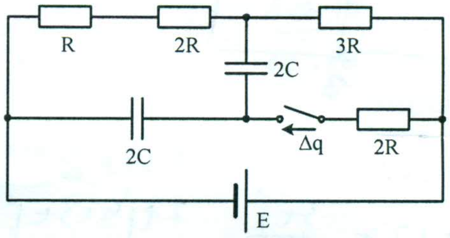
Elektrotehnika

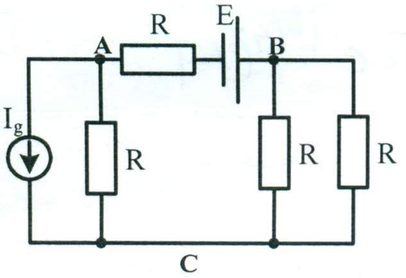
21. februar 2022.

1. U kolu na Slici 1 poznate su vrednosti E, R i C. Kondenzatori su bili neopterećeni pre povezivanja u kolo. Prekidač je otvoren i uspostavljeno je stacionarno stanje. Odrediti količinu naelektrisanja Δq koja će proteći kroz granu sa prekidačem nakon njegovog zatvaranja. **(20 poena)**

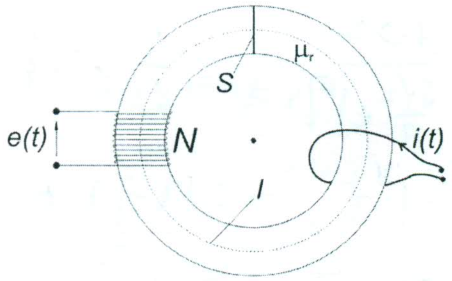


Slika 1

2. U kolu jednosmerne struje sa Slike 2 potrebno je izračunati napon U_{AC} i struju naponskog generatora E **primenom metode napona između čvorova**. Poznato je: $I_g = 20\,\text{A}$, $E = 40\,\text{V}$, $R = 1\,\Omega$. **(20 poena)**



Slika 2



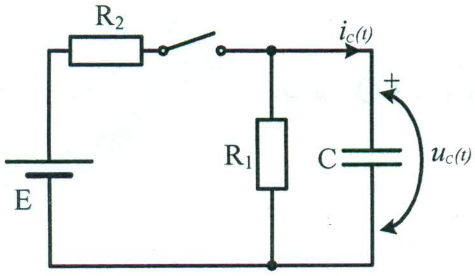
Slika 3

3. Na torus relativne magnetne permeabilnosti $\mu_r = 500$, površine poprečnog preseka $S = 1\,\text{cm}^2$ i dužine srednje linije $l = 10\,\text{cm}$, ravnomerno je namotano $N = 200$ navojaka žice. U provodniku proizvoljnog oblika koji obuhvata torus (Slika 3), postoji električna struja $i(t) = K \cdot t$, $t \geq 0$, gde je poznata konstanta $K = 10\,\text{A}/1\,\text{ms}$. Odrediti izraz za intenzitet vektora jačine magnetnog polja u torusu i indukovanu elektromotornu silu na krajevima namotaja. ($\mu_0 = 4\pi \cdot 10^{-7}$ H/m) **(20 poena)**

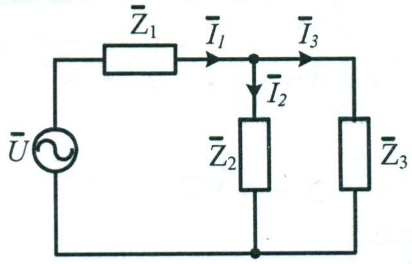
4. U kolu na Slici 4, poznate su vrednosti elemenata: $E = 200\,\text{V}$, $R_1 = 10\,\Omega$, $R_2 = 30\,\Omega$, $C = 2\,\mu\text{F}$. Prekidač je otvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se zatvara.

a) Odrediti izraze za napon i struju kondenzatora nakon zatvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame. **(15 poena)**

b) Odrediti trenutak, t_X, u kome će vrednost energije električnog polja kondenzatora biti jednaka četvrtini vrednosti koju će imati nakon završenog prelaznog procesa. **(5 poena)**



Slika 4



Slika 5

5. Na Slici 5 je prikazano kolo naizmenične struje koje čine naponski generator efektivne vrednosti napona $U = 10\,\text{V}$ i tri potrošača kompleksnih impedansi: $\overline{Z}_1 = (1 + j2)\,\Omega$, $\overline{Z}_2 = (1 - j3)\,\Omega$ i $\overline{Z}_3 = (1 + j)\,\Omega$.

