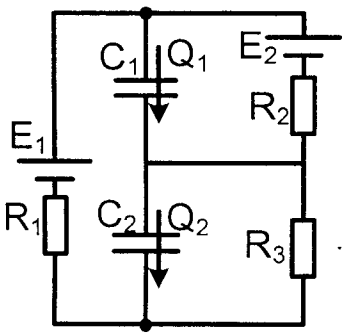
Elektrotehnika

01. februar 2023.

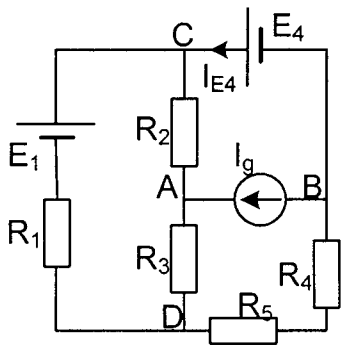
1. U kolu na Slici 1 uspostavljeno je stacionarno stanje. Poznato je $R_1 = R_2 = R_3 = 10\Omega$, $C_1 = C_2 = 10$ nF, $E_1 = 40$V i $E_2 = 10$V.

a) Odrediti količine naelektrisanja kondenzatora C_1 i C_2. **(10 poena)**

b) Odrediti površinu ploča kondenzatora S_1 i S_2 ako polje u svakom od kondenzatora iznosi $K=10^4$ V/m, a dielektrična konstanta je ε=10 nF/m. **(10 poena)**



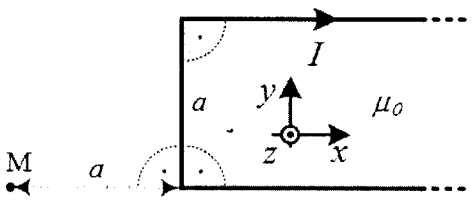
Slika 1



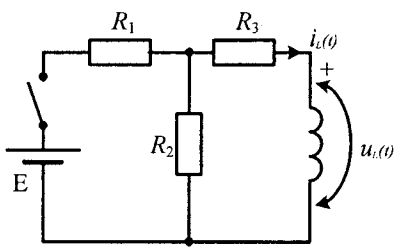
Slika 2

2. U kolu na Slici 2 poznate su vrednosti $R_1 = R_2 = R_3 = 3\Omega$, $R_4 = 1\Omega$, $R_5 = 2\Omega$, $I_g = 1$A, $E_1 = 6$V, $E_4 = 18$V. Primenom Tevenenove teoreme odrediti intenzitet struje kroz naponski generator E_4. **(20 poena)**

3. Na Slici 3 je prikazan izlomljeni pravolinijski provodnik, koji se sastoji od segmenta dužine ***a*** i dva veoma dugačka segmenta. Provodnik se nalazi u vazduhu, a kroz njega protiče struja intenziteta ***I***. Odrediti i skicirati vektor magnetne indukcije u tački M. **(20 poena)**



Slika 3



Slika 4

4. U kolu na Slici 4 poznate su vrednosti elemenata: $E = 12$V, $R_1 = 1\Omega$, $R_2 = R_3 = 2\Omega$ i $L = 4\mu$H. Prekidač je zatvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se otvara.

a) Odrediti izraze za struju i napon kalema nakon otvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame. **(15 poena)**

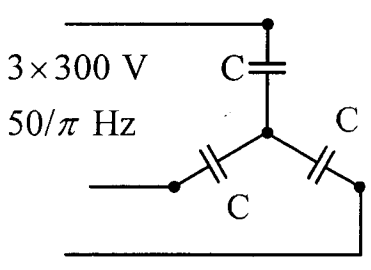
b) Odrediti trenutak t_1 u kome struja kalema opadne na 50% od svoje maksimalne vrednosti **(5 poena)**

5. Na Slici 5 prikazan je simetrični trofazni, čisto kapacitivni potrošač, priključen na trofazni sistem napona 3×300V, učestanosti $f = \frac{50}{\pi}$Hz. Kapacitivnost kondenzatora iznosi $C = 0.1$mF. Odrediti:

