi/4$

(ili: $\theta = \frac{\pi}{4} + \pi = +5\pi/4$)

$\Leftrightarrow \bar{U} = 10e^{-j3\pi/4}$ V

1) $U_m = U\sqrt{2} = 10\sqrt{2}$ V

2) $\omega = 2\pi f = 628 \frac{rad}{s}$

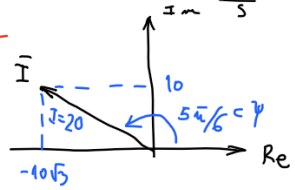
3) $\theta = -3\pi/4$ rad

$u(t) = \underset{1)}{10\sqrt{2}}\ \sin\left(\underset{2)}{628t} - \underset{3)}{\frac{3\pi}{4}}\right)$ V

koristimo sinus !

PRIMER 3: $\bar{I} = -10\sqrt{3} + j10$ A, $\omega = 314 \frac{rad}{s}$

II

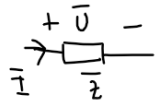


$I = |\bar{I}| = \sqrt{(-10\sqrt{3})^2 + 10^2} = \sqrt{10^2\cdot(3+1)} = \sqrt{10^2\cdot 2^2} = 20$ A $\Rightarrow I_m = 20\sqrt{2}$ A

$\psi = \operatorname{arctg}\frac{10}{-10\sqrt{3}} + k\pi = -\operatorname{arctg}\frac{\sqrt{3}}{3} + k\pi = -\pi/6 + k\pi$

II kv. $\psi = -\pi/6 + \pi = 5\pi/6$

$i(t) = 20\sqrt{2}\sin(314t + 5\pi/6)$ A



$\bar{Z} = Z e^{j\varphi} = R + jX$

$\bar{Z}$ - kompleksna impedansa [Ω]

Z - impedansa [Ω]

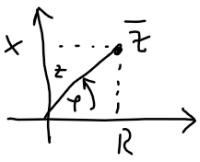
R - rezistansa [Ω]

X - reaktansa [Ω]

X - induktivni

$X < 0$ - kapacitivni

$X = 0$ - rezistivni



SNAGA: $\bar{S} = S e^{j\varphi} = P + jQ$

$\bar{S}$ - kompleksna prividna snaga [VA]

S - prividna snaga [VA]

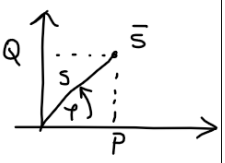
P - aktivna snaga [W]

Q - reaktivna snaga [VAR]

$Q > 0$ - induktivni

$Q < 0$ - kapacitivni

$Q = 0$ - rezistivni

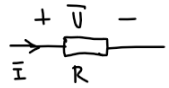


$\bar{S} = \bar{U}\bar{I}^* = \bar{Z}\cdot\bar{I}\cdot\bar{I}^* = \bar{Z}I^2 = RI^2 + jXI^2$

$\bar{S} = ZI^2 e^{j\varphi}$

S $\quad$ P $\quad$ Q

OTPORNIK:



$u(t) = Ri(t)$

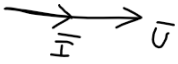
$\bar{Z}_R = R$ [Ω]

$\bar{U} = \bar{Z}_R\bar{I} = R\bar{I}$

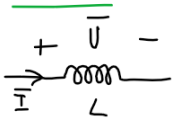
$Ue^{j\theta} = RIe^{j\psi} \Rightarrow U = RI,\ \theta = \psi$

$\bar{S}_R = \bar{Z}_R\cdot I^2 = RI^2 \Rightarrow P_R = RI^2$ [W]

$Q_R = 0$ [VAR]



KALEM:



$u(t) = L\frac{di(t)}{dt}$

$X_L = \omega L$ [Ω]

$\bar{Z}_L = +jX_L = j\omega L = \omega L e^{j\pi/2}$

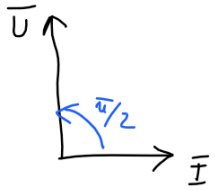
$\bar{U} = \bar{Z}_L\bar{I}$

$Ue^{j\theta} = \omega LIe^{j(\psi+\pi/2)} \Rightarrow U = X_L I,\ \theta = \psi + \pi/2$

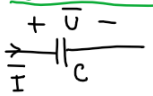
$\bar{S}_L = \bar{Z}_L\cdot I^2 = jX_L I^2 = j\omega LI^2$ [VA]

$P_L = 0$ W

$Q_L = \omega LI^2$ [var] > 0



KONDENZATOR:



$i(t) = C\frac{du(t)}{dt}$

$X_C = \frac{1}{\omega C}$ [Ω]

$\bar{Z}_C = -jX_C = -j\frac{1}{\omega C} = \frac{1}{\omega C}e^{-j\pi/2}$

$\bar{U} = \bar{Z}_C\bar{I} = \frac{I}{\omega C}e^{j(\psi-\pi/2)} \Rightarrow U = X_C I,\ \theta = \psi - \pi/2$

$\bar{S}_C = \bar{Z}_C I^2 = -jX_C I^2 = -j\frac{I^2}{\omega C}$ [VA]

$P_C = 0$ W

$Q_C = -X_C I^2 = -\frac{I^2}{\omega C}$ [VAR] < 0

---

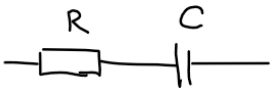
1. Odrediti kompleksnu impedansu $\bar{Z}$, ako je poznato da je vremenski oblik napona na impedansi $u(t) = 30\sin(314t - 15°)$ V i struje $i(t) = 15\sin\left(314t + \frac{\pi}{12}\right)$ A.

$\bar{U} = \frac{30}{\sqrt{2}}e^{-j\pi/12}$ V

$\bar{I} = \frac{15}{\sqrt{2}}e^{+j\pi/12}$ A

$\bar{Z} = \frac{\bar{U}}{\bar{I}} = \frac{30/\sqrt{2}}{15/\sqrt{2}}e^{-j(\pi/12+\pi/12)} = 2e^{-j\pi/6}$ Ω

$\bar{Z} = 2\cos\frac{\pi}{6} - j2\sin\frac{\pi}{6} = \sqrt{3} - j$ Ω



$\bar{Z} = R - j\frac{1}{\omega C}$

$R = \sqrt{3}$ Ω

$\frac{1}{\omega C} = 1 \Rightarrow C = \frac{1}{314} = 3{,}18$ mF