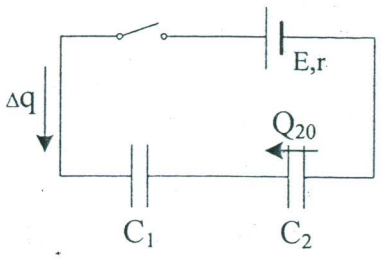
Elektrotehnika

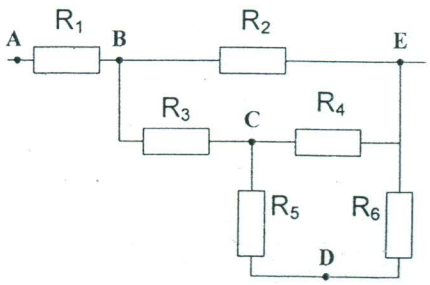
1. septembar 2022.

1. U kolu na Slici 1 poznato je $C_1 = C_2 = 20\,\text{mF}$, $E = 8\,\text{V}$, $r = 1\,\Omega$. Prekidač je otvoren, kondenzator C_1 je neopterećen, dok je kondenzator C_2 opterećen početnom količinom naelektrisanja $Q_{20} = 80\,\text{mC}$. Odrediti količinu naelektrisanja Δq koja će proteći kroz kolo nakon zatvaranja prekidača. Označiti referentne smerove i odrediti napon na kondenzatoru C_2 i količinu naelektrisanja na kondenzatoru C_1 nakon uspostavljanja stacionarnog stanja.
(20 poena)



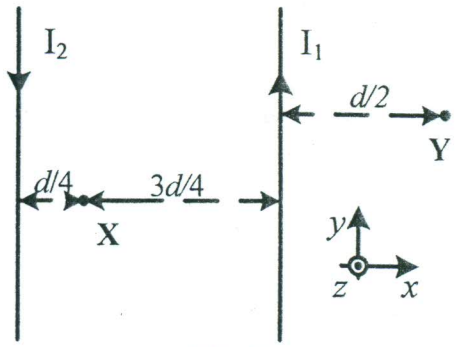
Slika 1

2. Na Slici 2 prikazana je grupa od šest otpornika poznatih otpornosti: $R_1 = R_2 = R_3 = R_5 = R_6 = R = 30\,\Omega$, $R_4 = 2R = 60\,\Omega$. Odrediti ekvivalentnu otpornost između tačaka B i C. **(15 poena)**



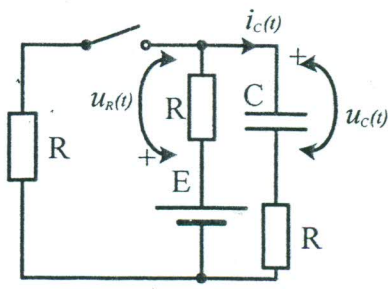
Slika 2

3. Na Slici 3 prikazana su dva pravolinijska, paralelna, veoma dugačka provodnika sa strujama intenziteta $I_1 = 4I$ i $I_2 = 3I$, gde je $I > 0$, koji se nalaze u vazduhu ($\mu = \mu_0$). Rastojanje između provodnika iznosi d.
a) Odrediti i skicirati **vektor** jačine magnetnog polja u tački X. **(10 poena)**
b) Odrediti zapreminsku gustinu energije magnetnog polja u tački Y. **(15 poena)**



Slika 3

4. U kolu na Slici 4 poznate su parametri elemenata E, R i C. Prekidač je otvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se zatvara.
a) Odrediti izraz za napon i struju kondenzatora nakon zatvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame. **(15 poena)**
b) Odrediti vrednost napona u_R u trenutku $t_1 = 6RC$. **(5 poena)**



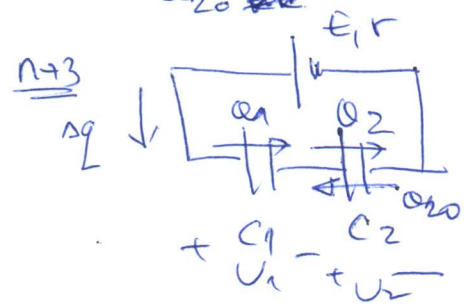
Slika 4

5. Paralelno pretežno kapacitivnom potrošaču aktivne snage $P_1 = 1\,\text{kW}$ i faktora snage $\cos\varphi_1 = 1/\sqrt{2}$, vezan je pretežno induktivni potrošač aktivne snage $P_2 = 3\,\text{kW}$ i faktora snage $\cos\varphi_2 = 0.6$. Ako je napon mreže $U = 100\,\text{V}$, $f = 50\,\text{Hz}$, izračunati:
a) ukupnu aktivnu, reaktivnu i prividnu snagu paralelne veze potrošača; **(12 poena)**
b) efektivnu vrednost struje koju paralelna veza potrošača uzima iz mreže i kompleksnu impedansu celokupnog potrošača. **(8 poena)**

