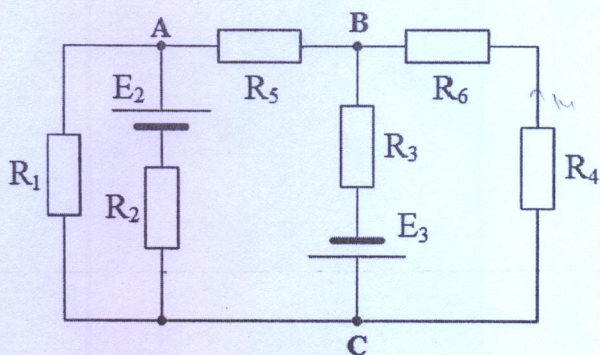


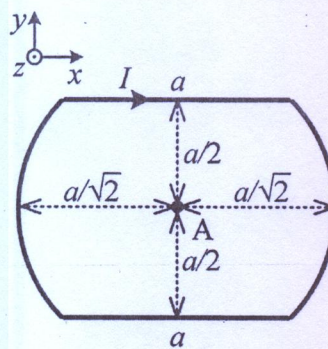
# Elektrotehnika

3. jul 2023.

1. Poznate su sve otpornosti i ems u kolu prikazanom na Slici 1:  $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 5\Omega$ ,  $E_2 = 20\text{ V}$ ,  $E_3 = 40\text{ V}$ . Rešiti kolo primenom metode napona između čvorova i odrediti struju kroz otpornik  $R_4$ . (20 poena)



Slika 1



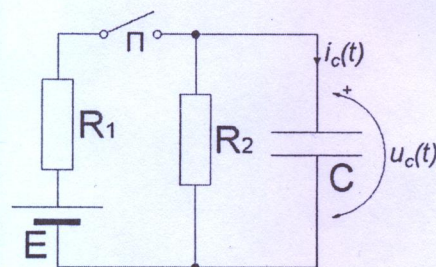
Slika 2

2. Kroz konturu u ravni koja se sastoji iz dva dela kružnice poluprečnika  $a/\sqrt{2}$  i dva pravolinijska provodnika dužine  $a$ , protiče struja intenziteta  $I$  u smeru kao na slici 2. Odrediti i skicirati vektor magnetne indukcije u tački A. Kontura se nalazi u vazduhu. (20 poena)

3. U kolu na Slici 3 poznate su vrednosti elemenata:  $E$ ,  $R_1=R$ ,  $R_2=2R$  i  $C$ . Prekidač  $\Pi$  je zatvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku  $t=0$ , prekidač se otvara.

a) Odrediti izraz za napon i struju kondenzatora nakon otvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame. (15 poena)

b) Odrediti trenutak  $t_1$  u kom je napon kondenzatora jednak trećini maksimalne vrednosti. (5 poena)



Slika 3

4. Kroz potrošač nepoznate impedanse  $\bar{Z}$  protiče naizmjenična struja trenutne vrednosti  $i(t) = 20\sin(\omega t + \pi/2)\text{ A}$ , gde je  $\omega = 200\text{ rad/s}$ . Kompleksna vrednost napona na potrošaču iznosi  $\bar{U} = 100 + j100\sqrt{3}\text{ V}$ .

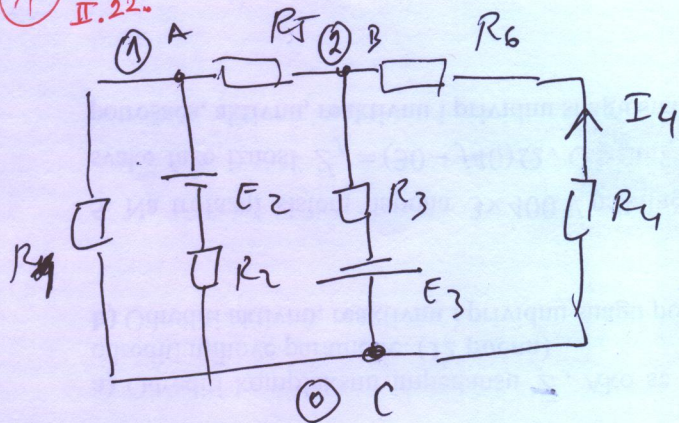
a) Odrediti kompleksnu impedansu  $\bar{Z}$ . Ako se potrošač može predstaviti rednom vezom dva elementa, odrediti njihove parametre. (12 poena)

b) Odrediti aktivnu, reaktivnu i prividnu snagu potrošača. (8 poena)

