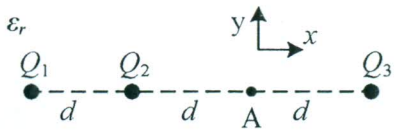
Elektrotehnika

12. septembar 2024.

1. Na Slici 1 su prikazana tri tačkasta naelektisanja Q_1=Q, Q_2=-2Q i Q_3=-3Q ($Q > 0$) koja se nalaze u dielektriku relativne dielektrične konstante ε_r.

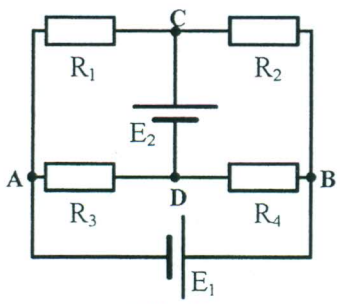
a) Odrediti i nacrtati vektor električnog polja u tački A. **(15 p.)**

b) Ako bi se u tačku A postavilo tačkasto naelektrisanje Q_A=-Q, odrediti i nacrtati vektor sile koja bi delovala na to naelektrisanje. **(5 p.)**

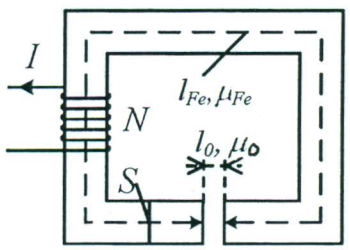


Slika 1

2. U kolu na Slici 2, odrediti snagu generatora E_1. Poznato je $E_1 = 14\text{V}, E_2 = 10\text{V}$ $R_1 = 15\Omega, R_2 = 10\Omega$, $R_3 = 10\Omega$, $R_4 = 15\Omega$. **(20 poena)**



Slika 2



Slika 3

3. Na jezgro magnetne permeabilnosti $\mu_{Fe} = \pi \cdot 10^{-4}\,\text{H/m}$, dužine srednje linije $l_{Fe} = 50\,\text{cm}$, debljine vazdušnog procepa $l_0 = 5\,\text{mm}$ i površine poprečnog preseka $S = 1\,\text{cm}^2$, namotan je provodnik sa $N = 100$ navojaka, kao što je prikazano na Slici 3.

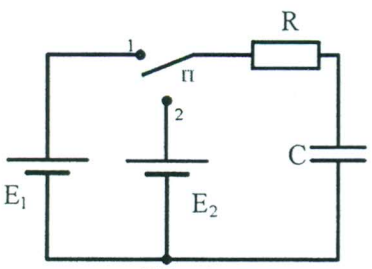
a) Odrediti induktivnost namotaja. **(10 poena)**

b) Ako kroz namotaj protiče struja intenziteta $I = 0.1\,\text{A}$, odrediti intezitet vektora jačine magnetnog polja u vazdušnom procepu. **(10 poena)**

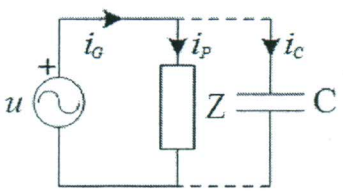
4. U kolu na Slici 4, poznate su vrednosti elemenata: $E_1 = 20\text{V}$, $E_2 = 10\text{V}$, $R = 10\Omega$ i $C = 2\mu\text{F}$. Preklopnik Π je u položaju 1 i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, preklopnik se prebacuje u položaj 2.

a) Odrediti izraze za napon i struju kondenzatora nakon prebacivanja preklopnika u položaj 2 i nacrtati odgovarajuće vremenske dijagrame. **(15 poena)**

b) Odrediti trenutak, t_1, u kome će vrednost napona kondenzatora biti jednaka 15V. **(5 poena)**



Slika 4



Slika 5

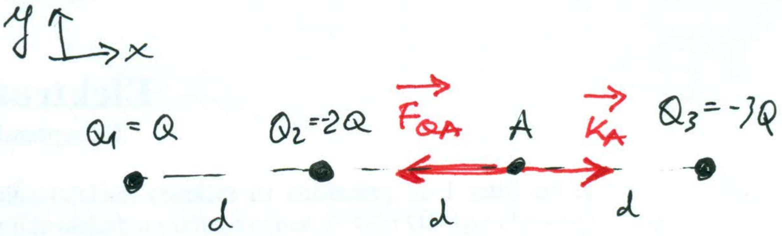
5. U kolu naizmenične struje sa Slike 5, potrošač kompleksne impedanse $\overline{Z} = (10 + j10)\Omega$ priključen je na naponski izvor $u(t)$, pri čemu je poznata struja potrošača $i_P(t) = 5\sqrt{2} \cdot \sin(2000 \cdot t)\,\text{A}$.