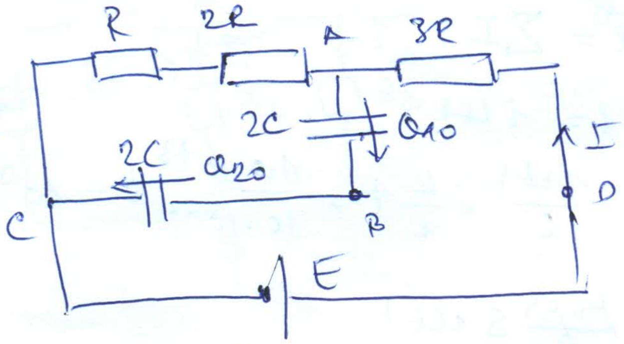
Odrediti:

a) kompleksne izraze za struje u svim granama kola; **(10 poena)**

b) aktivnu snagu potrošača Z_2 i kompleksnu prividnu snagu potrošača Z_3. **(10 poena)**

1

$n \to 0$

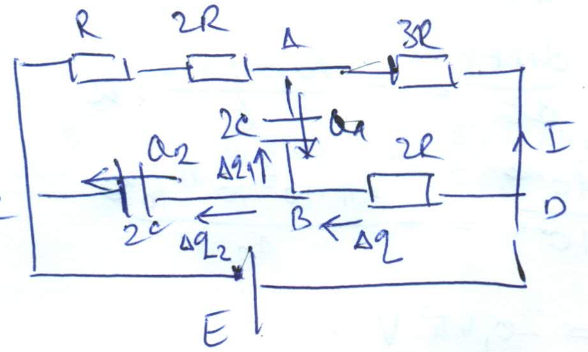


$$I = \frac{E}{6R}$$

$$Q_{10} = 2C \cdot U_{AB} = 2C \cdot \frac{U_{AC}}{2} = C \cdot U_{AC}$$
$$= C \cdot 3R \cdot I = \frac{C \cdot E}{2}$$

$$Q_{20} = 2C \cdot U_{BC} = 2C \cdot \frac{U_{AC}}{2} = \frac{CE}{2}$$

$n \to 3$



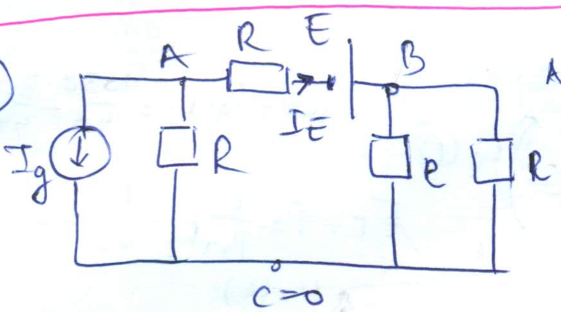
$$I = \frac{E}{6R}$$

$$Q_1 = 2C \cdot U_{AB} = 2C \cdot U_{AD} = 2C \cdot (-3RI)$$
$$= 2C\left(-\frac{E}{2}\right) = -CE$$

$$Q_2 = 2C \cdot U_{BC} = 2C \cdot U_{DC} = 2C \cdot E$$

$$Q_1 = Q_{10} - \Delta q_1 \Rightarrow \Delta q_1 = Q_{10} - Q_1 = \frac{CE}{2} - (-CE) = \frac{3CE}{2}$$
$$Q_2 = Q_{20} + \Delta q_2 \Rightarrow \Delta q_2 = Q_2 - Q_{20} = 2CE - \frac{CE}{2} = \frac{3CE}{2}$$
$$\Delta q = \Delta q_1 + \Delta q_2 = \frac{3CE}{2} + \frac{3CE}{2} = \boxed{3CE = \Delta q}$$

