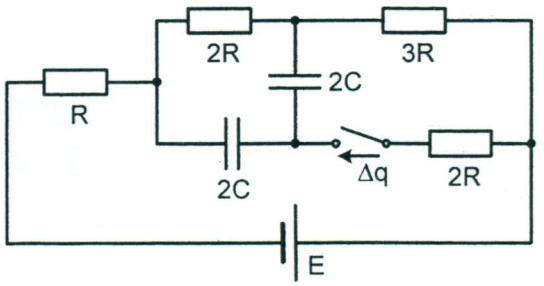
Elektrotehnika

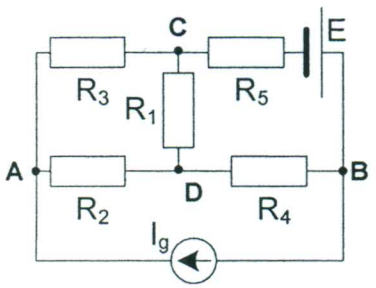
02. februar 2022.

1. U kolu na Slici 1 poznate su vrednosti E, R i C. Kondenzatori su bili neopterećeni pre povezivanja u kolo. Prekidač je otvoren i uspostavljeno je stacionarno stanje. Odrediti količinu naelektrisanja Δq koja će proteći kroz granu sa prekidačem nakon njegovog zatvaranja. **(20 poena)**

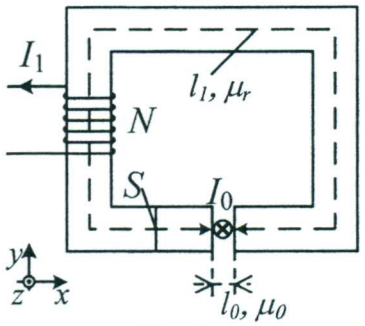


Slika 1

2. U kolu jednosmerne struje sa Slike 2, potrebno je izračunati intenzitete struja koje protiču kroz otpornik R_2 i generator E, **primenom metode konturnih struja**. Poznato je: $I_G = 2\,\mathrm{A}$, $E = 80\,\mathrm{V}$, $R_1 = R_2 = R_3 = R_4 = R_5 = 10\,\Omega$. **(20 poena)**



Slika 2



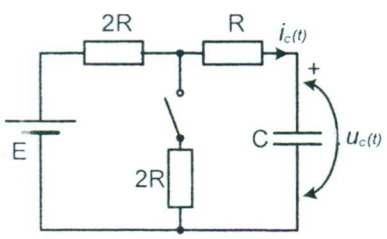
Slika 3

3. Na pravougaonom jezgru od feromagnetnog materijala (Slika 3), relativne magnetne permeabilnosti $\mu_r = 1000$, sa vazdušnim procepom debljine $l_0 = 0.2\,mm$, ravnomerno je namotano $N = 200$ navojaka kroz koje protiče struja intentziteta $I_1 = 5\,\mathrm{A}$. Dužina srednje linije magnetnog jezgra je $l_1 = 20cm$, a površina poprečnog preseka jezgra, oblika kvadrata, iznosi $S = 4\,\mathrm{cm}^2$. Odrediti i skicirati vektor sile koja deluje na tanak pravolinijski provodnik, postavljen u vazdušnom procepu na način prikazan na slici, kroz koji protiče struja intenziteta $I_0 = 1\,\mathrm{A}$. ($\mu_0 = 4\pi \cdot 10^{-7}\,\mathrm{H/m}$) **(20 poena)**

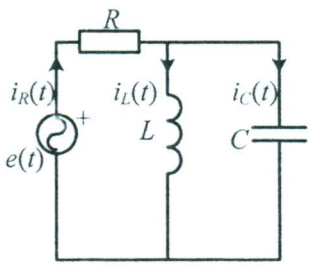
4. U kolu na Slici 4, poznate su vrednosti elemenata: $E = 200\,\mathrm{V}$, $R = 10\,\Omega$, $C = 2\,\mu\mathrm{F}$. Prekidač je otvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se zatvara.

a) Odrediti izraze za struju i napon kondenzatora nakon zatvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame. **(15 poena)**

b) Odrediti vrednost energije električnog polja kondenzatora u trenutku $t_1 = 120\,\mu\mathrm{s}$. **(5 poena)**



Slika 4



Slika 5

5. U kolu na Slici 5 poznato je: $e(t) = 200\sin(1000t + \pi/4)\,\mathrm{V}$, $R = 10\,\Omega$, $L = 10\,\mathrm{mH}$, $C = 200\,\mu\mathrm{F}$.