a) Odrediti kompleksne predstavnike napona i struje potrošača $\overline{Z}$, nacrtati njihov fazorski dijagram i odrediti vremenski oblik napona $u(t)$. **(7 poena)**

b) Odrediti aktivnu, reaktivnu i prividnu snagu potrošača $\overline{Z}$. **(3 poena)**

c) Odrediti kapacitivnost kondenzatora C koji je potrebno priključiti paralelno potrošaču $\overline{Z}$ (kao na Slici 5) da bi faktor snage ekvivalentnog potrošača iznosio $\cos\varphi = 1$. **(10 poena)**

(1)



a) $\vec{K}_{A1} = \dfrac{Q_1}{4\pi\varepsilon_0\varepsilon_r(2d)^2}\vec{i} = \dfrac{Q}{4\pi\varepsilon_0\varepsilon_r 4d^2}\vec{i}$

$$\boxed{\vec{K}_{A1} = \frac{Q}{16\pi\varepsilon_0\varepsilon_r d^2}\vec{i}}$$

$\vec{K}_{A2} = \dfrac{Q_2}{4\pi\varepsilon_0\varepsilon_r d^2}\vec{i} = \boxed{\dfrac{-2Q}{4\pi\varepsilon_0\varepsilon_r d^2}\vec{i} = \vec{K}_{A2}}$

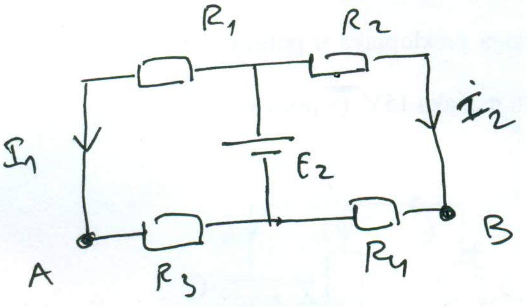
$\vec{K}_{A3} = \dfrac{Q_3}{4\pi\varepsilon_0\varepsilon_r d^2}(-\vec{i}) = \dfrac{-3Q}{4\pi\varepsilon_0\varepsilon_r d^2}(-\vec{i}) = \boxed{\dfrac{3Q}{4\pi\varepsilon_0\varepsilon_r d^2}\vec{i} = \vec{K}_{A3}}$

$\vec{K}_A = \vec{K}_{A1} + \vec{K}_{A2} + \vec{K}_{A3} = \dfrac{Q}{4\pi\varepsilon_0\varepsilon_r d^2}\vec{i}\left(\underbrace{\frac{1}{4} - 2 + 3}_{5/4}\right)$

$$\boxed{\vec{K}_A = \frac{5Q}{\pi 16\varepsilon_0\varepsilon_r d^2}\vec{i}}$$

b) $\boxed{\vec{F}_{Q_A} = Q_A \cdot \vec{K}_A = -\dfrac{5Q^2}{\pi 16\varepsilon_0\varepsilon_r d^2}\vec{i}}$

(2) Zadatak se može raditi na više načina. U nastavku je dato rešenje na bazi Tevenenove teoreme.

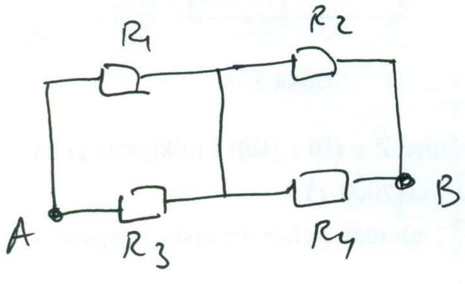


$I_1 = \dfrac{E_2}{R_1 + R_3} = \dfrac{10}{15+10} = \dfrac{10}{25} = 0{,}4\text{A}$