5. Na trofazni sistem napona  $3 \times 400\text{ V}$  priključen je trofazni potrošač, povezan u trougao. Impedansa svake faze iznosi  $\bar{Z}_f = (30 - j40)\Omega$ . Odrediti efektivnu vrednost faznih i linijskih struja, faktor snage potrošača, aktivnu, reaktivnu i prividnu snagu potrošača. (20 poena)

# Rešenja

1) VIŠETI ZADATAK  
II.22.



$$U_{AC} = U_{10}, \quad U_{BC} = U_{20}$$

$$\textcircled{1}: G_{11} U_{10} - G_{12} U_{20} = \frac{E_2}{R_2}$$

$$\textcircled{2}: -G_{21} U_{10} + G_{22} U_{20} = -\frac{E_3}{R_3}$$

$$G_{11} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5} = \frac{3}{R} = \frac{3}{5} \text{ S}$$

$$G_{12} = G_{21} = \frac{1}{R_5} = \frac{1}{R} = \frac{1}{5} \text{ S}$$

$$G_{22} = \frac{1}{R_5} + \frac{1}{R_3} + \frac{1}{R_6 + R_4} = \frac{2}{R} + \frac{1}{2R} = \frac{5}{2R} = \frac{1}{2} \text{ S}$$

$$\textcircled{1}: \frac{3}{5} U_{10} - \frac{1}{5} U_{20} = \frac{20}{5} = 4$$

$$\textcircled{2}: -\frac{1}{5} U_{10} + \frac{1}{2} U_{20} = -\frac{40}{5} = -8$$

$$\textcircled{1}: 3U_{10} - U_{20} = 20$$

$$\textcircled{2}: -2U_{10} + 5U_{20} = -80$$

$$U_{20} = 3U_{10} - 20$$

$$-2U_{10} + 5(3U_{10} - 20) = -80$$

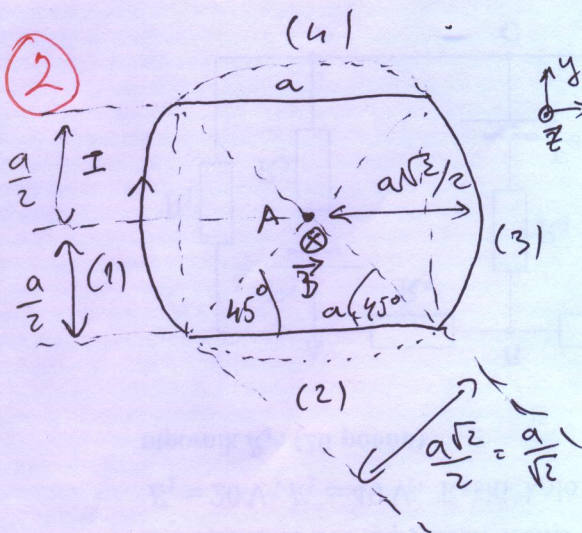
$$13U_{10} = 20 \Rightarrow U_{10} = \frac{20}{13} \text{ V}$$

$$U_{20} = 3 \cdot \frac{20}{13} - 20 = -\frac{200}{13} \text{ V}$$

$$U_{20} = -(R_4 + R_6) I_4 \Rightarrow$$

$$I_4 = \frac{-U_{20}}{R_4 + R_6} = \frac{20 \cdot 13}{10} = \frac{20}{13} \text{ A}$$

