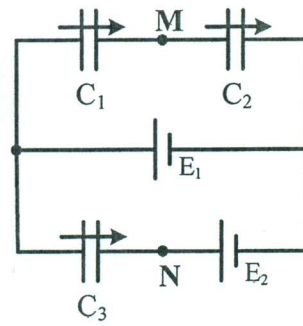


# Elektrotehnika

16. jun 2022.

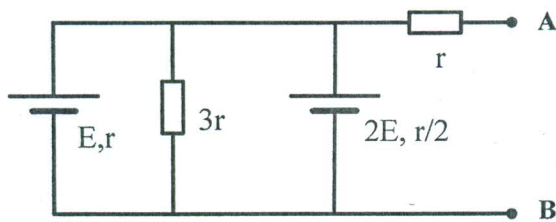
1. Kondenzatori kapacitivnosti  $C_1 = 20 \text{ nF}$ ,  $C_2 = 30 \text{ nF}$  i  $C_3 = 10 \text{ nF}$  i idealni naponski generatori elektromotornih sila  $E_1 = 10 \text{ V}$  i  $E_2 = 6 \text{ V}$ , povezani su kao na Slici 1. Pre povezivanja, kondenzatori su bili neopterećeni.

- a) Odrediti napone, količine naelektrisanja, kao i elektrostatičku energiju svakog kondenzatora. (15 p.)  
 b) Odrediti napon  $U_{MN}$ . (5 poena)

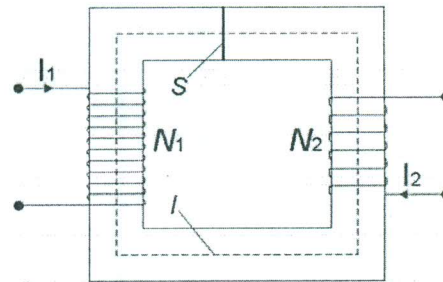


Slika 1

2. Za deo složenog kola, levo od tačaka A i B, (Slika 2) nacrtati ekvivalentni Tevenenov generator i odrediti njegove parametre. Poznato je  $E = 12 \text{ V}$  i  $r = 10 \Omega$ . (20 poena)



Slika 2

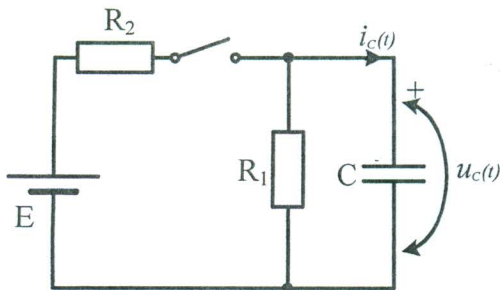


Slika 3

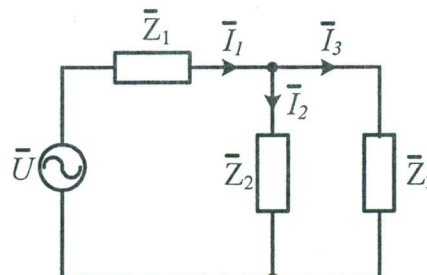
3. Na Slici 3 je prikazano magnetno kolo, koje je sačinjeno od jezgra relativne magnetne permeabilnosti  $\mu_r = 1000$ , dužine srednje linije  $l = 10 \text{ cm}$  i površine poprečnog preseka  $S = 1 \text{ cm}^2$  i dva namotaja sa  $N_1 = 40$  i  $N_2 = 20$  navojaka, kroz koje protiču struje intenziteta  $I_1 = 1 \text{ A}$  i  $I_2 = 2 \text{ A}$ . Odrediti intenzitet vektora magnetne indukcije u jezgru i međusobnu induktivnost namotaja. ( $\mu_0 = 4\pi \cdot 10^{-7} \text{ H/m}$ ) (20 poena)

4. U kolu na Slici 4, poznate su vrednosti elemenata:  $E = 200 \text{ V}$ ,  $R_1 = 10 \Omega$ ,  $R_2 = 30 \Omega$ ,  $C = 2 \mu\text{F}$ . Prekidač je zatvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku  $t = 0$ , prekidač se otvara.