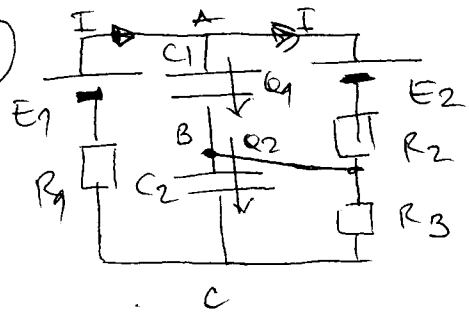
a) efektivnu vrednost linijske struje. **(10 poena)**

b) prividnu i reaktivnu snagu potrošača. **(6 poena)**

c) faktor snage potrošača. **(4 poena)**



Slika 5

① 

a) $I = \frac{E_1 - E_2}{R_1+R_2+R_3} = \frac{40-10}{30} = 1A$

$U_1 = U_{AB} = E_2 + R_2 I = 10+10 = 20V$

$Q_1 = U_1 \cdot C_1 = 200\mu C$

$U_2 = U_{BC} = R_3 \cdot I = 10V$

$Q_2 = U_2 C_2 = 100\mu C$

b) $K_1 = \frac{U_1}{d_1} \Rightarrow d_1 = \frac{U_1}{K_1} = \frac{20}{10^4} = 2\cdot 10^{-3} m$

$K_2 = \frac{U_2}{d_2} \Rightarrow d_2 = \frac{U_2}{K_2} = \frac{10}{10^4} = 10^{-3} m$

$C_1 = \varepsilon \frac{S_1}{d_1} \Rightarrow S_1 = \frac{C_1 \cdot d_1}{\varepsilon} = \frac{10\mu \cdot 2\cdot 10^{-3}}{10\mu} = 2\cdot 10^{-3} m^2 = S_1$

$C_2 = \varepsilon \frac{S_2}{d_2} \Rightarrow S_2 = \frac{C_2 d_2}{\varepsilon} = \frac{10\mu \cdot 10^{-3}}{10\mu} = 1\cdot 10^{-3} m^2 = S_2$

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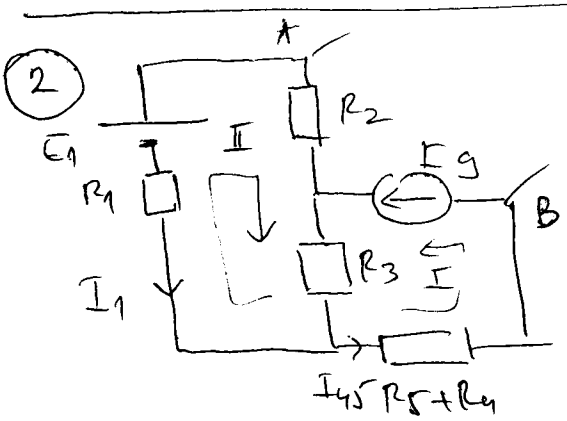
② 

$E_T = U_{AB}^{OV} = E_1 + R_1 I_1 + (R_4+R_5) I_{45}$

$I_I = I_g$

$R_3 I_I + (R_1+R_2+R_3) I_{II} = E_1$

$I_{II} = \frac{E_1 - R_3 I_g}{R_1+R_2+R_3}$ $\quad I_I = I_g$

$R_1 = R_2 = R_3 = R =$

$R_4 + R_5 = R$

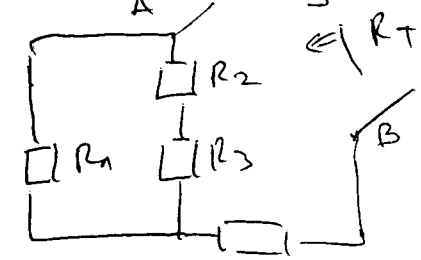
$I_1 = -I_{II} = \frac{R_3 I_g - E_1}{R_1+R_2+R_3}$

$I_{45} = I_I = I_g$

$E_T = 8V$

$E_T = E_1 + R_1 \frac{R_3 I_g - E_1}{R_1+R_2+R_3} + (R_4+R_5) I_g = E_1 + R \cdot \frac{RI_g - E_1}{3R} + RI_g$

