PRELAZNI PROCESI

JEDNOSMERNE STRUJE

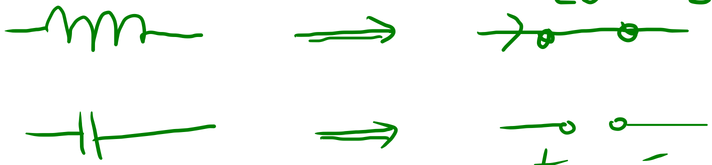
DC (direct current) $\Rightarrow$ $u(t) = U = const$, $i(t) = I = const$

	VEZA $U \leftrightarrow I$	DC
R $\rightarrow i_R$ + u_R −	OMOV ZAKON $u_R = R\, i_R$ $i_R = \frac{1}{R} u_R$	$U_R = R\, I_R$ $I_R = \frac{1}{R} U_R$
L $\rightarrow i_L$ + u_L −	$u_L = \frac{d\Psi}{dt}$, $\Psi = Li$ $u_L = L \frac{di_L}{dt}$ $i_L = \frac{1}{L} \int u_L\, dt$	$i_L(t) = I_L \doteq const \Rightarrow u_L(t) = L \frac{dI_L}{dt} = 0$! $I_L = const$ $\boxed{U_L = 0\,V \,!}$ NEMA INDUKCIJE KALEM JE KRATAK SPOJ $I_L = const$ + $U_L = 0$ −
C $\rightarrow i_C$ + u_C −	$i_C = \frac{dq_C}{dt}$, $q_C = Cu_C$ $i_C = C \frac{du_C}{dt}$ $u_C = \frac{1}{C} \int i_C\, dt$	$u_C(t) = U_C = const \Rightarrow i_C = C \frac{dU_C}{dt} = 0$! $U_C = const$ $\boxed{I_C = 0\,A \,!}$ NEMA PROTOKA STRUJE KONDENZATOR JE OTVORENA VEZA $I_C = 0$ + − $U_C = const$

PROCEDURA ZA REŠAVANJE ZADATAKA

1) $t<0$: KOLO SE NALAZI U POČETNOM STACIONARNOM STANJU.

$I_{L0} = i_L(0^-)$



REŠAVA SE KOLO DC STRUJE

$I_{L0} = ?$ i/ili $U_{C0} = ?$

$+\ u_{C0}\ - = u_C(0^-)$

2) $t>0$: KOLO JE U PRELAZNOM PROCESU (PP)

* PIŠU SE DIFERENCIJALNE JEDNAČINE I REŠAVAJU $i_L(t)$ i/ili $u_C(t)$
* VAŽI DA JE: (POČETNI USLOV) $\boxed{t=0}$

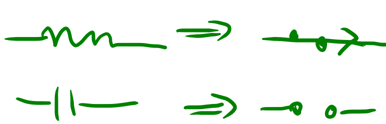
  $i_L(0^+) = i_L(0^-) = I_{L0}$ PAZI: $u_L(0^+) \neq u_L(0^-) = 0$

  $u_C(0^+) = u_C(0^-) = U_{C0}$ PAZI: $i_C(0^+) \neq i_C(0^-) = 0$

* $u_L(t) = L\dfrac{di_L(t)}{dt} = \ldots$, $i_C(t) = C\dfrac{du_C(t)}{dt} = \ldots$

3) $t\to\infty$: KOLO NALAZI U NOVO STACIONARNO STANJE

OPCIONO PROVERA



$I_{L\infty} = i_L(t\to\infty)$

$U_{C\infty} = u_C(t\to\infty)$

7. U kolu na Slici 7 poznate su vrednosti elemenata: $E = 6V$, $R = 1\Omega$, $L = 2\mu H$.

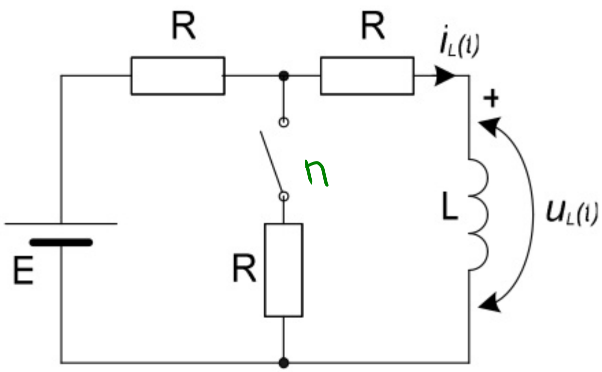
~~I)~~ Prekidač je zatvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se otvara.

