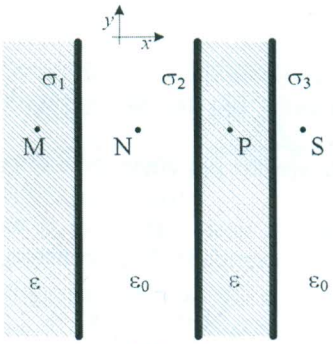
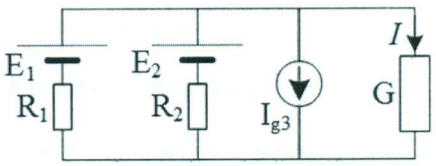
Elektrotehnika

15. jun 2023.

1. Na Slici 1 prikazane su tri paralelne, veoma velike, ravnomerno naelektrisane površi, površinskih gustina naelektisanja $\sigma_1 = \sigma > 0$, $\sigma_2 = -\sigma$ i $\sigma_3 = -\sigma/2$. Odrediti i skicirati vektore električnog polja u tačkama M, N, P i S. Poznato je: $\varepsilon = 4 \cdot \varepsilon_0$. **(20 poena)** – MODIFIKOVAN ZADATAK II-3 IZ ZBIRKE I ZADATAK SA VEZBI



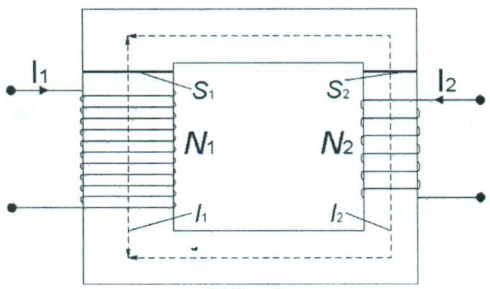
Slika 1



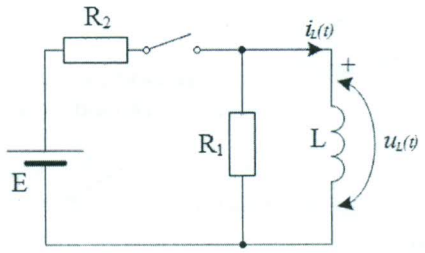
Slika 2

2. U kolu na Slici 2 poznato je: $R_1 = 10\,\Omega$, $R_2 = 5\,\Omega$, $E_1 = 50\,\text{V}$, $E_2 = 20\,\text{V}$, $I_{g3} = 3\,\text{A}$, $G = 0.2\,\text{S}$. Odrediti struju I primenom Tevenenove teoreme. **(20 poena)**

3. Na Slici 3 je prikazano magnetno kolo, koje je sačinjeno od jezgra relativne magnetne permeabilnosti μ_r. Jezgro se sastoji iz dva dela koji imaju površine poprečnih preseka S_1 i S_2 i čije su dužine srednjih linija l_1 i l_2. Na jezgro su namotana dva namotaja sa N_1 i N_2 navojaka, kroz koje protiču struje intenziteta I_1 i I_2. Odrediti magnetni fluks u jezgru, gustinu energije magnetnog polja u delu jezgra dužine l_1 i međusobnu induktivnost namotaja. **(20 poena)** ZADATAK III-13 IZ ZBIRKE + MEĐUSOBNA INDUKTIVNOST



Slika 3



Slika 4

4. U kolu na Slici 4, poznate su vrednosti elemenata: E, $R_1 = R$, $R_2 = 2R$, L. Prekidač je zatvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se otvara.

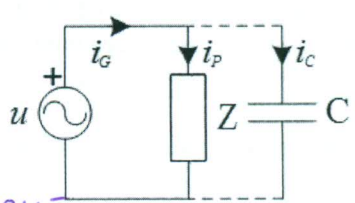
a) Odrediti izraze za struju i napon kalema nakon otvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame. **(15 poena)** ZADATAK BAZIRAN NA IV-4 IZ ZBIRKE

b) Odrediti trenutak t_1 u kome struja kalema dostiže 25% svoje maksimalne vrednosti. **(5 poena)**

5. U kolu naizmenične struje (Slika 5), potrošač kompleksne impedanse $\underline{Z} = (1 + j)\Omega$ priključen je na naponski izvor poznatih parametara $U_m = 200\,\text{V}$, $\omega = 100\,\text{rad/s}$.

