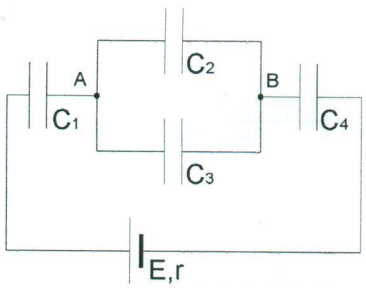
# Elektrotehnika

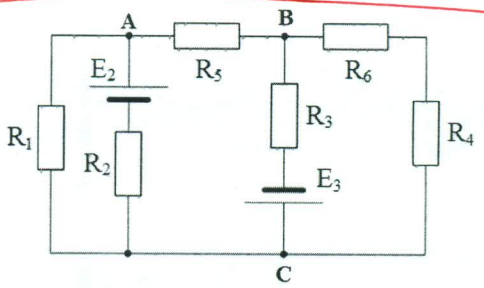
**31. avgust 2023.**

**1.** U kolu na Slici 1 odrediti napon $U_{AB}$ i elektrostatičku energiju kondenzatora $C_1$ i $C_2$. Parametri elemenata su: $E = 30V$, $C_1 = 20nF$, $C_2 = 10nF$, $C_3 = 10nF$, $C_4 = 20nF$. Pre povezivanja u kolo, kondenzatori su bili neopterećeni. **(20 poena)**

REŠENJE ZAD I.5. IZ ZBIRKE



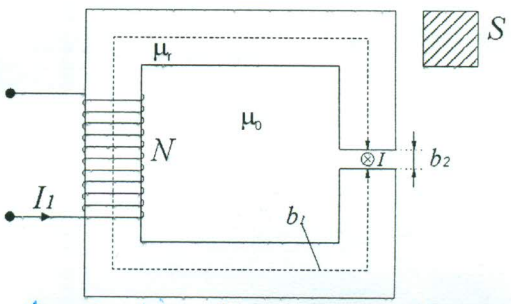
Slika 1



Slika 2

**2.** Poznate su sve otpornosti i ems u kolu prikazanom na Slici 2: $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 5\Omega$, $E_2 = 20\,\mathrm{V}$, $E_3 = 40\,\mathrm{V}$. Rešiti kolo primenom metode napona između čvorova, a nakon toga odrediti struju kroz otpornik $R_5$ i snagu generatora $E_3$. **(20 poena)**

**3.** Na pravougaonom jezgru od feromagnetnog materijala, relativne magnetne permeabilnosti $\mu_r = 1000$, sa vazdušnim procepom širine $b_2 = 2mm$, ravnomerno je namotano $N = 20000$ navojaka, tako da nema magnetnog rasipanja (Slika 3). Dužina srednje linije magnetnog jezgra je $b_1 = 20cm$, a površina kvadratnog poprečnog preseka jezgra iznosi $S = 9cm^2$. Mehanička sila koja deluje na tanak pravolinijski provodnik, postavljen paralelno gornjoj i donjoj ivici proreza, kroz koji protiče struja intenziteta $I = 1A$, iznosi $F = 3 \cdot 10^{-2} N$. Izračunati struju $I_1$ u navojcima. **(20 poena)**



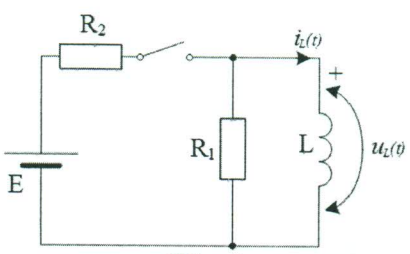
Slika 3

REŠENJE ZADATAK III.12 IZ ZBIRKE

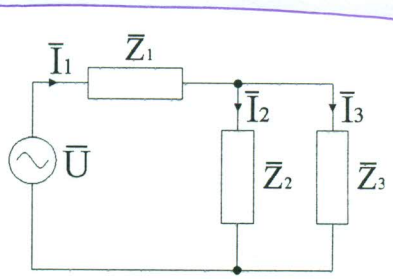
**4.** U kolu na Slici 4, poznate su vrednosti elemenata: $E$, $R_1 = R$, $R_2 = 2R$, $L$. Prekidač $\Pi$ je otvoren i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se zatvara.

a) Odrediti izraze za struju i napon kalema nakon zatvaranja prekidača i nacrtati odgovarajuće vremenske dijagrame. **(15 poena)**

b) Odrediti trenutak $t_1$ u kome energija kalema dostiže 25% svoje maksimalne vrednosti. **(5 poena)**