- a) Odrediti izraze za napon i struju kondenzatora nakon otvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame. (15 poena)  
 b) Odrediti trenutak,  $t_x$ , u kome će vrednost energije električnog polja kondenzatora biti jednaka četvrtini početne vrednosti. (5 poena)



Slika 4

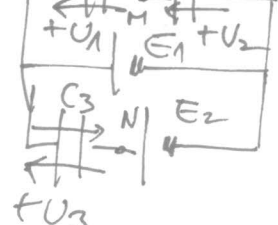


Slika 5

5. Na Slici 5 je prikazano kolo naizmenične struje koje čine naponski generator efektivne vrednosti napona  $U = 100 \text{ V}$  i tri potrošača kompleksnih impedansi:  $\bar{Z}_1 = (1 + j2) \Omega$ ,  $\bar{Z}_2 = (1 - j3) \Omega$  i  $\bar{Z}_3 = (1 + j) \Omega$ .

Odrediti:

- a) kompleksne izraze za struje u svim granama kola; (10 poena)  
 b) reaktivnu snagu potrošača  $Z_2$  i kompleksnu prividnu snagu potrošača  $Z_1$ . (10 poena)

1) 

$$\left. \begin{aligned} U_1 + U_2 &= E_1 \\ Q_1 &= Q_2 \\ Q_1 &= U_1 C_1 \\ Q_2 &= U_2 C_2 \end{aligned} \right\} \Rightarrow \frac{Q_1}{C_1} + \frac{Q_1}{C_2} = E_1$$

$$Q_1 = \frac{E_1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{10}{\frac{1}{20\mu} + \frac{1}{30\mu}} = \frac{10}{\frac{5}{60\mu}} = 120\mu\text{C}$$

$$Q_2 = Q_1 = \frac{600\mu\text{C}}{5} = 120\mu\text{C}$$

$$U_1 = \frac{Q_1}{C_1} = \frac{120\mu\text{C}}{20\mu\text{F}} = 6\text{V}$$

$$W_1 = \frac{1}{2} Q_1 U_1 = \frac{1}{2} \cdot 120\mu \cdot 6 = 360\mu\text{J}$$

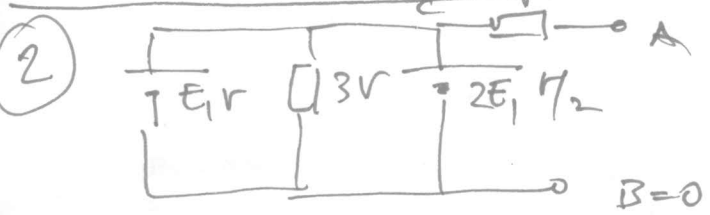
$$U_2 = \frac{Q_2}{C_2} = \frac{120\mu\text{C}}{30\mu\text{F}} = 4\text{V}$$

$$W_2 = \frac{1}{2} Q_2 U_2 = \frac{1}{2} \cdot 120\mu \cdot 4 = 240\mu\text{J}$$

$$E_1 = U_3 + E_2 \Rightarrow U_3 = E_1 - E_2 = 4\text{V} \quad Q_3 = U_3 C_3 = 40\mu\text{C}$$

$$W_3 = \frac{1}{2} Q_3 U_3 = \frac{1}{2} \cdot 40\mu \cdot 4 = 80\mu\text{J}$$

b) 
$$U_{MN} = -E_2 + U_2 = -6 + 4 = -2\text{V} = U_{MN}$$



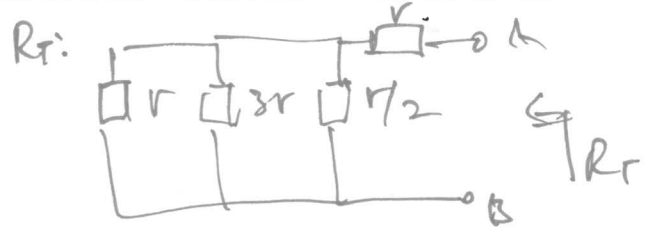
$$U_{co} \left( \frac{1}{r} + \frac{1}{3r} + \frac{1}{r/2} \right) = \frac{E}{r} + \frac{2E}{r/2}$$