2



A: $U_{AO}\left(\frac{1}{R} + \frac{1}{R}\right) - U_{BO}\frac{1}{R} = -I_g - \frac{E}{R}$ $\quad / R$

B: $-U_{AO}\frac{1}{R} + \left(\frac{1}{R} + \frac{1}{R} + \frac{1}{R}\right)U_{BO} = \frac{E}{R}$ $\quad / R$

$$2U_{AO} - U_{BO} = -I_gR - E$$
$$-U_{AO} + 3U_{BO} = E$$

$$U_{BO} = 2U_{AO} + I_gR + E$$
$$-U_{AO} + 3(U_{AO} + I_gR + E) = E$$

$$U_{BO} = 2U_{AO} + I_gR + E$$
$$-U_{AO} + 6U_{AO} + 3I_gR + 3E = E$$

$$U_{BO} = 2U_{AO} + I_gR + E$$
$$5U_{AO} = -2E - 3I_gR$$

$$U_{AO} = -\frac{2E + 3I_gR}{5} = \frac{-(80+60)}{5} = -\frac{140}{5} = -28V$$
$$U_{BO} = 2U_{AO} + RI_g + E = -56 + 20 + 40 = 4\,V$$

$$U_{AO} = -28V$$
$$U_{BO} = 4V$$

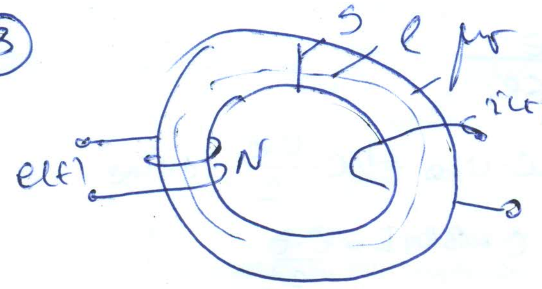
$$\boxed{U_{AC} = U_{AO} = -28V}$$

$$U_{BA} = U_{BO} - U_{AO} = 4 + 28 = 32$$
$$= E - RI_E$$

$$I_E = \frac{E - U_{BA}}{R} = \frac{40 - 32}{1}$$

$$\boxed{I_E = 8A}$$

(3)



$$\oint \vec{H}\,d\vec{l} = \sum I$$

$$H(t)\cdot l = i(t)$$

$$H(t) = \frac{i(t)}{l} = \frac{k}{l}t = \frac{10\cdot 10^{+3}}{10\cdot 10^{-2}}t = 10^5 t\left[\frac{A}{m}\right]$$

$$\phi(t) = B(t)\cdot S = \mu_0\mu_r H(t)\cdot S = \frac{\mu_0\mu_r}{l} S\, i(t)$$

$$e_{ind} = -\frac{d(N\phi(t))}{dt} = -\frac{N\mu_0\mu_r S}{l}\frac{di(t)}{dt} = -\frac{N\mu_0\mu_r S}{l}\cdot k$$

$$= -\frac{200\cdot 4\pi\cdot 10^{-7}\cdot 500\cdot 10^{-4}}{10\cdot 10^{-2}}\frac{10}{10^{-3}} = -\frac{4\pi\cdot 10^{5}\cdot 10^{-11}\cdot 10}{10^{-4}}$$

$$= -4\pi\cdot 10^{10}\cdot 10^{-11} = -4\pi\cdot 10^{-1} = -0{,}4\pi\ V$$

$$\boxed{H(t) = 10^5 t\left[\frac{A}{m}\right]} \qquad \boxed{e_{ind} = -0{,}4\pi\ [V]}$$