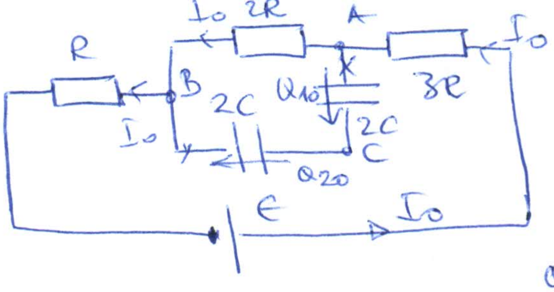
a) Odrediti kompleksne struje u svim granama i kompleksne napone na otporniku, kalemu i kondenzatoru, **(10 poena)**

b) Odrediti vremenski oblik struje kondenzatora, **(4 poena)**

c) Odrediti aktivnu, reaktivnu i prividnu snagu celokupnog potrošača. **(6 poena)**

(1)

n→0



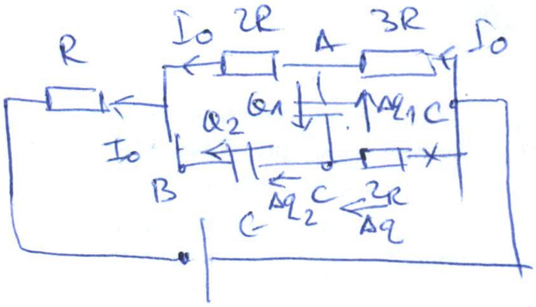
$$I_0 = \frac{\epsilon}{3R+2R+R} = \frac{\epsilon}{6R}$$

$$U_{AB} = 2R I_0 = \frac{\epsilon}{3}$$

$$U_{AC} = U_{CB} = \frac{U_{AB}}{2} = \frac{\epsilon}{6}$$

$$Q_{10} = Q_{20} = 2C \cdot U_{AC} = \frac{C\epsilon}{3}$$

n→3



$$I_0 = \frac{\epsilon}{3R+2R+R} = \frac{\epsilon}{6R}$$

$$U_{AC} = -3R I_0 = -\frac{\epsilon}{2}$$

$$U_{CB} = 5R I_0 = \frac{5\epsilon}{6}$$

$$Q_1 = U_{AC} \cdot 2C = -C\epsilon$$

$$Q_2 = U_{CB} \cdot 2C = \frac{5\epsilon C}{3}$$

$$Q_1 = Q_{10} - \Delta q_1 \Rightarrow \Delta q_1 = Q_{10} - Q_1 = \frac{4}{3} C\epsilon$$

$$Q_2 = Q_{20} + \Delta q_2 \Rightarrow \Delta q_2 = Q_2 - Q_{20} = \frac{4}{3} C\epsilon$$

$$\boxed{\Delta q = \Delta q_1 + \Delta q_2 = \frac{8}{3} C\epsilon}$$

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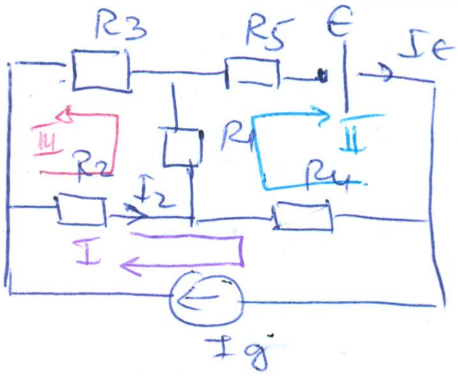
(2)