$$U_{co} \frac{3+1+6}{3r} = \frac{E}{r} (1+4)$$

$$E_T = U_{AB} = U_{CB} = U_{co} = \frac{3}{2} E = 18\text{V}$$

$$\frac{10}{3} U_{co} = 5E$$

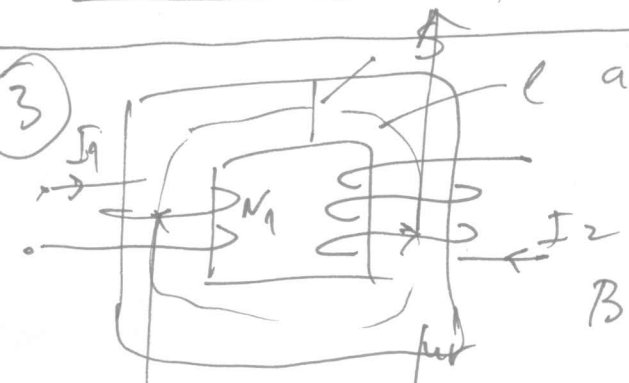
$$U_{co} = \frac{15E}{10} = \frac{3}{2} E$$



$$R_T = r + \frac{1}{\frac{1}{3r} + \frac{1}{r/2}}$$

$$R_T = r + \frac{3+1+6}{3r} = r + \frac{3V}{10}$$

$$R_T = \frac{13}{10} r = 13\Omega$$



a) 
$$\oint \vec{H} \cdot d\vec{l} = \Sigma I$$

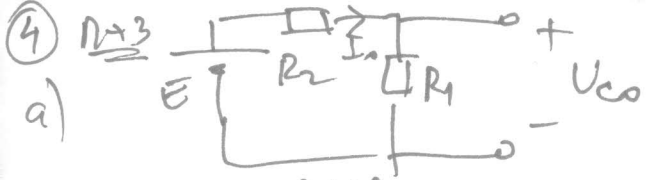
$$H \cdot l = N_1 I_1 + N_2 I_2$$

$$\frac{B}{\mu_0 \mu_r} l = N_1 I_1 + N_2 I_2$$

$$B = \frac{N_1 I_1 + N_2 I_2}{e/\mu_0 \mu_r} = \frac{(40 \cdot 1 + 20 \cdot 2) \cdot 10^3 \cdot 4\pi \cdot 10^{-7}}{10 \cdot 10^{-2}}$$

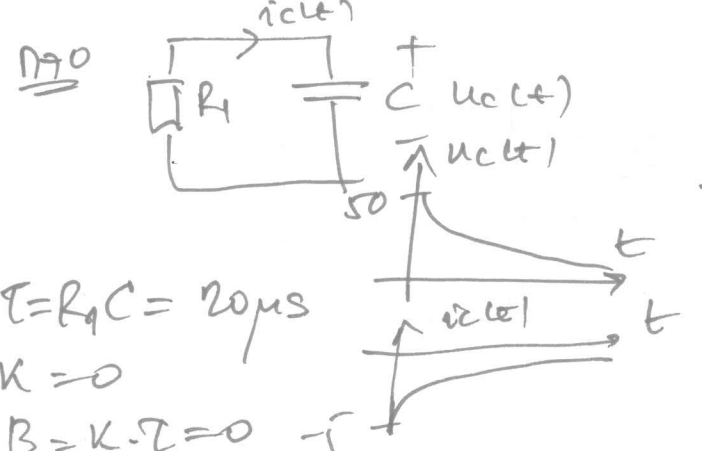
$$= \frac{80 \cdot 4\pi \cdot 10^{-4}}{10^{-1}} = 320\pi \cdot 10^{-3} \text{ T}$$

b) 
$$L_{12} = L_{21} = \frac{N_1 N_2}{R_m} = \frac{N_1 N_2}{e/\mu_0 \mu_r S} = \frac{10^{-1}}{0.32\pi} \text{ mH}$$