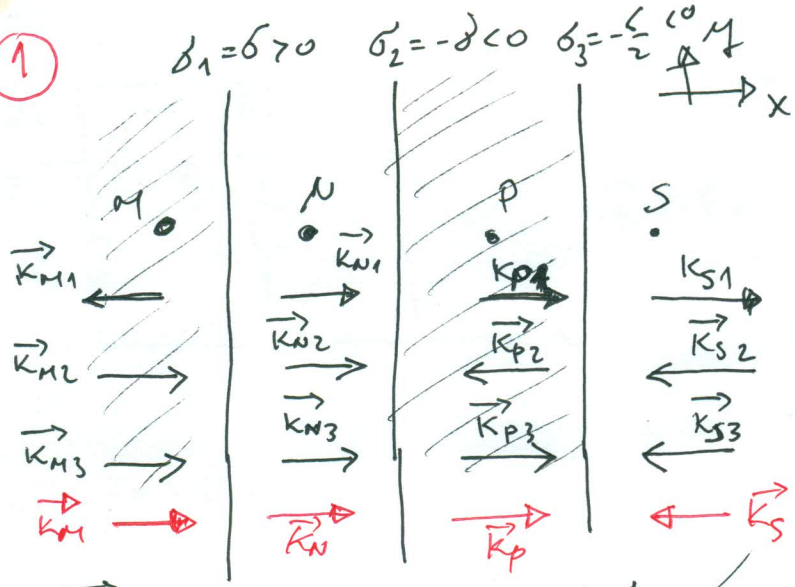
a) Odrediti kapacitivnost kondenzatora koji je potrebno priključiti paralelno potrošaču da bi se faktor snage popravio na vrednost $\cos\varphi = 1$. **(10 poena)** MODIFIKOVAN ZADATAK V-14 IZ ZBIRKE

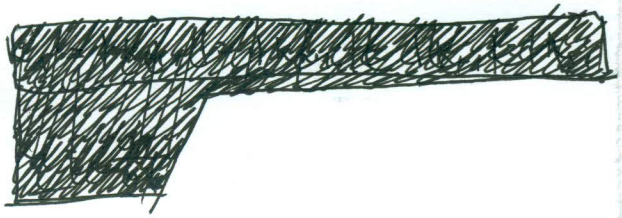
b) Nacrtati fazorski dijagram za sve električne veličine označene na slici, u situaciji nakon priključenja kondenzatora. **(10 poena)**



Slika 5

1





$$\vec{K}_M = \vec{K}_{M1} + \vec{K}_{M2} + \vec{K}_{M3} = \frac{\sigma}{2\varepsilon}(-\vec{\imath}) + \frac{\sigma}{2\varepsilon}\vec{\imath} + \frac{\sigma/2}{2\varepsilon}\vec{\imath} = \frac{\sigma}{4\varepsilon}\vec{\imath}$$

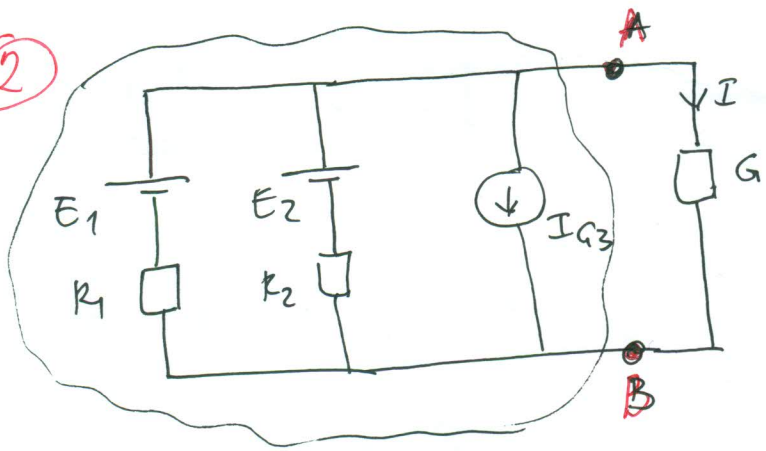
$$\boxed{\vec{K}_M = \frac{\sigma}{16\varepsilon_0}\vec{\imath}}$$