Slika 4



Slika 5

**5.** Na Slici 5 je prikazano kolo naizmenične struje koje čine naponski generator efektivne vrednosti napona $U = 50V$ i tri potrošača impedansi $Z_1 = (1-2j)\Omega$, $Z_2 = (1-j)\Omega$ i $Z_3 = (1+3j)\Omega$. Odrediti:

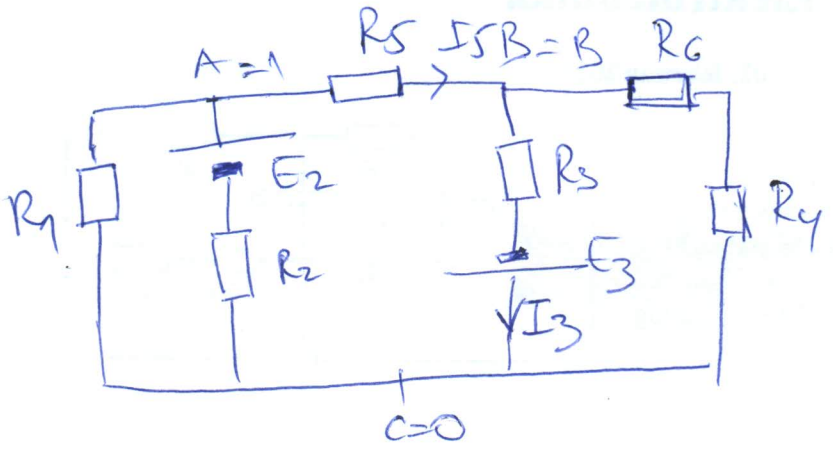
a) kompleksne izraze za struje u svim granama kola; **(8 poena)**

b) efektivnu vrednost napona na potrošaču $Z_1$; **(6 poena)**

c) aktivnu snagu potrošača $Z_2$ i kompleksnu prividnu snagu potrošača $Z_3$. **(8 poena)**

REŠENJE ZADATAK V.6 IZ ZBIRKE

(2)




$E_2 = 20\,V$

$E_3 = 40\,V$

$R_1 = R_2 = R_5 = R_3 = R_4 = R_6 = 5\,\Omega$

1: $U_{10}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_5}\right) - U_{20}\frac{1}{R_5} = \frac{E_2}{R_2}$

2: $-U_{10}\frac{1}{R_5} + \left(\frac{1}{R_5}+\frac{1}{R_3}+\frac{1}{R_4+R_6}\right)U_{20} = \frac{-E_3}{R_3}$

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$\frac{3U_{10}}{R} - \frac{U_{20}}{R} = \frac{E_2}{R}$

$-\frac{U_{10}}{R} + U_{20}\left(\frac{1}{R}+\frac{1}{R}+\frac{1}{2R}\right) = \frac{-E_3}{R}$

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$3U_{10} - U_{20} = E_2 \Rightarrow U_{20} = 3U_{10} - E_2$

$-U_{10} + \frac{5}{2}U_{20} = -E_3$

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$U_{20} = 3U_{10} - E_2$

$-U_{10} + \frac{15}{2}U_{10} - \frac{5}{2}E_2 = -E_3 \Rightarrow \frac{13}{2}U_{10} = \frac{5}{2}E_2 - E_3$

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$U_{10} = \frac{2}{13}\left(\frac{5}{2}E_2 - E_3\right) = \frac{2}{13}\left(\frac{5}{2}\cdot 20 - 40\right) = \frac{2}{13}(50-40) = \frac{20}{13}\,V$

$U_{20} = 3U_{10} - E_2 = \frac{60}{13} - 20 = \frac{60}{13} - \frac{260}{13} = -\frac{200}{13}\,V$

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$U_{AB} = U_{12} = U_{10} - U_{20} = \frac{20}{13} + \frac{200}{13} = \frac{220}{13}\,V$

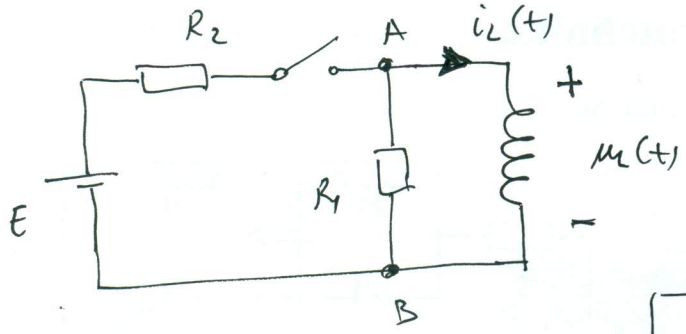
$I_5 = \frac{U_{AB}}{R_5} = \frac{220}{13\cdot 5} = \frac{440}{130} = \frac{44}{13}\,A$