II) Prekidač je otvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se zatvara.

a) Odrediti izraze za struju i napon kalema nakon otvaranja (zatvaranja) prekidača i nacrtati odgovarajuće vremenske dijagrame;

b) Odrediti vreme, nakon otvaranja (zatvaranja) prekidača, posle koga će se struja kalema dostići 90% svoje ~~krajnje~~ MAKSIMALNE vrednosti;

c) Odrediti minimalnu i maksimalnu energiju kalema.

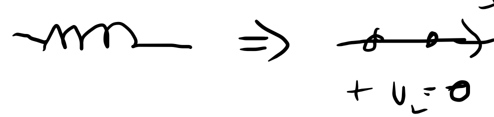


Slika 7

I) DOMAĆI RAD $\left(\text{n} \xrightarrow{t=0} \text{n}\right)$

II) RADIMO NA ČASU $\left(\text{n} \xrightarrow{t=0} \text{n}\right)$

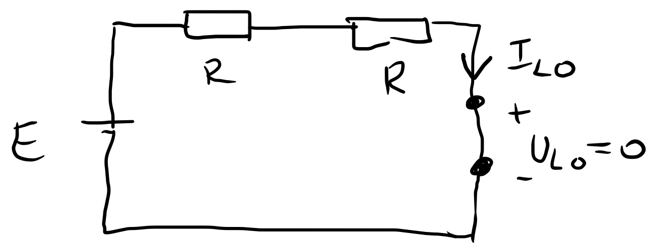
a) 1) $t<0$: POČETNO STACIONARNO STANJE



$u_L(0^-) = 0\ V$

$i_L(0^-) = I_{L0} = ?$

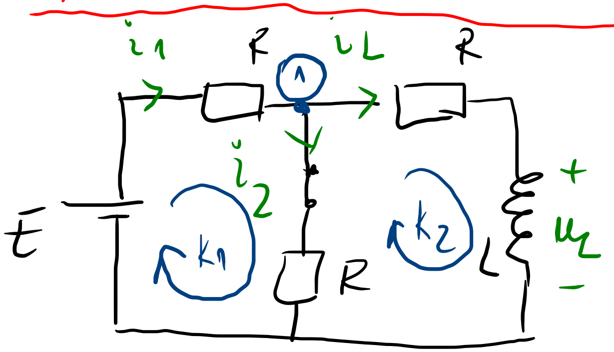
$t<0$: PREKIDAČ OTVOREN (slučaj II)

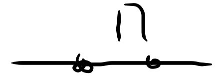


$$I_{L0} = \frac{E}{2R} = \frac{6}{2 \cdot 1} = 3\ A$$

$$i_L(0^-) = I_{L0} = 3A$$

2) $t > 0$: PRELAZNI PROCES





PREKIDAČ ZATVOREN

ČVOR 1 (I KZ): $\quad i_1(t) = i_2(t) + i_L(t) \quad (1)$

K1 (II KZ): $\quad E - R i_1(t) - R i_2(t) = 0 \quad (2)$

K2 (II KZ): $\quad R i_2(t) - R i_L(t) - u_L(t) = 0 \quad (3)$

$$u_L(t) = L \frac{di_L(t)}{dt} \quad (4)$$

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* REŠAVA SE PRVO PO u_L i i_L iz (1), (2) i (3), PA SE ZAMENI (4):

(1) → (2) : $\quad E - R(i_2 + i_L) - R i_2 = 0 \Rightarrow E - 2 R i_2 - R i_L = 0$