$$I_2 = I_I + I_{III} = 2 + (-2) = 0A$$

$$I_\epsilon = I_{II} = 4A$$

$$I_I = I_g$$

$$-R_4 I_I + (R_1+R_4+R_5) I_{II} + R_1 I_{III} = E$$

$$R_2 I_I + R_1 I_{II} + (R_2+R_1+R_3) I_{III} = 0$$

$$-R I_I + 3R I_{II} + R I_{III} = E \quad /:R$$

$$R I_I + R I_{II} + 3R I_{III} = 0 \quad /:R$$

$$I_I = I_g$$

$$3I_{II} + I_{III} = \frac{E}{R} + I_g$$

$$I_{II} + 3I_{III} = -I_g \Rightarrow I_{II} = -3I_{III} - I_g$$

$$3(-3I_{III} - I_g) + I_{III} = \frac{E}{R} + I_g$$

$$I_{II} = -3I_{III} - I_g$$

$$I_{II} = -3I_{III} - I_g$$

$$-9I_{III} - 3I_g + I_{III} = \frac{E}{R} + I_g$$

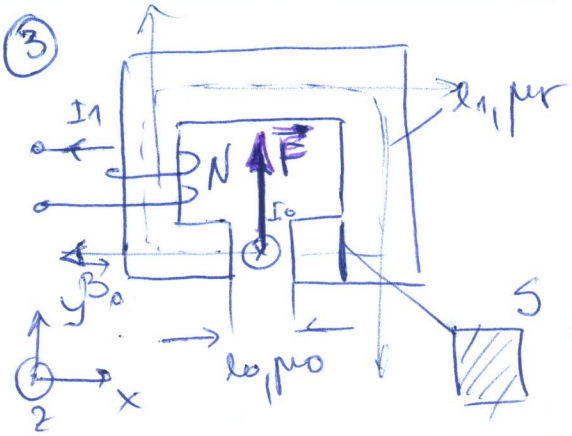
$$I_{II} = -3I_{III} - I_g$$

$$-8I_{III} = \frac{E}{R} + 4I_g$$

$$I_{III} = -\frac{I_g}{2} - \frac{E}{8R} = \frac{-2}{2} - \frac{80}{80} = -2A$$

$$I_{II} = -3I_{III} - I_g = -3(-2) - 2 = 6 - 2 = 4A$$

③



$$\oint \vec{H} d\vec{l} = \sum I$$

$$H_1 l_1 + H_0 l_0 = N I_1$$

$$\frac{B l_1}{\mu_0 \mu_r} + \frac{B l_0}{\mu_0} = N I_1$$

$$\vec{B}_0 = B_0 (-\vec{i})$$

$$B = \frac{N I_1 \mu_0}{\frac{l_1}{\mu_r} + l_0} = 1 T$$

$$\vec{F} = I_0 \vec{l} \times \vec{B}_0 = I_0 \sqrt{S} (-\vec{k}) \times B_0 (-\vec{i})$$

$$\vec{F} = I_0 B \sqrt{S}\, \vec{k} \times \vec{i} = I_0 B \sqrt{S}\, \vec{j}$$

$$\boxed{\vec{F} = 20\pi \vec{j}\ mN = 0{,}02\pi \vec{j}\ N}$$

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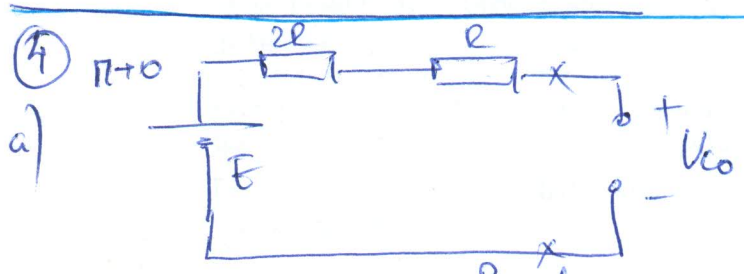
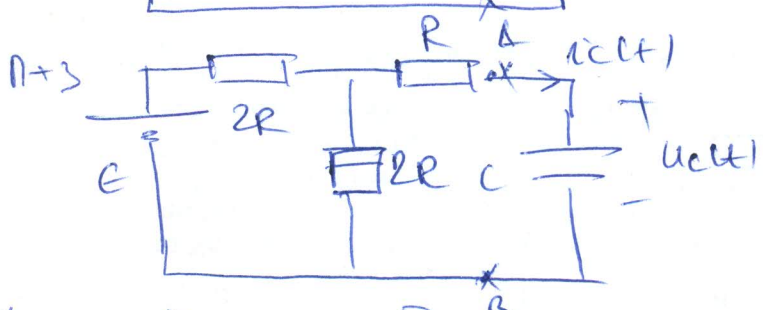
④

a)

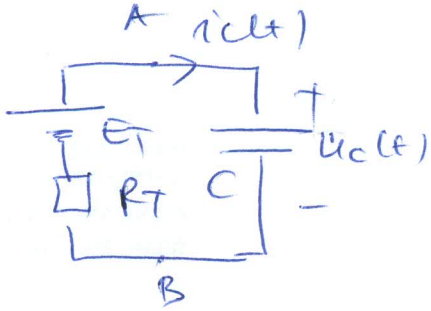


$$U_{C0} = E$$

$t = 0^+$



$\Leftrightarrow$



$$U_{AB}^{0V} = E_T = \frac{E}{2R+2R} \cdot 2R = \frac{E}{2}$$

$$R_T = R + 2R \| 2R = R + R = 2R$$

$$i_C(t) = C \frac{du_C(t)}{dt}$$

$$E_T - R_T i_C(t) - u_C(t) = 0$$

$$E_T - R_T C \frac{du_C(t)}{dt} - u_C(t) = 0$$

$$\frac{du_C(t)}{dt} + \frac{u_C(t)}{R_T C} = \frac{E_T}{R_T C}$$

$$\tau = R_T C = 2RC = 40\mu C$$

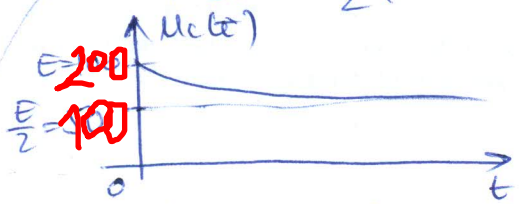
$$K = \frac{E_T}{R_T C} = \frac{E}{4RC}$$