$$\vec{K}_N = \vec{K}_{N1} + \vec{K}_{N2} + \vec{K}_{N3} = \frac{\sigma}{2\varepsilon_0}\vec{\imath} + \frac{\sigma}{2\varepsilon_0}\vec{\imath} + \frac{\sigma}{4\varepsilon_0}\vec{\imath} = \boxed{\frac{5\sigma}{4\varepsilon_0}\vec{\imath} = \vec{K}_N}$$

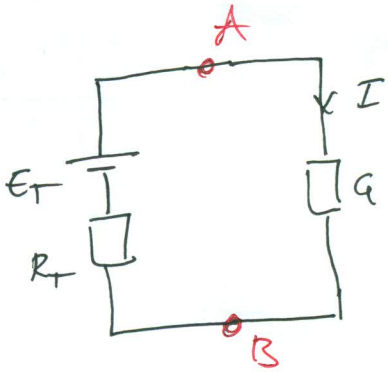
$$\vec{K}_P = \vec{K}_{P1} + \vec{K}_{P2} + \vec{K}_{P3} = \frac{\sigma}{2\varepsilon}\vec{\imath} - \frac{\sigma}{2\varepsilon}\vec{\imath} + \frac{\sigma}{4\varepsilon}\vec{\imath} = \boxed{\frac{\sigma}{16\varepsilon_0}\vec{\imath} = \vec{K}_P}$$

$$\vec{K}_S = \vec{K}_{S1} + \vec{K}_{S2} + \vec{K}_{S3} = \frac{\sigma}{2\varepsilon_0}\vec{\imath} - \frac{\sigma}{2\varepsilon_0}\vec{\imath} - \frac{\sigma}{4\varepsilon_0}\vec{\imath} = \boxed{-\frac{\sigma}{4\varepsilon_0}\vec{\imath} = \vec{K}_S}$$

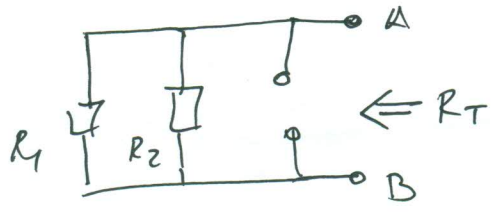
2



E_T, R_T

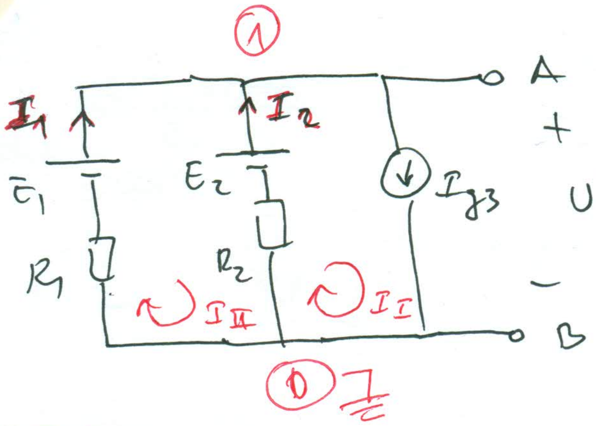


$$I = \frac{E_T}{R_T + \frac{1}{G}}$$



$$R_T = R_1 + R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{10 \cdot 5}{10 + 5} = \frac{10}{3}\ \Omega$$

(1)