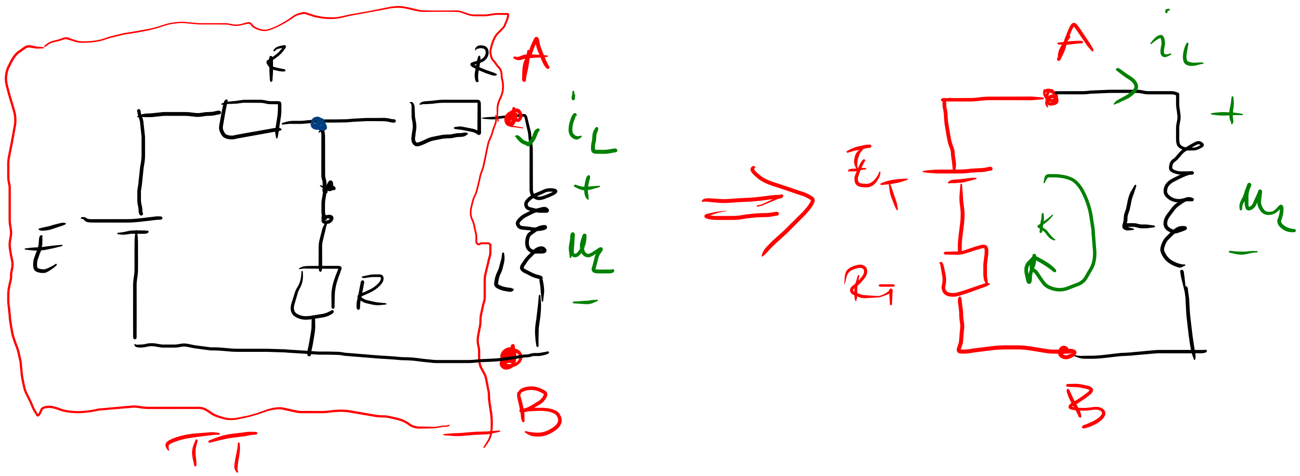
$$i_2 = \frac{E - R i_L}{2R} = \frac{E}{2R} - \frac{1}{2} i_L \quad (5)$$

(5) → (3) : $\quad R \dfrac{E - R i_L}{2R} - R i_L - u_L = 0$

$$E/2 - \frac{3}{2} R i_L - u_L = 0 \quad (6)$$

(4) → (6) : $\quad \dfrac{E}{2} - \dfrac{3}{2} R i_L - L \dfrac{di_L}{dt} = 0 \; / : (-L) \Rightarrow \boxed{\dfrac{di_L}{dt} + \dfrac{3R}{2L} i_L = \dfrac{E}{2L}}$

ALTERNATIVNO REŠENJE PRIMENOM TEVENENOVE TEOREME:



$$E_T = \frac{E}{2} = 3\ V$$

$$R_T = R + R/2 = \frac{3}{2} R$$

$$R_T = 1{,}5\ \Omega$$

K: $E_T - R_T i_L - u_L = 0$

$$\frac{E}{2} - \frac{3}{2} R i_L - L \frac{di_L}{dt} = 0 \quad / : (-L)$$

$$\boxed{\frac{di_L}{dt} + \frac{3R}{2L} i_L = \frac{E}{2L}}$$

ist a dif. jed.

$$\boxed{\tau = \frac{L}{R_T}} = \frac{L}{\frac{3}{2}R} = \frac{2L}{3R}$$

REŠAVANJE DIFERENCIJALNE JEDNAČINE 1. REDA:

$$\boxed{\frac{di_L}{dt} + \frac{i_L}{\tau} = K}\ ,\qquad K = \frac{E}{2L}\ ,\ \tau = \frac{2L}{3R}$$ — VREMENSKA KONSTANTA KOLA

KANONSKI OBLIK

$$\tau = \frac{2L}{3R} = \frac{2\cdot 2\mu}{3\cdot 1} = 4/3\ \mu s$$

REŠENJE: $i_L(t) = A e^{-t/\tau} + B$

ALTERNATIVNO PAMĆENJE KONSTANTI:

$$i_L(\infty) = \boxed{I_{L\infty} = B}$$

$$i_L(0) = I_{L0} = A + B$$

$$\boxed{A = I_{L0} - I_{L\infty}}$$

$$B = K\tau = \frac{E}{2L}\,\frac{2L}{3R} = \frac{E}{3R} = \frac{6}{3\cdot 1} = 2\,A$$ (PARTIKULARNO REŠENJE)

$$i_L(0^+) = A + B \quad (t \to 0^+)$$

$$i_L(0^+) = i_L(0^-) = I_{L0} = \frac{E}{2R} = 3A$$

(POČETNI USLOV)