$I_2 = \dfrac{E_2}{R_2 + R_4} = \dfrac{10}{10+15} = 0{,}4\text{A}$

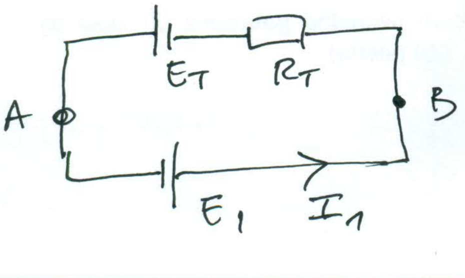
$E_T = U_{AB}^{ov} = R_3 I_1 - R_4 I_2$

$$\boxed{E_T = 10 \cdot 0{,}4 - 15 \cdot 0{,}4 = -2\text{V}}$$



$R_T = R_{AB}^{ov} = R_1 \| R_3 + R_2 \| R_4 = 2 \cdot \dfrac{10 \cdot 15}{10+15}$

$$\boxed{R_T = 12\,\Omega}$$



$I_1 = \dfrac{E_1 + E_T}{R_T} = \dfrac{14-2}{12} = \boxed{1\text{A} = I_1}$

$$\boxed{P_{E1} = E_1 I_1 = 14\text{W}}$$

(3)

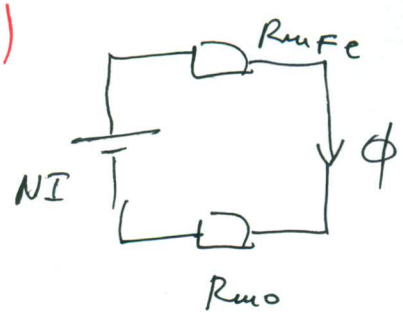
a) $$L = \frac{N^2}{R_{m0} + R_{mFe}} = \frac{N^2}{\frac{l_0}{\mu_0 S} + \frac{l_{Fe}}{\mu_{Fe} S}} = \frac{10^4}{\frac{5\cdot 10^{-3}}{4\pi\cdot 10^{-7}\cdot 10^{-4}\cdot 10^{-1}} + \frac{50\cdot 10^{-2}\cdot 10^{-4}}{\pi\cdot 10^{-4}\cdot 10^{-4}\cdot 10^{-3}}}$$

$$L = \frac{10^4}{\frac{5}{4\pi\cdot 10^{-8}} + \frac{5}{\pi\cdot 10^{-7}}} = \frac{10^4}{\frac{5\cdot 10^7}{\pi}\left(\frac{10}{4}+1\right)} = \frac{\pi}{17{,}5}\cdot 10^{-3}\ H$$

($\frac{10}{4}+1 = 3{,}5$)

$$L = \frac{\pi}{17{,}5}\ mH = \frac{4\pi}{70}\ mH$$

b)



$$\phi = \frac{NI}{R_{mFe} + R_{m0}}, \quad \phi = BS \Rightarrow B = \frac{\phi}{S}$$

$$B = \frac{NI}{S(R_{mFe} + R_{m0})}, \quad B = \mu H$$

$$H_0 = \frac{B}{\mu_0} = \frac{NI}{\mu_0 S(R_{mFe} + R_{m0})}$$

$$H_0 = \frac{NI}{\frac{\mu_0 l_{Fe}}{\mu_{Fe}} + l_0}$$

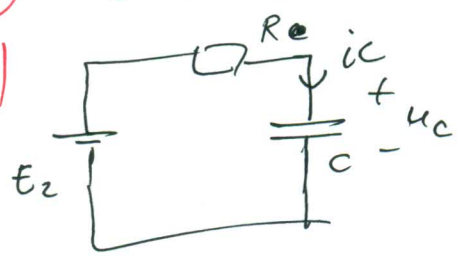
$$H_0 = \frac{100\cdot 0{,}1}{\frac{4\pi\cdot 10^{-7}\cdot 50\cdot 10^{-2}}{\pi\cdot 10^{-4}} + 5\cdot 10^{-3}} = \frac{10}{2\cdot 10^{-3} + 5\cdot 10^{-3}} = \frac{10000}{7}\ \frac{H}{m}$$

H_0

(4)

$u_c(0^+) = u_c(0^-) = U_{c0} = E_1 = 20V$ ← POČETNÍ USLOV

a)



$$-E_2 + R i_C + u_C = 0$$

$$RC\frac{du_C}{dt} + u_C = E_2$$

$$\frac{du_C}{dt} + \frac{u_C}{RC} = \frac{E_2}{RC} = K, \quad \tau = RC = 20\mu s$$