$U_{AB}^{ov} = E_T$

I) NAČIN

$$I_1 + I_2 = I_{g3} \Rightarrow I_2 = I_{g3} - I_1$$

$$E_1 - E_2 + R_2 I_2 - R_1 I_1 = 0$$

$$E_1 - E_2 + R_2 (I_{g3} - I_1) - R_1 I_1 = 0$$

$$E_1 - E_2 + R_2 I_{g3} - (R_1 + R_2) I_1 = 0$$

$$I_1 = \frac{E_1 - E_2 + R_2 I_{g3}}{R_1 + R_2} = \frac{45}{15} = 3A$$

$$E_T = U_{AB}^{ov} = E_1 - R_1 I_1 = 20V$$

II) NAČIN

$$I_I = I_{g3}$$

$$-R_2 I_I + (R_1 + R_2) I_{II} = E_1 - E_2$$

$$I_1 = I_{II} = \frac{E_1 - E_2 + R_2 I_{g3}}{R_1 + R_2} = 3A$$

$$E_T = U_{AB}^{ov} = E_1 - R_1 I_1 = 20V$$

III) NAČIN

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) U_{10} = \frac{E_1}{R_1} + \frac{E_2}{R_2} - I_{g3}$$

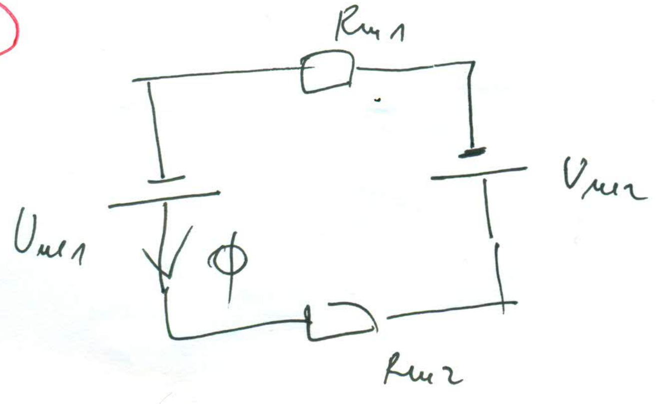
$$E_T = U_{AB}^{ov} = U_{10} = \frac{\frac{E_1}{R_1} + \frac{E_2}{R_2} - I_{g3}}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{5 + 4 - 3}{0,1 + 0,2}$$

$$E_T = \frac{60}{3} = 20V$$

$$I = \frac{E_T}{R_T + \frac{1}{G}} = \frac{20}{\frac{10}{3} + 5} = \frac{60}{10 + 15}$$

$$I = \frac{60}{25} = \frac{12}{5} = 2,4\ A$$

(3)



$$U_{m1} = N_1 I_1$$

$$U_{m2} = N_2 I_2$$

$$R_{m1} = \frac{l_1}{\mu_0 \mu_r S_1} \qquad R_{m2} = \frac{l_2}{\mu_0 \mu_r S_2}$$

$$\Phi = \frac{U_{m1} - U_{m2}}{R_{m1} + R_{m2}} = \frac{N_1 I_1 - N_2 I_2}{\frac{l_1}{\mu_0 \mu_r S_1} + \frac{l_2}{\mu_0 \mu_r S_2}}$$

$$W_{m1} = \frac{1}{2} B_1 H_1 = \frac{1}{2} B_1 \frac{B_1}{\mu_0 \mu_r} = \frac{B_1^2}{2 \mu_0 \mu_r}$$

$$w_{m1} = \frac{(\Phi / S_1)^2}{2 \mu_0 \mu_r} = \frac{\Phi^2}{2 \mu_0 \mu_r S_1^2} = \frac{(N_1 I_1 - N_2 I_2)^2}{2 \frac{\mu_0 \mu_r S_1^2}{(\mu_0 \mu_r)^2} \left(\frac{l_1}{S_1} + \frac{l_2}{S_2}\right)^2}$$