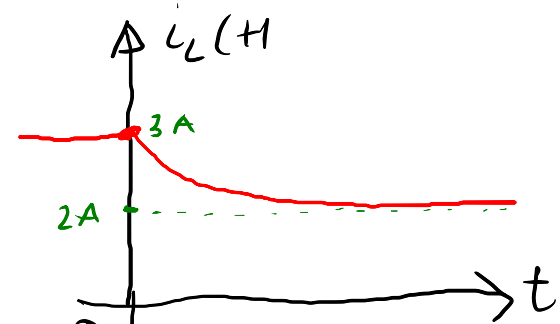
$$\frac{E}{2R} = A + \frac{E}{3R}$$

$$A = \frac{E}{2R} - \frac{E}{3R} = \frac{E}{6R} = 1A$$

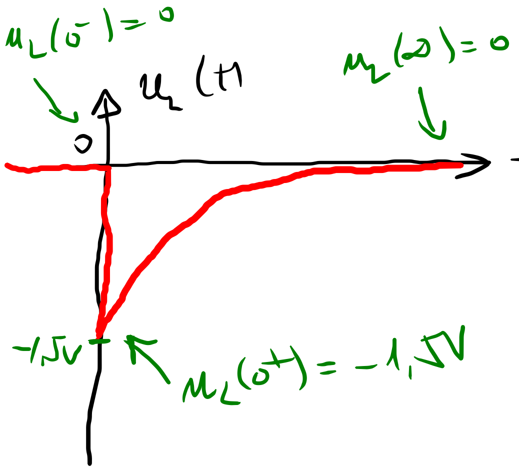
$$i_L(t) = \frac{E}{6R} e^{-t/\tau} + \frac{E}{3R} = \left(1\cdot e^{-\frac{3}{4}10^6 t} + 2\right) A$$

$$I_{L0} = i_L(0^+) = \lim_{t\to 0^+} i_L(t) = \frac{E}{6R} + \frac{E}{3R} = \frac{E}{2R} = 3A$$

$$I_{L\infty} = i_L(\infty) = \lim_{t\to\infty} i_L(t) = \frac{E}{3R} = 2A$$



NAPON KALEMA :



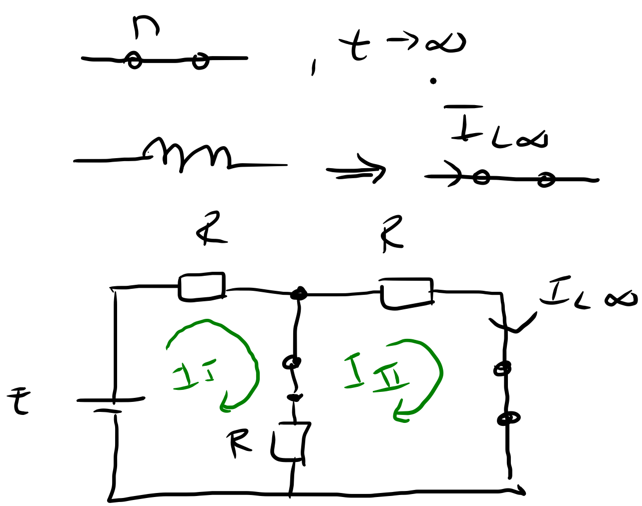
$$u_L(t)=L\frac{di_L(t)}{dt}=L\frac{d}{dt}\left[\frac{E}{6R}e^{-t/\tau}+\frac{E}{3R}\right]$$

$$u_L(t)=\cancel{L}\frac{E}{6\cancel{R}}\cdot\underbrace{\left(-\frac{1}{\tau}\right)}_{-\frac{3\cancel{R}}{2\cancel{L}}}e^{-t/\tau}\quad,\quad \tau=\frac{2L}{3R}$$

$$u_L(t)=-\frac{E}{4}e^{-t/\tau}=-1{,}5\,e^{-\frac{3}{4}10^6 t}\ V$$

(MALA PROVERA: i_L OPADA $\Rightarrow \frac{di_L}{dt}<0 \Rightarrow u_L<0$)