2)



$$\vec{B}_A = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 = (2B_2 + 2B_3) \cdot (-\vec{k})$$

$$B_2 = B_4 = \frac{\mu_0 I}{4\pi \left(\frac{a}{2}\right)} \left( \cos \frac{\pi}{4} + \cos \frac{\pi}{4} \right)$$

$$B_2 = B_4 = \frac{\mu_0 I}{2\pi a} \left( \frac{2\sqrt{2}}{2} \right) = \frac{\mu_0 I \sqrt{2}}{2\pi a}$$

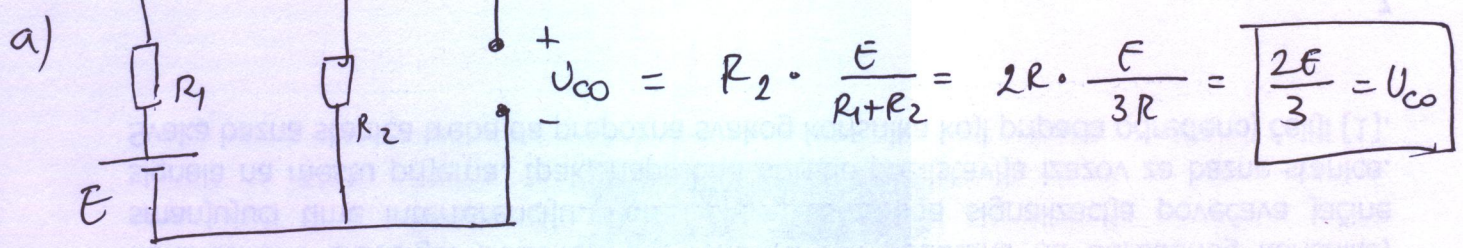
$$B_3 = B_1 = \frac{\mu_0 I}{2(a\sqrt{2})} \cdot \frac{1}{4} = \frac{\mu_0 I \sqrt{2}}{8a}$$

(ČETVRTINA KRUŽNICE)

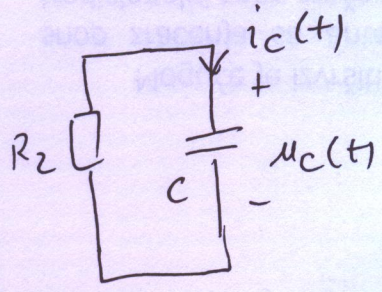
$$\vec{B}_A = -2 \left( \frac{\mu_0 I \sqrt{2}}{2\pi a} + \frac{\mu_0 I \sqrt{2}}{8a} \right) \vec{k}$$

$$\vec{B}_A = -\frac{\mu_0 I \sqrt{2}}{a} \left( \frac{1}{\pi} + \frac{1}{4} \right) \vec{k}$$

3) VIŠE TI ZADATAK 1V.5.



$$U_{co} = R_2 \cdot \frac{E}{R_1 + R_2} = 2R \cdot \frac{E}{3R} = \boxed{\frac{2E}{3} = U_{co}}$$



$$u_C(t) + R_2 i_C(t) = 0$$

$$u_C(t) + 2R \cdot C \frac{du_C}{dt} = 0 \Rightarrow \boxed{\frac{du_C(t)}{dt} + \frac{u_C(t)}{2RC} = 0}$$

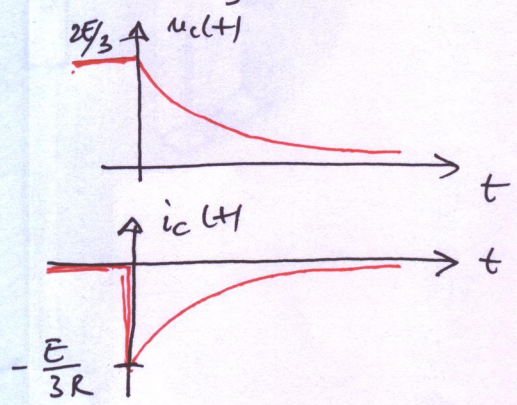
$$\boxed{\tau = 2RC}$$

$$u_C(t) = A e^{-t/\tau} + B, \quad B = 0, \quad A = u_C(0) - B = U_{co} = \frac{2E}{3}$$