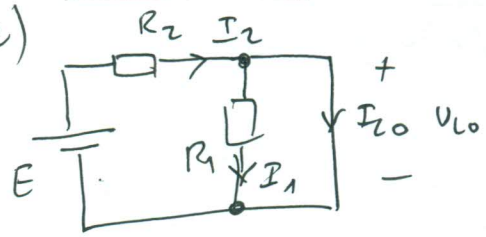
$$w_{m1} = \frac{\mu_0 \mu_r (N_1 I_1 - N_2 I_2)^2}{2 \left(l_1 + \frac{l_2 S_1}{S_2}\right)^2}$$

$$L_{12} = L_{21} = -\frac{N_1 N_2}{R_{m1} + R_{m2}} = \frac{-\mu_0 \mu_r N_1 N_2}{\frac{l_1}{S_1} + \frac{l_2}{S_2}}$$

4

STAC. STANJE

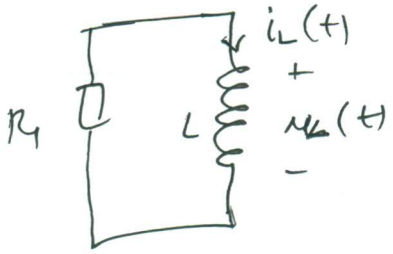
a)



$U_{L0} = 0 \Rightarrow I_1 = \frac{U_{L0}}{R_1} = 0 \Rightarrow I_2 = I_{L0}$

$$\boxed{I_{L0} = \frac{E}{R_2} = \frac{E}{2R}}$$

PREL. PROCES



$R_1 i_L(t) + u_L(t) = 0$

$u_L(t) = L \frac{di_L(t)}{dt}$

$R_1 i_L + L \frac{di_L}{dt} = 0 \Rightarrow$

$$\boxed{\frac{di_L}{dt} + \frac{R_1}{L} i_L = 0}$$

$\alpha = \frac{R_1}{L} = \frac{R}{L} \quad k = 0$

$\tau = \frac{1}{\alpha} = \frac{L}{R_1} = \frac{L}{R}$

$i_L(t) = A e^{-t/\tau} + B$

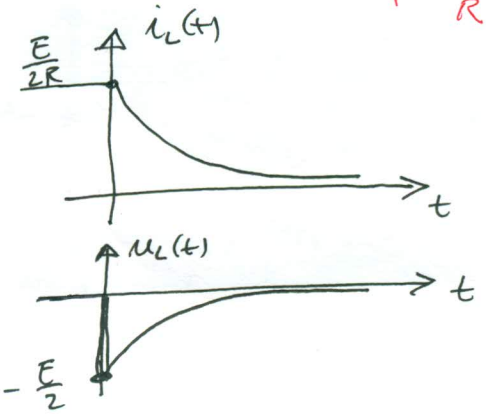
$B = k\tau = 0$

$A = i_L(0) - B = I_{L0} = \frac{E}{2R}$

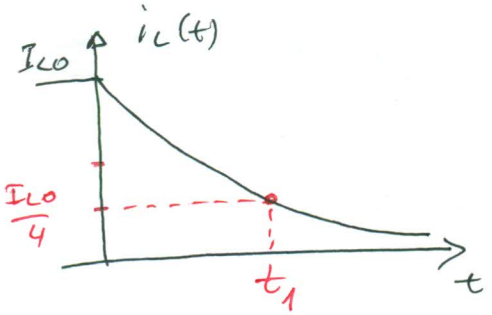
$$\boxed{i_L(t) = \frac{E}{2R} e^{-\frac{R}{L} t}}$$

$u_L(t) = L \frac{di_L}{dt} = L \frac{E}{2R} \left(-\frac{R}{L}\right) e^{-\frac{R}{L} t}$

$$\boxed{u_L(t) = -\frac{E}{2} e^{-\frac{R}{L} t}}$$



b)