3) $t\to\infty$: NOVO STACIONARNO STANJE (OPCIONO, PROVERA)



$$\left.\begin{aligned} I\ \text{kont}:&\quad 2RI_I-RI_{II}=E \qquad (1)\\ II\ \text{kont}:&\quad -RI_I+2RI_{II}=0 \qquad (2)\end{aligned}\right\}\ \text{MKS}$$

$$(1)+2\cdot(2)\Rightarrow\quad 3RI_{II}=E$$

$$I_{L\infty}=I_{II}=\frac{E}{3R}=\frac{6}{3\cdot 1}=2A\ \checkmark$$

b) $i_L(t_1) = 90\% \; I_{L\,max}$ $\qquad I_{L\,max} = I_{L0} = \dfrac{E}{2R}$

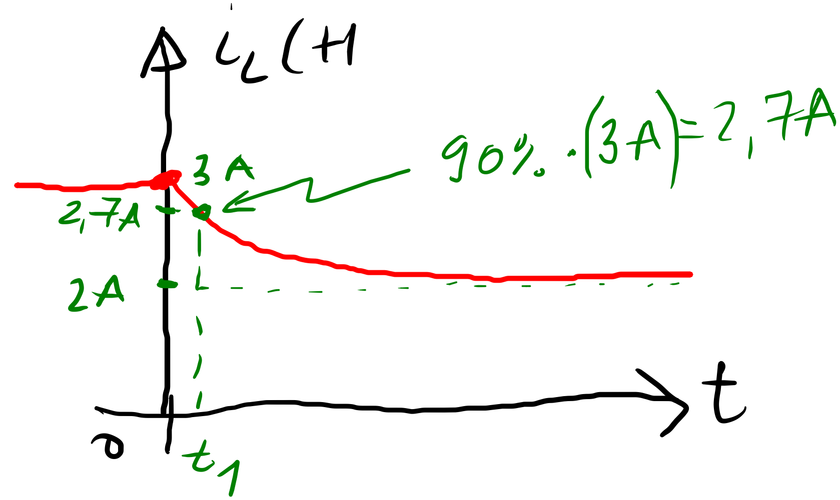
$$\frac{\cancel{E}}{6\cancel{R}} e^{-t_1/\tau} + \frac{\cancel{E}}{3\cancel{R}} = 0{,}9 \cdot \frac{\cancel{E}}{2\cancel{R}}$$

$$\frac{1}{6} e^{-t_1/\tau} = 0{,}45 - 0{,}33$$

$$e^{-t_1/\tau} = 0{,}12 \;/\ln(\cdot)$$

$$-t_1/\tau = \ln(0{,}12)$$

$$t_1 = -\tau \ln(0{,}12) = 2{,}12 \cdot \tau = 2{,}83\,\mu s$$



c) $W_L(t) = \dfrac{1}{2} L \, i_L^2(t)$

$$W_{L\,max} = \frac{1}{2} L I_{L\,max}^2 = \frac{1}{2} L I_{L0}^2 = \frac{1}{2} L \left(\frac{E}{2R}\right)^2 = 9\,\mu J$$

$$W_{L\,min} = \frac{1}{2} L I_{L\,min}^2 = \frac{1}{2} L I_{L\infty}^2 = \frac{1}{2} L \left(\frac{E}{3R}\right)^2 = 4\,\mu J$$

7. U kolu na Slici 7 poznate su vrednosti elemenata: $E = 6V$, $R = 1\Omega$, $L = 2\mu H$.

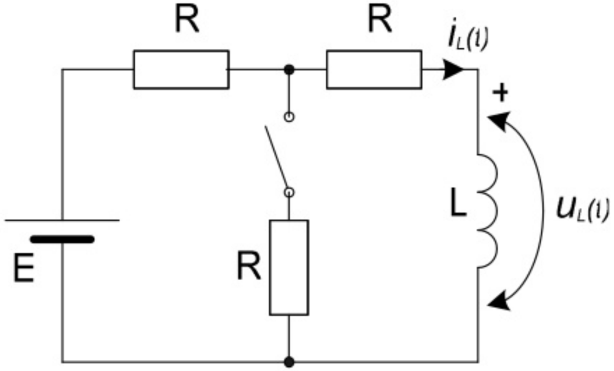
I) Prekidač je zatvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se otvara.