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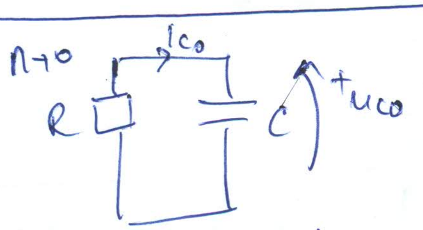
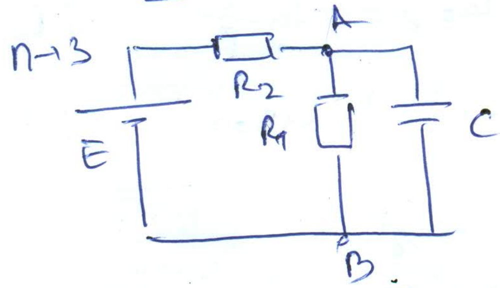
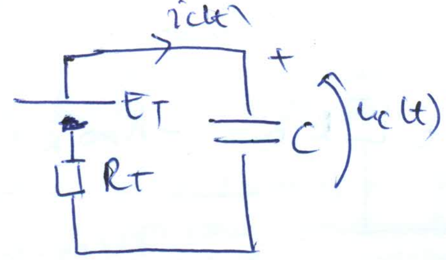
(4) $n\to 0$ $\qquad u_{c0} = 0$

a)



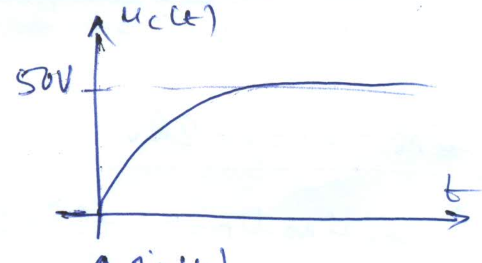
$n\to 3$





$$R_T = R_1 \| R_2 = \frac{10\cdot 30}{40} = \frac{30}{4}\,\Omega$$

$$E_T = R_1\cdot\frac{E}{R_1+R_2} = \frac{10}{40}\,200 = 50V$$



$$E_T - R_T i_c(t) - u_c(t) = 0$$

$$i_c(t) = C\frac{du_c(t)}{dt}$$

$$E_T - R_T C\frac{du_c(t)}{dt} - u_c(t) = 0$$

$$\frac{du_c(t)}{dt} + \frac{u_c(t)}{R_T C} = \frac{E_T}{R_T C}$$

$$\tau = R_T C = \frac{30}{4}\cdot 2\mu = 15\mu s$$

$$K = \frac{E_T}{R_T C} \qquad B = K\cdot\tau = E_T = 50V$$

$$A + B = u_{c0} \Rightarrow A = -B = -50V$$

$$\boxed{u_c(t) = Ae^{-t/\tau} + B = 50(1-e^{-t/\tau})\ [V]}$$

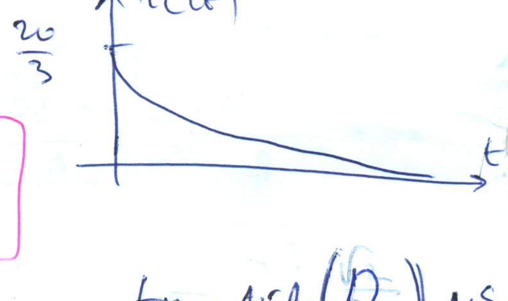
$$i_c(t) = C\frac{du_c(t)}{dt}$$

$$= -C\left(-\frac{1}{\tau}\right)E_T e^{-t/\tau}$$

$$= \frac{CE_T}{\tau}e^{-t/\tau}$$

$$= \frac{C\cdot E_T}{R_T C}e^{-t/\tau}$$

$$\boxed{i_c(t) = \frac{E_T}{R_T}e^{-\frac{t}{\tau}} = \frac{20}{3}e^{-t/\tau}\ [A]}$$



b) $W_c(t_x) = \frac{1}{2}C u_c^2(t_x) = \frac{E_T^2 C}{2}(1-e^{-t_x/\tau})^2 = \frac{1}{4}W_{cMAX} = \frac{1}{8}C\cdot E_T^2$