$i_L(t_1) = I_{L0} e^{-\frac{R}{L} t_1} = 25\% \, I_{L0}$

$e^{-\frac{R}{L} t_1} = \frac{1}{4} \quad / \ln(\cdot)$

$-\frac{R}{L} t_1 = \ln \frac{1}{4}$

$$\boxed{t_1 = \frac{L}{R} \ln 4}$$

5

$\bar{Z} = 1 + j\ \Omega$

$U_m = 200\ V$

$\omega = 100\ rad/s$

a) $\bar{Y} = \frac{1}{\bar{Z}} = \frac{1}{1+j} \cdot \frac{1-j}{1-j} = \frac{1}{2} - j\frac{1}{2}\ S$

$\bar{Y}_C = j\omega C$

$\bar{Y}_{ekv} = \bar{Y} + \bar{Y}_C = \frac{1}{2} + j(\omega C - 1/2)$

$\bar{Y}_{ekv} = Y_{ekv} e^{-j\varphi} = Y_{ekv}(\underbrace{\cos\varphi}_{=1} - j\underbrace{\sin\varphi}_{=0})$

$\Rightarrow \omega C = \frac{1}{2}$

b) $\bar{U} = U e^{j\theta}$, $U = \frac{U_m}{\sqrt{2}}$, $\theta = 0$

$\boxed{\bar{U} = 100\sqrt{2}\ V}$

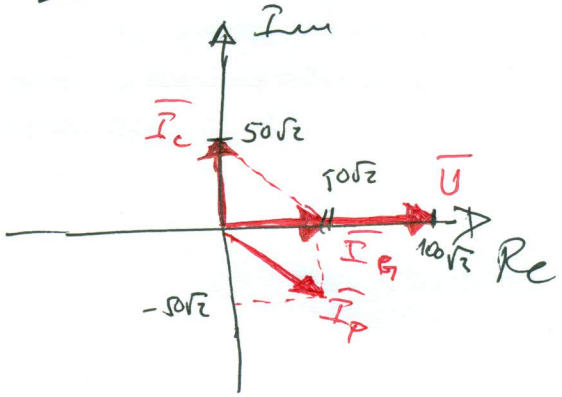
$\bar{I}_G = \frac{\bar{U}}{\bar{Z}_{ekv}} = \bar{U}\bar{Y}_{ekv} = \frac{100\sqrt{2}}{2}$

$\boxed{\bar{I}_G = 50\sqrt{2}\ A}$

$\Rightarrow \boxed{C = \frac{1}{2\omega} = \frac{1}{200} = 5\,mF}$

$\bar{I}_p = \frac{\bar{U}}{\bar{Z}} = \bar{U}\bar{Y} = 100\sqrt{2}\left(\frac{1}{2} - j\frac{1}{2}\right) = \boxed{50\sqrt{2} - j50\sqrt{2} = \bar{I}_p}$

$\bar{I}_C = \frac{\bar{U}}{\bar{Z}_C} = \bar{U}\bar{Y}_C = \bar{U} j\omega C = j\frac{100\sqrt{2}}{2} = \boxed{j50\sqrt{2} = \bar{I}_C}$



provera:

$\bar{I}_G = \bar{I}_p + \bar{I}_C$

$50\sqrt{2} = 50\sqrt{2} - j50\sqrt{2} + j50\sqrt{2}$

OK

a) II NAČIN (SNAGE)

$\bar{S} = \frac{|\bar{U}|^2}{\bar{Z}^*} = |\bar{U}|^2 \bar{Y}^* = (100\sqrt{2})^2\left(\frac{1}{2} + j\frac{1}{2}\right) = (10k + j10k)\ VA \Rightarrow P = 10\,kW,\ Q = 10\,kVAR$

$\bar{S}_{ekv} = P_{ekv} + jQ_{ekv}$ $\quad P_{ekv} = P = 10\,kW$

$\cos\varphi = 1 \Rightarrow Q_{ekv} = 0 \Rightarrow Q_{ekv} = Q + Q_C = 0$

$Q_C = -Q$

$Q_C = -U^2\omega C = -Q$

$C = \frac{Q}{\omega U^2} = \frac{\frac{1}{2}U^2}{\omega U^2} = \frac{1}{2\omega} = \frac{1}{200} = 5\,mF$