$U_{20} = R_3 I_3 - E_3 \Rightarrow I_3 = \frac{U_{20}+E_3}{R_3} = \frac{-\frac{200}{13}+40}{5} = -\frac{40}{13}+8 = \frac{64}{13}\,A$

$P_{E_3} = E_3 I_3 = \frac{64}{13}\cdot 40 = \frac{2560}{13}\,W$

(4)

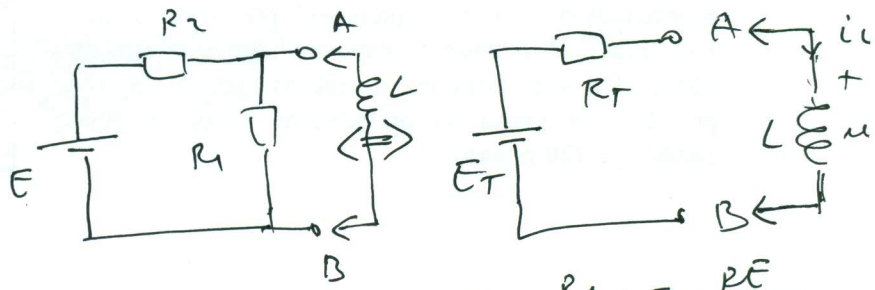
a)




STAC. STAMIE:

$$i_L(0) = I_{L0} = 0\,A$$

PREC. PROCES:




$$E_T = \frac{R_1}{R_1+R_2}E = \frac{RE}{3R}$$

$$E_T = E/3$$

$$R_T = R_1 \| R_2 = \frac{R\cdot 2R}{3R}$$

$$R_T = \frac{2}{3}R$$

$$-E_T + R_T i_L + u_L = 0$$

$$R_T i_L + L\frac{di_L}{dt} = E_T$$

$$\frac{di_L}{dt} + \frac{R_T}{L} i_L = \frac{E_T}{L}$$

$$\boxed{\frac{di_L}{dt} + \frac{2R}{3L} i_L = \frac{E}{3L}} \qquad \frac{1}{\tau} = a \qquad K$$

$$i_L(t) = Ae^{-t/\tau} + B$$

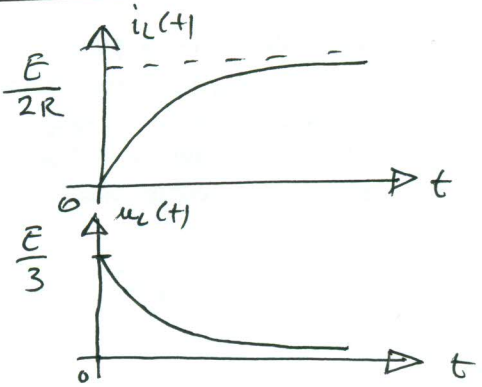
$$B = K\tau = \frac{E}{3L}\cdot\left(\frac{2R}{3L}\right)^{-1} = \frac{E}{2R} = B$$

$$I_{L0} = A + B \Rightarrow A = -B = -\frac{E}{2R}$$

$$\boxed{i_L(t) = -\frac{E}{2R}e^{-\frac{2R}{3L}t} + \frac{E}{2R}}$$

$$u_L(t) = L\frac{di}{dt} = -\frac{E}{2R}\cdot K\cdot\left(-\frac{2R}{3L}\right)e^{-\frac{2R}{3L}t} = \boxed{\frac{E}{3}e^{-\frac{2R}{3L}t} = u_L(t)}$$

b) $$W_L(t) = \frac{1}{2}Li_L^2(t) = \frac{LE^2}{8R^2}\left(1 - e^{-\frac{2R}{3L}t}\right)^2$$




$$W_{L\max} = \frac{LE^2}{8R^2}$$

$$\frac{LE^2}{8R^2}\left(1 - e^{-\frac{2R}{3L}t_1}\right)^2 = \frac{1}{4}\frac{LE^2}{8R^2} \quad / \sqrt{\ }$$

$$1 - e^{-\frac{2R}{3L}t_1} = \frac{1}{2}$$

$$e^{-\frac{2R}{3L}t_1} = 1/2 \quad / \ln(\cdot)$$

$$-\frac{2R}{3L}t_1 = -\ln(2)$$

$$\boxed{t_1 = \frac{3L}{2R}\ln(2)}$$