~~**II)**~~ Prekidač je otvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se zatvara.

a) Odrediti izraze za struju i napon kalema nakon otvaranja (zatvaranja) prekidača i nacrtati odgovarajuće vremenske dijagrame;

b) Odrediti vreme, nakon otvaranja (zatvaranja) prekidača, posle koga će se struja kalema dostići 90% svoje krajnje vrednosti;

c) Odrediti minimalnu i maksimalnu energiju kalema.



Slika 7

I) DOMAĆI RAD ($\xrightarrow{t=0}$)

RESULTAT ZA PROVERU :

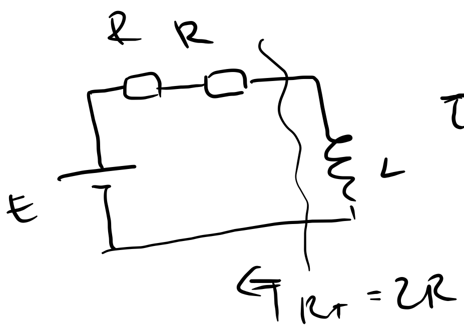
1) $t<0$: $I_{L0} = \frac{E}{3R} = 2A$

3) $t \to \infty$ $I_{L\infty} = \frac{E}{2R} = 3A$

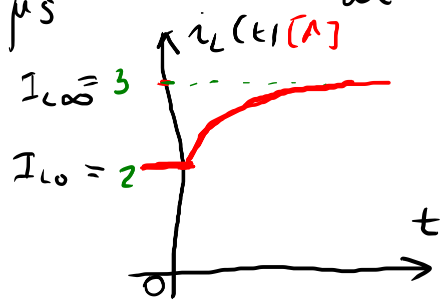
2) $t>0$: $i_L(t) = (I_{L0} - I_{L\infty})e^{-t/\tau} + I_{L\infty}$

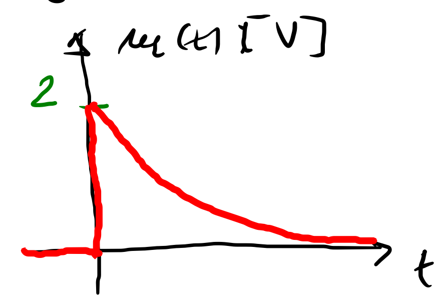
$i_L(t) = -\frac{E}{6R} e^{-t/\tau} + E/2R = -e^{-10^6 t} + 3A$

$u_L(t) = L\frac{di_L}{dt} = \dots = \frac{E}{3} e^{-t/\tau} = 2e^{-10^6 t}$ V

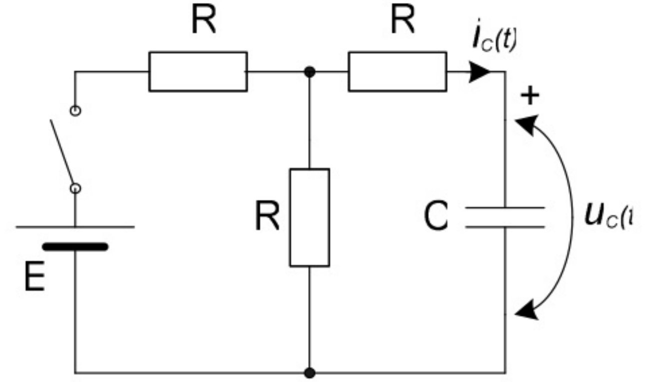


$\tau = \frac{L}{R_T} = \frac{L}{2R} = 1\mu s$





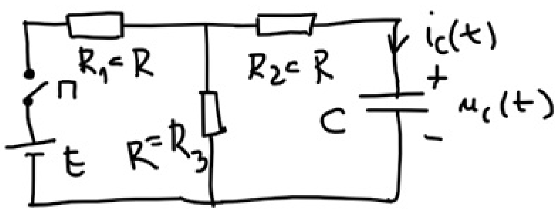
3. U kolu na Slici 2, poznate su vrednosti elemenata: $E = 6\,\mathrm{V}$, $R = 2\,\Omega$, $C = 2\,\mathrm{mF}$. Prekidač je ~~otvoren~~ ZATVOREN i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se ~~zatvara~~ OTVARA.
a) Odrediti izraze za struju i napon kondenzatora nakon zatvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame;
b) Odrediti vreme, nakon zatvaranja prekidača, posle koga će napon kondenzatora dostići 95% ~~krajnje~~ MAKSIMALNE vrednosti.
c) Odrediti minimalnu i maksimalnu elektrostatičku energiju kondenzatora.