($\frac{u_C}{RC} = \frac{1}{\tau}$)

$$u_C(t) = Ae^{-t/\tau} + B$$

$$B = K\tau = \frac{E_2 RC}{RC} = E_2 = 10V$$

$$u_C(0) = A + B \Rightarrow A = -B + u_C(0)$$

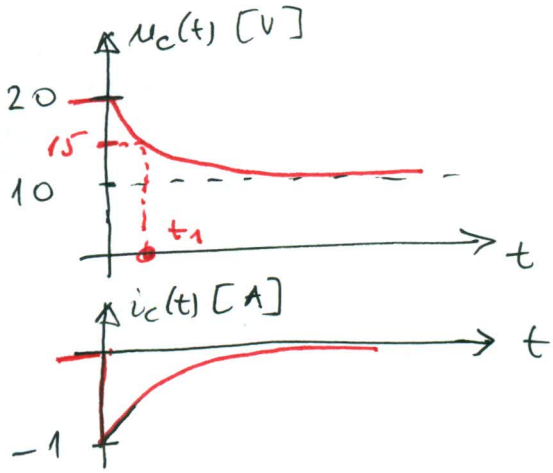
$$A = -E_2 + E_1 = -20 + 10 = +10\ V$$

$$\Rightarrow \quad u_C(t) = (E_1 - E_2)e^{-t/\tau} + E_2$$

$$u_C(t) = \left(10e^{-\frac{t}{20\mu s}} + 10\right) V$$

$$i_C(t) = C\frac{du_C}{dt} = C\cdot\frac{E_1-E_2}{-\tau}e^{-t/\tau} = \frac{E_2-E_1}{R}e^{-t/\tau}$$

$$\boxed{i_C(t) = \left(-e^{-\frac{t}{20\mu s}}\right) A}$$



b) $u_C(t_1) = 15\ V$

$$10e^{-\frac{t_1}{\tau}} + 10 = 15$$

$$10e^{-t_1/\tau} = 5$$

$$e^{-t_1/\tau} = \frac{1}{2} \;/\; ln(\cdot)$$

$$-t_1/\tau = ln\left(\frac{1}{2}\right) = -ln(2)$$

$$\boxed{t_1 = \tau\, ln(2) = 20\, ln(2)\ \mu s}$$

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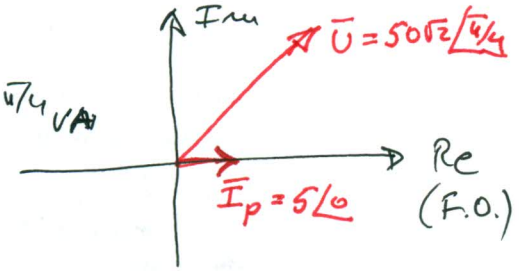
(5) a) $\boxed{\bar{I}_p = 5e^{j0^\circ} = 5\ A}$

$$\bar{U} = \bar{Z}\bar{I}_p = (10+j10)\cdot 5 = (50+j50)V = \boxed{50\sqrt{2}\,e^{j\pi/4}\,V = \bar{U}}$$

$$\boxed{u(t) = 50\sqrt{2}\cdot\sqrt{2}\,sin(2000t + \pi/4)\,V = 100\,sin(2000t+\pi/4)\,V}$$



b) $\bar{S}_p = \bar{Z}I_p^2 = \bar{U}\bar{I}_p^* = 250 + j250 = 250\sqrt{2}\,e^{j\pi/4}\,VA$

$$\boxed{\begin{aligned} P_p &= 250\ W \\ Q_p &= 250\ VAR \\ S_p &= 250\sqrt{2}\ VA \end{aligned}}$$

c) $\cos\varphi = 1 \Rightarrow S = P \Rightarrow Q = 0$

$Q = Q_p + Q_c$

$\Big\}\ Q_c = -Q_p = -250\ VAR \quad (1)$

$$Q_c = -\omega C\cdot U^2 \quad (2)$$

$$(1),(2) \Rightarrow -Q_p = -\omega C U^2 \Rightarrow C = \frac{Q_p}{\omega U^2} = \frac{250}{2000\cdot(50\sqrt{2})^2}$$

$$C = \frac{\cancel{250}^{1}}{2000\cdot\cancel{2500}_{10}\cdot 2} = \frac{1}{40\cdot 10^3}$$

$$\boxed{C = 25\ \mu F}$$