① $n \to 0$ $\quad Q_{10} = 0$

Q_{20}

$n \to \infty$



$$U_1 + U_2 = E$$

$$\frac{Q_1}{C_1} + \frac{Q_2}{C_2} = E$$

$$Q_1 = \Delta q + Q_{10} = \Delta q$$

$$Q_2 = -Q_{20} + \Delta q$$

$$C_1 = C_2 = C$$

$$\frac{\Delta q}{C} + \frac{-Q_{20} + \Delta q}{C} = E$$

$$\frac{2\Delta q}{C} = E + \frac{Q_{20}}{C}$$

$$\boxed{\Delta q = \frac{CE}{2} + \frac{Q_{20}}{2}}$$

$$\Delta q = \frac{20 \cdot 10^{-3} \cdot 8}{2} + \frac{80 \cdot 10^{-3}}{2} = 80\,mC + 40\,mC = 120\,mC$$

$$\boxed{\Delta q = 120\,mC}$$

$$U_2 = \frac{Q_2}{C_2} = \frac{-Q_{20} + \Delta q}{C_2} = \frac{-80 + 120}{20} = \frac{40}{20} = 2V$$

$$Q_4 = \Delta q = 120\,mC$$

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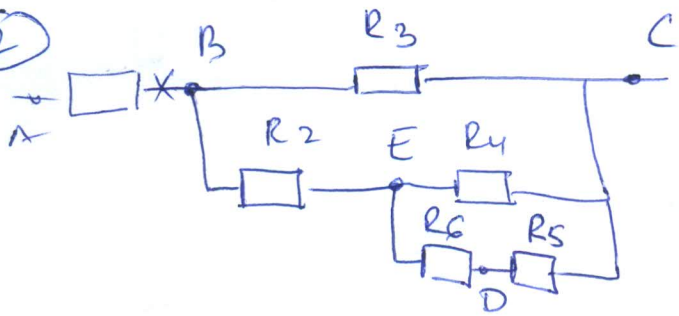
②



$$R_{BC} = R_3 \| (R_2 + R_4 \| (R_5 + R_6))$$

$$R_{BC} = R \| (R + 2R \| (R + R))$$

$$= R \| (R + 2R \| 2R) = R \| (R + R)$$

$$= R \| 2R = \frac{R \cdot 2R}{3R} = \frac{2}{3}R = 20\,\Omega$$

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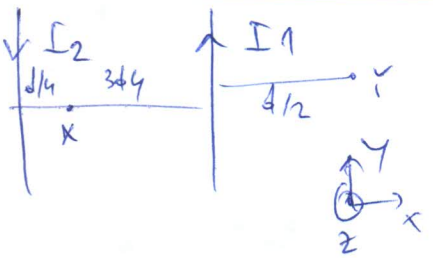
③



a) $\vec{B}_x = \vec{B}_{x1} + \vec{B}_{x2}$

$$= \frac{\mu_0 I_1}{2\pi \frac{3d}{4}} \vec{k} + \frac{\mu_0 I_2}{2\pi \frac{d}{4}} \vec{k} = \frac{4\mu_0 \cdot 4I}{6\pi d} \vec{k} + \frac{4\mu_0 3I}{2\pi d} \vec{k}$$

$$= \frac{24\mu_0 I}{2\pi d} \vec{k} \left(\frac{4}{3} + 3\right) = \frac{2\mu_0 I}{\pi d} \cdot \frac{13}{3} \vec{k}$$

$$\vec{B}_x = \frac{26\mu_0 I}{3\pi d} \vec{k}$$

$$\boxed{\vec{H}_x = \frac{\vec{B}_x}{\mu_0} = \frac{26I}{3\pi d} \vec{k}}$$

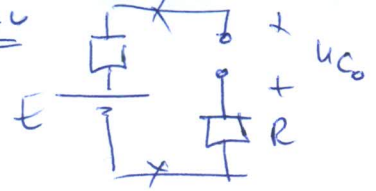
b) $w_y = \frac{1}{2} B_x H_x = \frac{1}{2} \frac{B_x^2}{\mu_0}$

$$\vec{B}_y = \vec{B}_{y1} + \vec{B}_{y2} = \frac{\mu_0 I_1}{2\pi \frac{d}{2}} (-\vec{k}) + \frac{\mu_0 I_2}{2\pi \frac{3d}{2}} \vec{k} = \frac{2 \cdot \mu_0 4I}{2\pi d} \vec{k} + \frac{2\mu_0 3I}{6\pi d} \vec{k}$$

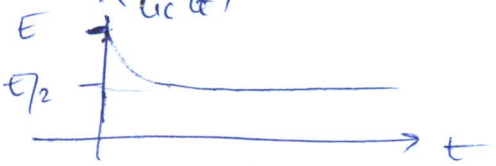
$$= -\frac{4\mu_0 I}{\pi d} \vec{k} + \frac{\mu_0 I}{\pi d} \vec{k} = -\frac{3\mu_0 I}{\pi d} \vec{k}$$

$$w_y = \frac{1}{2} \frac{3^2 \mu_0^2 I^2}{\mu_0 \pi^2 d^2} = \frac{9\mu_0}{2} \left(\frac{I}{\pi d}\right)^2$$