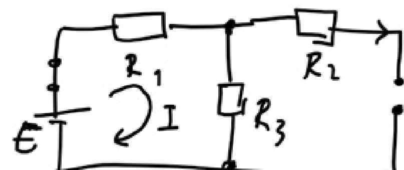
Slika 2

a)

REŠENJE:



1) $t < 0$ П = ON $\Rightarrow$ stac. stanje



$I_{C0} = 0$

$+ \; U_{C0} = ?$ (POČETNI USLOV)

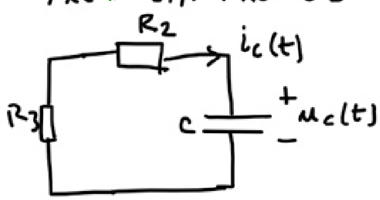
$-$

$I = \dfrac{E}{R_1 + R_3} \Rightarrow U_{C0} = +R_3 I - R_2 \underbrace{I_{C0}}_{=0} = R_3 I = \dfrac{R_3 E}{R_1 + R_3}$

$\boxed{U_{C0} = \dfrac{E}{2} = 3\,\mathrm{V}}$ POČETNI USLOV

2) $t > 0$ П = OFF

PRELAZNI PROCES



$i_C = C\dfrac{du_C}{dt}$

$-R_3 i_C - R_2 i_C - u_C = 0$

$u_C + (R_2 + R_3) i_C = 0$

$u_C + (R_2 + R_3) C \dfrac{du_C}{dt} = 0 \; / : C(R_2 + R_3)$

$\dfrac{du_C}{dt} + \dfrac{u_C}{\underbrace{(R_2 + R_3)C}_{=\tau}} = 0 = k$

$\tau = (R_2 + R_3)C = 2RC = 8\,\mathrm{ms}$

$k = 0$

$u_C(t) = A e^{-\frac{t}{\tau}} + B$

$B = k\tau = 0$

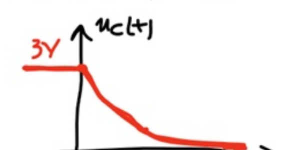
$A = U_{C0} - B = \dfrac{E}{2} = 3\,\mathrm{V}$

$u_C(t) = \dfrac{E}{2} e^{-t/\tau}$

$u_C(t) = 3 e^{-\frac{t}{8\,\mathrm{ms}}}$ [V]

$t = 0$: $u_C(0) = E/2 = 3\,\mathrm{V}$

$t \to \infty$: $u_C(\infty) = 0\,\mathrm{V}$



$$i_c(t) = C\frac{du_c}{dt}$$

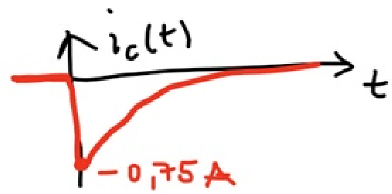
$$i_c(t) = C\frac{E}{2} e^{-\frac{t}{\tau}} \cdot \left(-\frac{1}{\tau}\right)$$

$$i_c(t) = -\frac{E}{4R} e^{-t/\tau} \ [A]$$

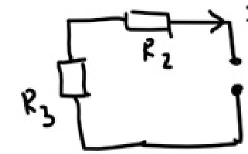
$$i_c(t) = -0{,}75 e^{-\frac{t}{8ms}} \ [A]$$

$t=0: \quad i_c(0^+) = -0{,}75\,A$

$t\to\infty: \quad i_c(\infty) = 0\,A$



3) $t\to\infty$ PROVERA (NOVOSTAC. STANJE)