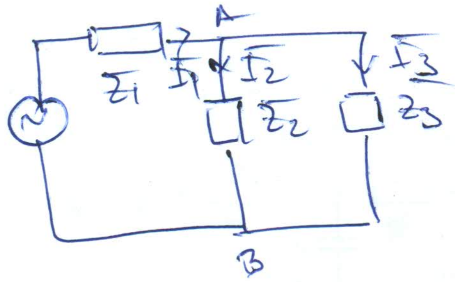
$$1 - e^{-\frac{t_x}{\tau}} = \frac{1}{2} \Rightarrow e^{-t_x/\tau} = 1 - \frac{1}{2} = \frac{1}{2} \Rightarrow \boxed{t_x = \tau\ln(2)}$$

$$t_x = 15\ln(2)\ \mu s$$

$$\boxed{t_x \cong 10{,}39\ \mu s}$$

5

a)



$$\bar{Z}_{23} = \frac{\bar{Z}_2 \cdot \bar{Z}_3}{\bar{Z}_2 + \bar{Z}_3} = \frac{(1-j3)(1+j)}{1-j3+1+j} = \frac{1-j3+j-j^2 3}{2-j2}$$

$$= \frac{(1-j2+3)(1+j)}{2(1-j)(1+j)} = \frac{(4-j2)(1+j)}{2(1+1)}$$

$$= \frac{2(2-j)(1+j)}{4} = \frac{2-j+2j-j^2}{2}$$

$$= \frac{2+j+1}{2} = \boxed{\frac{3+j}{2} = \bar{Z}_{23}}$$

$$\bar{Z}_e = \bar{Z}_1 + \bar{Z}_{23}$$

$$= 1+j2+\frac{3+j}{2} = \frac{2+j4+3+j}{2}$$

$$\bar{Z}_e = \frac{5+j5}{2} = \frac{5}{2}(1+j) = \frac{5\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2}\right) = \frac{5\sqrt{2}}{2} e^{j\pi/4}\ \Omega$$

$$\bar{I}_1 = \frac{\bar{U}}{\bar{Z}_e} = \frac{10\cdot 2}{5\sqrt{2}\, e^{j\pi/4}} = \frac{4}{\sqrt{2}} e^{-j\frac{\pi}{4}} = 2\sqrt{2}\, e^{-j\pi/4}\ [A] \qquad \bar{U} = U = 10\ V$$

$$\boxed{\bar{I}_1 = 2\sqrt{2}\left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) = 2(1-j)\ A}$$

$$\bar{U}_{AB} = \bar{U}_2 = \bar{U}_3 = \bar{Z}_{23} \cdot \bar{I}_1 = \frac{3+j}{2}\, 2(1-j) = 3+j-3j-j^2 = 3-2j+1 = 4-2j$$

$$= 2(2-j)\ V$$

$$\bar{I}_2 = \frac{\bar{U}_{AB}}{\bar{Z}_2} = \frac{2(2-j)}{1-j3}\,\frac{(1+j3)}{1+j3} = \frac{2(2-j+j6-j^2 3)}{1^2+9} = \frac{2(2+j5+3)}{10} = \frac{5\cdot 2(1+j)}{10}$$

$$\boxed{\bar{I}_2 = (1+j)\ A = \sqrt{2}\left(\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2}\right) = \sqrt{2}\, e^{j\pi/4}\ A}$$

$$\bar{I}_3 = \frac{\bar{U}_{AB}}{\bar{Z}_3} = \frac{2(2-j)}{1+j} \cdot \frac{1-j}{1-j} = \frac{2(2-j-2j+j^2)}{1^2+1} = 2-3j-1 = \boxed{(1-3j)\ A = \bar{I}_3}$$

b) $\boxed{P_2 = R_2 I_2^2 = 1\cdot(\sqrt{2})^2 = 2W}$

$$\bar{S}_3 = \bar{Z}_3 I_3^2 = (1+j)\left(\sqrt{1^2+3^2}\right)^2 = (1+j)10\ VA$$

$$\boxed{\bar{S}_3 = (10+j10)\ VA}$$