$= E_1 + \frac{RI_g}{3} - \frac{E_1}{3} + RI_g = \frac{2}{3}E_1 + \frac{4}{3}RI_g = \frac{2}{3}\cdot 6 + \frac{4}{3}\cdot 3\cdot 1 = 8V$

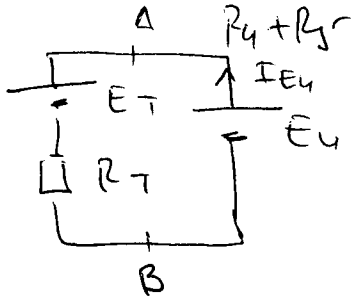


$\Leftarrow R_T$

$R_T = R_4 + R_5 + R_1 \,||\, (R_2+R_3)$

$= R + R \,||\, 2R = R + \frac{R\cdot 2R}{3R} = \frac{5}{3}R = 5\Omega$

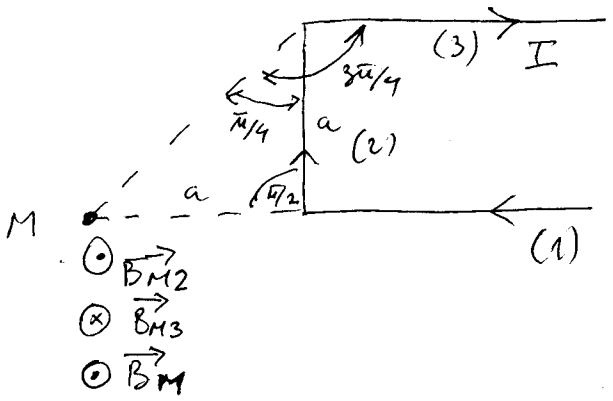
$R_T = 5\Omega$



$I_{E4} = \frac{E_4 - E_T}{R_T} = \frac{18-8}{5} = \frac{10}{5} = 2A$

$I_{E4} = 2A$

(3)



$$\vec{B}_M = \vec{B}_{M1} + \vec{B}_{M2} + \vec{B}_{M3}$$

$$\vec{B}_{M1} = \vec{0}$$

$$\vec{B}_{M2} = \frac{\mu_0 I}{4\pi a}\left(\cos\frac{\pi}{2} + \cos\frac{\pi}{4}\right)\cdot\vec{k}$$

$$\vec{B}_{M2} = \frac{\mu_0 I\sqrt{2}}{8\pi a}\vec{k}$$

$$\vec{B}_{M3} = \frac{\mu_0 I}{4\pi a}\left(\cos\frac{3\pi}{4} + \cos 0\right)(-\vec{k})$$

$$\vec{B}_{M3} = \frac{\mu_0 I(2-\sqrt{2})}{8\pi a}(-\vec{k})$$

$$\vec{B}_M = \vec{0} + \frac{\mu_0 I\sqrt{2}}{8\pi a}\vec{k} - \frac{\mu_0 I(2-\sqrt{2})}{8\pi a}\vec{k}$$

$$\vec{B}_M = \frac{\mu_0 I}{4\pi a}(\sqrt{2}-1)\vec{k}$$

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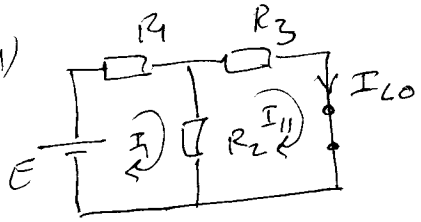
(4) a)