$$B = K \cdot \tau = \frac{E}{2}$$

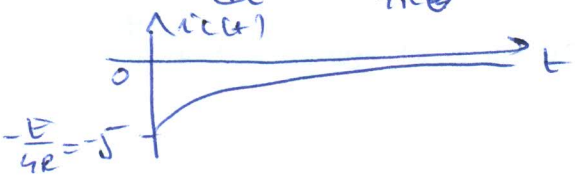
$$A + B = U_{C0} \Rightarrow A = \frac{E}{2}$$

$$u_C(t) = A e^{-t/\tau} + B$$

$$u_C(t) = \frac{E}{2}(e^{-t/\tau} + 1) = 100(e^{-t/\tau} + 1)\ [V]$$



$$i_C(t) = C \frac{du_C(t)}{dt} = \frac{-E}{4R} e^{-t/\tau} = -5 e^{-t/\tau}\ [A]$$



b) $t_1 = 120\mu s \quad \tau = 40\mu s$

$$W_C(t_1) = \frac{1}{2} C u_C^2(t_1) = \frac{1}{2} \cdot 2\mu \cdot \left(100\left(1 + e^{-\frac{120}{40}}\right)\right)^2 = 10^{-6} \cdot 10^4 (1 + e^{-3})^2$$

$$\boxed{W_C(t_1) = 10^{-2}(1 + e^{-3})^2 \approx 11{,}02\ mJ}$$

$e(t) = 200 \sin(1000t + \frac{\pi}{4})$ [V]

$\bar{E} = \frac{200}{\sqrt{2}} e^{j\pi/4} = 100\sqrt{2}\, e^{j\pi/4}$ [V]

a)

$R = 10\,\Omega \quad L = 10\,mH \quad C = 200\,\mu F$

$\bar{Z}_L = j\omega L = j \cdot 1000 \cdot 10 \cdot 10^{-3} = j10\,\Omega$

$\bar{Z}_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 10^3 \cdot 200 \cdot 10^{-6}} = -j\frac{10^3}{200} = -j5\,\Omega$

$\bar{Z}_{LC} = \frac{\bar{Z}_L \cdot \bar{Z}_C}{\bar{Z}_L + \bar{Z}_C} = \frac{j10 \cdot (-j5)}{j10 - j5} = -j10\,\Omega$

$\bar{Z}_e = R + \bar{Z}_{LC} = 10(1-j)\,\Omega = 10\sqrt{2}\left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) = 10\sqrt{2}\, e^{j\pi/4}\,\Omega$

$\bar{I}_R = \frac{\bar{E}}{\bar{Z}_e} = \frac{100\sqrt{2}\, e^{j\pi/4}}{10\sqrt{2}\, e^{-j\pi/4}} = 10\, e^{j\pi/2} = j10\,A = \bar{I}_R$

$\bar{U}_{LC} = \bar{Z}_{LC} \cdot \bar{I}_R = -j10 \cdot j10 = 100\,V = \bar{U}_L = \bar{U}_C$

$\bar{I}_L = \frac{\bar{U}_{LC}}{\bar{Z}_L} = \frac{100}{j10} = -j10\,A$

$\bar{I}_C = \frac{\bar{U}_{LC}}{\bar{Z}_C} = \frac{100}{-j5} = j20\,A$

$\bar{U}_R = R \cdot \bar{I}_R = j100\,V$

b) $\bar{I}_C = j20\,A \Rightarrow I_C = 20\,A \quad I_{cm} = 20\sqrt{2}\,A$

$\bar{I}_C = 20\, e^{j\pi/2} \quad \psi_C = \pi/2$

$i_C(t) = 20\sqrt{2} \sin(1000t + \pi/2)$ [A]

c) $\bar{S} = \bar{E}\,\bar{I}_e^* = \bar{Z}_e I_e^2 = 10\sqrt{2}\, e^{-j\pi/4} \cdot 100 = 1000\sqrt{2}\, e^{-j\pi/4}$ VA

$= 1000\sqrt{2}\left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right)$

$= (1000 - j1000)$ VA

$S = 1000\sqrt{2}\,VA = \sqrt{2}\,kVA$

$P = 1000\,W = 1\,kW$

$Q = -1000\,VAr = -1\,kVAr$