$$I_0 = \frac{E}{R_2 + R_1}$$

$$U_{co} = R_1 \cdot I_0 = \frac{R_1}{R_1 + R_2} E = 50V$$



$$R_1 i_c(t) + u_c(t) = 0$$

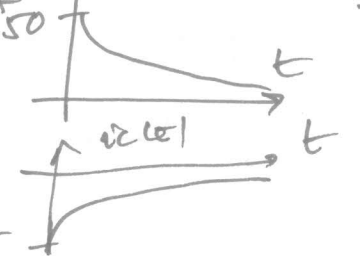
$$i_c(t) = C \frac{du_c(t)}{dt}$$

$\tau = R_1 C = 20 \mu s$

$K = 0$

$B = K \cdot \tau = 0$

$A + B = U_{co} \Rightarrow A = U_{co} = 50V$



$$R_1 C \frac{du_c(t)}{dt} + u_c(t) = 0$$

$$\frac{du_c(t)}{dt} + \frac{u_c(t)}{R_1 C} = 0$$

$$u_c(t) = A e^{-t/\tau} + B = 50 e^{-t/\tau} [V]$$

$$i_c(t) = C \frac{du_c(t)}{dt} = C \cdot \frac{R_1 E}{R_1 + R_2} \left( \frac{-1}{\tau} \right) e^{-t/\tau}$$

$$i_c(t) = -\frac{E}{R_1 + R_2} e^{-t/\tau} = -5 e^{-t/\tau} [A]$$

b)  $W_c(t_x) = \frac{1}{2} C u_c^2(t_x) = \frac{1}{4} W_c(0) = \frac{1}{4} \cdot \frac{1}{2} C U_{co}^2(0)$

$u_c^2(t_x) = \frac{u_c^2(0)}{4} \Rightarrow u_c(t_x) = \frac{u_c(0)}{2}$

$50 e^{-t_x/\tau} = 25 \Rightarrow -\frac{t_x}{\tau} = \ln \frac{1}{2} \Rightarrow t_x = \tau \ln 2 = 13,86 \mu s$

5)  $\bar{z}_{23} = \frac{\bar{z}_2 \cdot \bar{z}_3}{\bar{z}_2 + \bar{z}_3} = \frac{(1-j3)(1+j)}{1-j3+1+j} = \frac{1-j3+j-j^2 3}{2-j2} = \frac{4-j2}{2(1-j)} = \frac{2-j}{1-j} \frac{1+j}{1+j}$

$= \frac{2-j+2j-j^2}{2} = \frac{2+j+1}{2} = \frac{3+j}{2}$

$\bar{I}_1 = \frac{\bar{U}}{\bar{z}_1 + \bar{z}_{23}} = \frac{100}{1+j2 + \frac{3+j}{2}} = \frac{200}{2+j4+3+j} = \frac{200}{5+j5} = \frac{40}{1+j} \frac{1-j}{1-j}$

$= \frac{40(1-j)}{2} = \boxed{20(1-j) = \bar{I}_1}$

$\bar{U}_{23} = \bar{z}_{23} \bar{I}_1 = \frac{3+j}{2} \cdot 20(1-j) = 10(3+j-3j-j^2) = 10(4-2j)$

$= 20(2-j)$

$\bar{I}_2 = \frac{\bar{U}_{23}}{\bar{z}_2} = \frac{20(2-j)}{1-j3} \frac{1+j3}{1+j3} = 20 \frac{2-j+j6-j^2 3}{10} = 2(5+j5) = 10(1+j)$

$\bar{I}_3 = \frac{\bar{U}_{23}}{\bar{z}_3} = \frac{20(2-j)}{1+j} \frac{1-j}{1-j} = \frac{20(2-j-2j+j^2)}{2} = \boxed{10(1-3j) = \bar{I}_3}$

b)  $\bar{S}_2 = \bar{U}_{23} \bar{I}_2^* = \bar{z}_2 I_2^2 \quad Q_2 = \text{Im} \{ \bar{z}_2 \} I_2^2 = -3 \cdot 100 \sqrt{2}^2 = -600 \text{ VAR}$

$\bar{S}_3 = \bar{U}_{23} \bar{I}_3^* = \bar{z}_3 I_3^2 = (1+j2) 100 (\sqrt{1+4})^2 = (1+j2) 800 \text{ VA}$