$$(R_1+R_2)I_1 - R_2 I_{II} = E$$

$$-R_2 I_1 + (R_2+R_3)I_{II} = 0$$

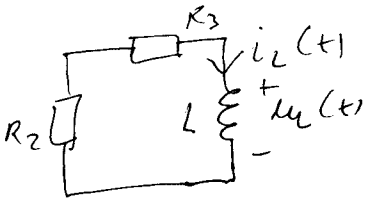
$$3I_1 - 2I_{II} = 12$$

$$-2I_1 + 4I_{II} = 0 \Rightarrow I_1 = 2I_{II}$$

$$\Rightarrow 6I_{II} - 2I_{II} = 12$$

$$I_{II} = 3\,A$$

$$\boxed{I_{L0} = I_{II} = 3\,A}$$



$$(R_2+R_3)i_L + u_L = 0$$

$$(R_2+R_3)i_L + L\frac{di_L}{dt} = 0 \;/:L$$

$$\boxed{\frac{di_L}{dt} + \left(\frac{R_2+R_3}{L}\right)i_L = 0}$$

$$\alpha = \frac{R_2+R_3}{L} = 10^6\,\frac{1}{s}$$

$$\boxed{\tau = \frac{1}{\alpha} = \frac{L}{R_2+R_3} = 1\,\mu s}$$

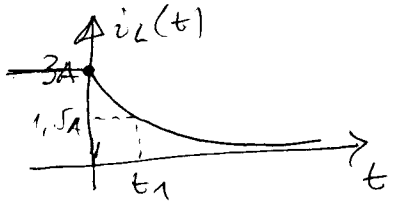
$$i_L(t) = Ae^{-\frac{t}{\tau}} + B$$

$$B = 0$$

$$I_{L0} = A + B \Rightarrow A = I_{L0}$$

$$\Rightarrow \boxed{i_L(t) = I_{L0}e^{-\frac{t}{\tau}}}$$

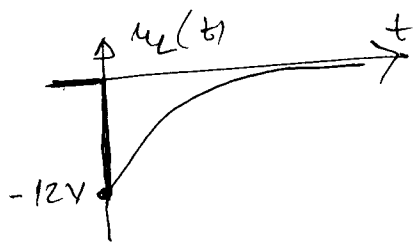
$$\boxed{i_L(t) = 3e^{-\frac{t}{1\mu s}}\,A}$$



$$u_L(t) = L\frac{di_L}{dt} = L\cdot I_{L0}\cdot\left(-\frac{1}{\tau}\right)e^{-t/\tau}$$

$$u_L(t) = -I_{L0}\cancel{L}\cdot\frac{R_2+R_3}{\cancel{L}}e^{-t/\tau} = -(R_2+R_3)I_{L0}e^{-t/\tau}$$

$$\boxed{u_L(t) = -12e^{-\frac{t}{1\mu s}}\,V}$$



b)

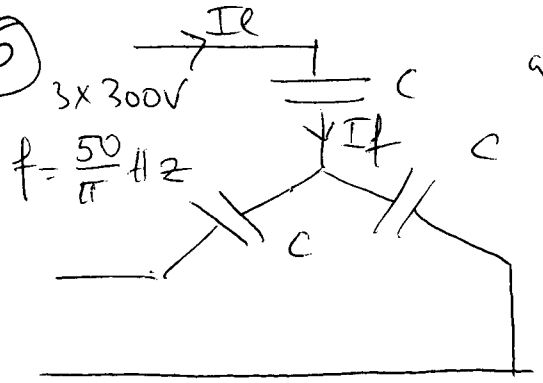
$$i_L(t_1) = 0{,}5\cdot I_{L0}$$

$$i_L(t_1) = I_{L0}e^{-t_1/\tau}$$

$$\Rightarrow e^{-t_1/\tau} = 0{,}5$$

$$t_1 = -\tau\ln(0{,}5) = \tau\ln(2) = 0{,}69\,\mu s$$

(5)



a) $C = 0{,}1\,\mu F$

$$Z_f = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \cdot \frac{50}{\pi} \cdot 0{,}1 \cdot 10^{-3}} = \frac{10^4}{100} = 100\,\Omega$$

$$\boxed{Z_f = 100\,\Omega}$$

$U_l = 300\,V$

$$U_f = \frac{U_l}{\sqrt{3}} = \frac{300}{\sqrt{3}} = 100\sqrt{3}\,V$$

$$\boxed{I_l = I_f = \frac{U_f}{Z_f} = \sqrt{3}\,A}$$

b) $S = 3 U_f I_f = 3 \cdot 100\sqrt{3} \cdot \sqrt{3} = \boxed{900\,VA = S}$

$P = 0 \Rightarrow \boxed{Q = -900\,VAr}$

c) $\boxed{\cos\varphi = \frac{P}{S} = 0}$