$I_{c\infty} = 0\,A$

$u_{c\infty} = 0\,V$

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b) $$u_c(t_1) = \frac{E}{2} e^{-\frac{t_1}{\tau}}$$

$$u_c(t_1) = 95\%\, U_{c\,max} = 0{,}95 \cdot \frac{E}{2} \ (= 2{,}85\,V)$$

$$\frac{E}{2} e^{-t_1/\tau} = 0{,}95\frac{E}{2}$$

$$t_1 = -\tau \cdot \ln(0{,}95) \approx 0{,}4\,ms$$

c) $$W_c(t) = \frac{1}{2} C u_c^2(t)$$

$$W_{c\,max} = \frac{1}{2} C U_{c\,max}^2 = \frac{1}{8} CE^2 = 9\,mJ \quad \left(U_{c\,max} = \frac{E}{2}\right)$$

$$W_{c\,min} = \frac{1}{2} C U_{c\,min}^2 = 0\,J \quad (U_{c\,min} = 0)$$

NAPOMENA: $\boxed{Q = CU} \Rightarrow q_c(t) = C\,u_c(t)$

$$i_c(t) = C\frac{du_c(t)}{dt}$$

$$i_c(t) = \frac{dq_c(t)}{dt}$$

$U_{c0} = E/2 \Rightarrow Q_{c0} = CE/2$

$$\frac{du_c}{dt} + \frac{u_c}{\tau} = K \ /\cdot C$$

$$\frac{dq_c}{dt} + \frac{q_c}{\tau} = K\cdot C$$

2. (DOMAĆI) U kolu na Slici 2, poznate su vrednosti elemenata: $E = 6V$, $R = 2\Omega$, $C = 2mF$. Prekidač je **otvoren** i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se **zatvara**.

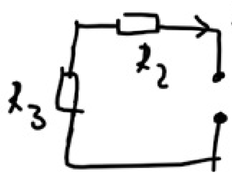
a) Odrediti izraze za struju i napon kondenzatora nakon zatvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame;

b) Odrediti vreme, nakon zatvaranja prekidača, posle koga će napon kondenzatora dostići 95% maksimalne vrednosti.

c) Odrediti minimalnu i maksimalnu elektrostatičku energiju kondenzatora.

a)

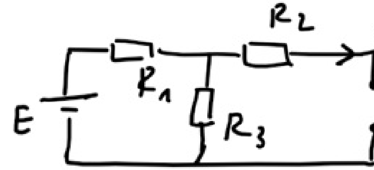
1) $t<0$ Π=OFF (STAC. ST.)



$I_{C0-} = 0$

$U_{C0} = 0$

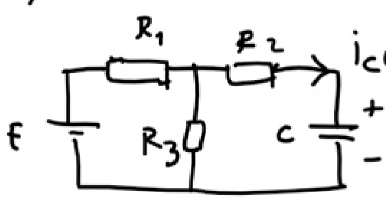
3) $t \to \infty$ Π=ON (NOVO STAC. ST.)



$I_{C\infty} = 0$

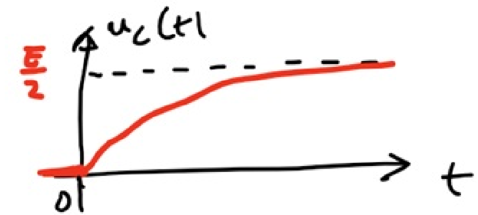
$U_{C\infty} = \dfrac{R_3 E}{R_1 + R_3} = E/2$

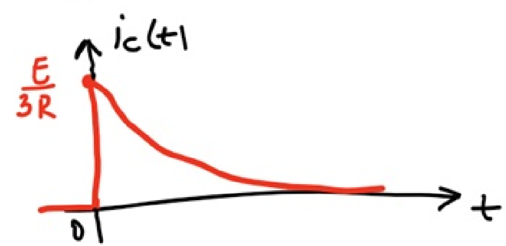
2) $t>0$ Π=ON (PREL. PR.)



$\tau = (R_2 + R_1 \| R_3)C$

$\tau = \frac{3}{2} RC$





⋮

$u_c(t) = \frac{E}{2}\left(1 - e^{-t/\tau}\right)$ [V]

$i_c(t) = \frac{E}{3R} e^{-t/\tau}$ [A]

b) ...

c) ...