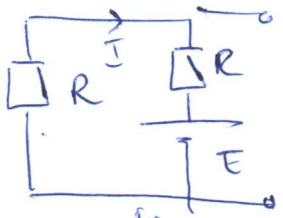
4) $n+2$



$u_{c0} = E$



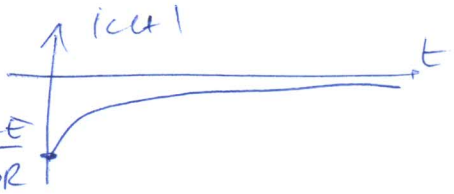
$n+3$



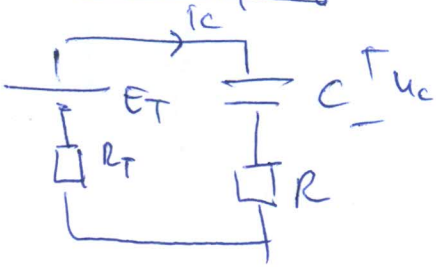
$\Leftrightarrow E_T, R_T$

$$I = \frac{-E}{2R}$$

$$E_T = -RI = \frac{E}{2}$$



$$R_T = R \| R = R/2$$



$$E_T - R_T i_c(t) - R i_c(t) + u_c(t) = 0$$

$$\frac{E}{2} - \frac{3R}{2} i_c(t) + u_c(t) = 0$$

$$i_c(t) = C \frac{du_c(t)}{dt}$$

$$E/2 - 3R/2 \cdot C \frac{du_c(t)}{dt} + u_c(t) = 0$$

$$\frac{du_c(t)}{dt} + \frac{u_c(t)}{3RC/2} = \frac{E}{3RC}$$

$$\tau = \frac{3RC}{2} \qquad K = \frac{E}{3RC}$$

$$B = K \cdot \tau = \frac{3RC}{2} \cdot \frac{E}{3RC} = E/2$$

$$A + B = U_{c0} \Rightarrow A = E - E/2 = E/2$$

$$\boxed{u_c(t) = \frac{E}{2}\left(1 + e^{-t/\tau}\right)}$$

$$i_c(t) = \frac{E}{2} \cdot \frac{-1}{\tau} \cdot C e^{-t/\tau} = \frac{E}{2} \cdot \frac{-2C}{3RC} e^{-t/\tau} = \boxed{-\frac{E}{3R} e^{-t/\tau} = i_c(t)}$$

b) $u_c(t) + R i_c(t) + E + u_R(t) = 0$

$$u_R(t) = -R i_c(t) + E - u_c(t)$$

$$u_R(t) = +\frac{E}{3} e^{-t/\tau} + E - \frac{E}{2} - \frac{E}{2} e^{-t/\tau} = +\frac{E}{2} - \frac{E}{6} e^{-t/\tau}$$

$$u_R(t_1) = +\frac{E}{2} - \frac{E}{6} e^{-\frac{6RC \cdot 2}{3RC}} = +\frac{E}{2} - \frac{E}{6} e^{-4}$$

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5) a) $P_1 = 1\,kW$ $\quad \cos\varphi_1 = 1/\sqrt{2}$ kapacitivan $\quad P_2 = 3\,kW$ $\quad \cos\varphi_2 = 0{,}6$ induktivan $\quad U = 100\,V$ $\quad f = 50\,Hz$

$$P = P_1 + P_2 = 4\,kW$$

$$\sin\varphi_1 = -\sqrt{1 - \cos^2\varphi_1} = -\sqrt{2}/2$$

$$\sin\varphi_2 = +\sqrt{1 - \cos^2\varphi_2} = 0{,}8$$

$$Q_1 = S_1 \sin\varphi_1 = P_1 \operatorname{tg}\varphi_1 = -1\,kVAr$$

$$Q_2 = P_2 \operatorname{tg}\varphi_2 = 3 \cdot \frac{0{,}8}{0{,}6} = 4\,kVAr$$

$$\Big\} \; Q = Q_1 + Q_2 = 3\,kVAr$$

$$S = \sqrt{P^2 + Q^2} = 5\,kVA$$

b) $I = \frac{S}{U} = \frac{5000}{100} = 50\,A$ $\qquad Z = \frac{U}{I} = \frac{100}{50} = 2\,\Omega$

$$\cos\varphi = \frac{P}{S} = \frac{4}{5} = 0{,}8$$

$$\sin\varphi = \frac{Q}{S} = \frac{3}{5} = 0{,}6$$

$$\underline{Z} = Z\cos\varphi + jZ\sin\varphi = (1{,}6 + j1{,}2)\,\Omega$$