$$\boxed{u_C(t) = \frac{2E}{3} e^{-\frac{t}{2RC}} [V]}$$

$$i_C(t) = C \frac{du_C}{dt} = C \frac{2E}{3} \cdot \left(-\frac{1}{2RC}\right) e^{-\frac{t}{2RC}}$$

$$\boxed{i_C(t) = -\frac{E}{3R} e^{-\frac{t}{2RC}} [A]}$$



b)

$$u_C(t_1) = \frac{2E}{3} e^{-\frac{t_1}{2RC}} = \frac{1}{3} \cdot \underbrace{\left(\frac{2E}{3}\right)}_{U_{cmax}}$$

$$e^{-\frac{t_1}{2RC}} = \frac{1}{3} \quad | \ln(\cdot)$$

$$-\frac{t_1}{2RC} = \ln \frac{1}{3} \Rightarrow \boxed{t_1 = 2RC \ln(3)}$$

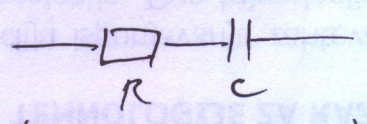
4)

$$i(t) = 20 \sin(\omega t + \pi/2) A \Rightarrow \bar{I} = \frac{20}{\sqrt{2}} e^{j\pi/2} = j10\sqrt{2} A$$

a)

$$\bar{U} = 100 + j100\sqrt{3} V$$

$$\Rightarrow \bar{Z} = \frac{\bar{U}}{\bar{I}} = \frac{100 + j100\sqrt{3}}{j10\sqrt{2}} \cdot \frac{(-j\sqrt{2})}{(-j\sqrt{2})} = \frac{100(-j\sqrt{2} + \sqrt{6})}{2 \cdot 10} = \boxed{5\sqrt{6} - j5\sqrt{2} \Omega}$$



$$5\sqrt{6} - j5\sqrt{2} = R - j \frac{1}{\omega C}$$

$$\boxed{R = 5\sqrt{6} \Omega}$$

$$5\sqrt{2} = \frac{1}{200C}$$

$$\boxed{C = \frac{1}{1000\sqrt{2}} \approx 0,71 \mu F}$$

$$\bar{Z} = R + j \frac{1}{\omega C}$$

$$b) \bar{S} = \bar{U} \bar{I}^* = (100 + j1000\sqrt{3}) \cdot (-j10\sqrt{2}) = 1000\sqrt{6} - j1000\sqrt{2} \text{ VA}$$

$$P = 1000\sqrt{6} \text{ W} = \sqrt{6} \text{ KW}$$

$$Q = -1000\sqrt{2} \text{ VAR} = -\sqrt{2} \text{ KYAR}$$

$$S = \sqrt{P^2 + Q^2} = \sqrt{(\sqrt{6})^2 + (\sqrt{2})^2} \text{ KVA} = \sqrt{8} \text{ KVA} = 2\sqrt{2} \text{ KVA}$$

5) Видеть задание V.25.

$$3 \times 400 \text{ V} \Rightarrow U_L = 400 \text{ V}, \text{ трос } \Rightarrow U_F = U_L = 400 \text{ V}$$

$$Z_F = \sqrt{30^2 + (-40)^2} = 50 \Omega$$

$$I_L = \sqrt{3} I_F$$

$$I_F = \frac{U_F}{Z_F} = \frac{400 \text{ V}}{50 \Omega} = 8 \text{ A} \Rightarrow I_L = 8\sqrt{3} \text{ A}$$

$$\cos \varphi = \frac{R_F}{Z_F} = \frac{30}{50} = \frac{3}{5} = 0,6$$

$$P = 3 U_F I_F \cos \varphi = 3 R_F I_F^2 = 3 \cdot 8^2 \cdot 30 = 5760 \text{ W} = P$$

$$Q = 3 U_F I_F \sin \varphi = 3 X_F I_F^2 = 3 \cdot 8^2 \cdot (-40) = -7680 \text{ VAR} = Q$$

$$S = 3 U_F I_F = 3 \cdot 400 \cdot 8 = 9600 \text{ VA} = S$$