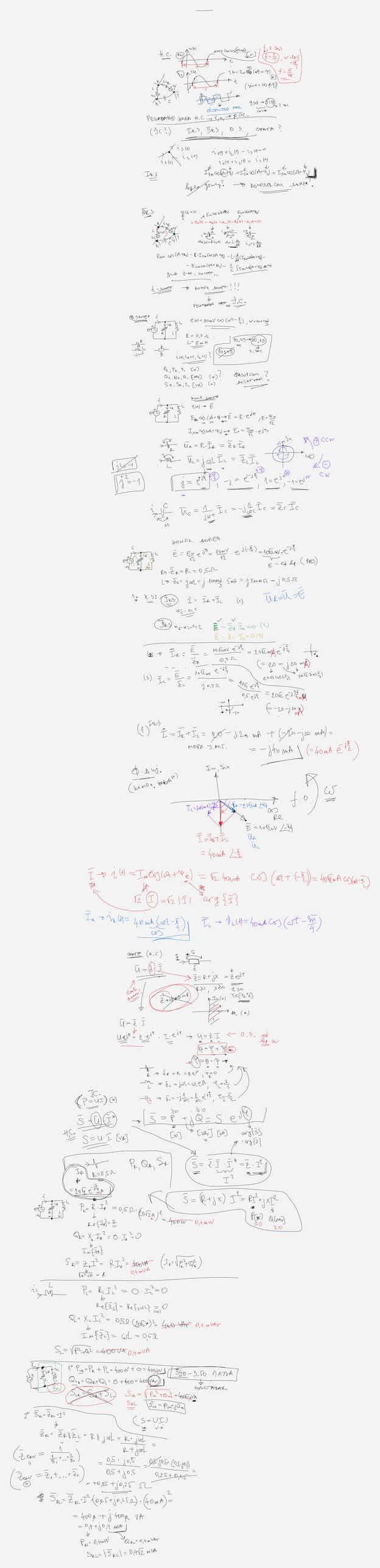
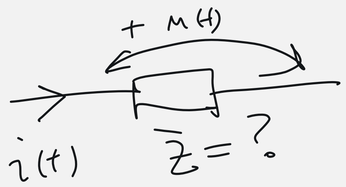


(*)



$i(t) = 10\sqrt{2}\cos(\omega t + \frac{\pi}{2})$ A

$u(t) = 200\cos(\omega t + \frac{\pi}{3})$ V

$\omega = 314$ rad/s

$(\omega = 2\pi f,\ f = 50$ Hz$)$

a) $\bar{Z} = ?$ (R, L, C) ← cocimbusion reg

b) P, Q, S → $\bar{Z}$

$i(t) \to \bar{I} = 10 \cdot e^{j\frac{\pi}{2}}$ A ; $u(t) \to \bar{U} = 100\sqrt{2} \cdot e^{j\frac{\pi}{3}}$ V

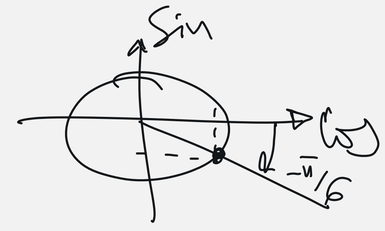
$10\sqrt{2}/\sqrt{2}$ $\qquad$ $200/\sqrt{2}$

$$\bar{Z} = \frac{\bar{U}}{\bar{I}} = \frac{100\sqrt{2}\, e^{j\frac{\pi}{3}}\ V}{10 \cdot e^{j\frac{\pi}{2}}\ A} = 10\sqrt{2}\, e^{j(\frac{\pi}{3} - \frac{\pi}{2})}\ \Omega$$

$\bar{Z}$

$\varphi = \theta - \psi = -\frac{\pi}{6}$

$$\bar{Z} = 10\sqrt{2}\, e^{-j\frac{\pi}{6}}\ \Omega$$



$$\bar{Z} = 10\sqrt{2}\left(\cos\frac{-\pi}{6} + j\sin\frac{-\pi}{6}\right)\Omega = 10\sqrt{2}\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)\Omega$$

c)

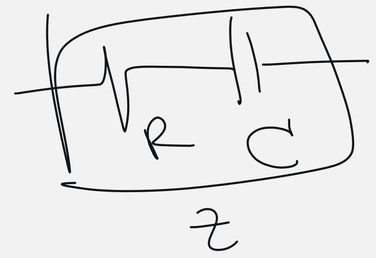
$$\bar{Z} = \underbrace{5\sqrt{6}}_{R} \underbrace{- j5\sqrt{2}}_{C}\ \Omega$$

$\bar{Z}_R = R$

$R = 5\sqrt{6}\ \Omega$

$\bar{Z}_C = -j\frac{1}{\omega C}$

$\frac{1}{\omega C} = 5\sqrt{2}\,\Omega \to C = \frac{1}{314 \cdot 5\sqrt{2}} = 0{,}45$ mF



d) $\bar{S} = \bar{U} \cdot \bar{I}^* = 100\sqrt{2}\, e^{j\frac{\pi}{3}} \cdot \left(10 \cdot e^{j\frac{\pi}{2}}\right)^* = 100\sqrt{2}\, e^{j\frac{\pi}{3}} \cdot 10\, e^{-j\frac{\pi}{2}} =$

$= 1000\sqrt{2}\, e^{-j\frac{\pi}{6}}$ VA

φ

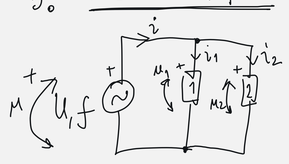
$S = 1000\sqrt{2}$ VA $= \sqrt{2}$ kVA

$P = S\cos\varphi = 500\sqrt{6}$ W

$Q = S\sin\varphi = 500\sqrt{2}$ VAr

15.40 – 15.50 – 16.30 – 16.40 → ?
1. 2. 3?

3. Без комп. раб. → сер. зад.



① $P_1 = 10\,kW$, $\cos\varphi_1 = \frac{1}{\sqrt{2}}$ (претеглено капацитивно!)

② $R_2 = 8\,\Omega$, $X_2 = 6\,\Omega$

$U = 400\,V$
$f = 50\,Hz$

а) I_1, I_2 ?
б) $P_{ук}$, $Q_{ук}$
в) I

а) $I_1 = ?$ $\quad U_1 = U_2 = U = 400\,V$ $\quad (u_1(t) = u_2(t) = u(t))$

? → $P_1, \cos\varphi_1$ (пред. капац.) $\quad \underline{Z}_1 = R_1 + jX_1 \rightarrow X_1 < 0$

$Q_1 = X_1 \cdot I_1^2 < 0$!

$\underline{Z}_C = -j\frac{1}{\omega C}$

$\underline{Z}_L = +j\omega L$

$\underline{S}_1 = U_1 \cdot I_1$

$\underline{S}_1 = P_1 + jQ_1 = S_1 \cdot e^{j\varphi_1}$

$S_1 \cdot \cos\varphi_1 = P_1, \quad S_1 \cdot \sin\varphi_1 = Q_1 < 0$

$S_1 = \frac{P_1}{\cos\varphi_1} = \frac{10\,kW}{1/\sqrt{2}} = 10\sqrt{2}\,kW$ (kVA)

$\varphi_1 = -\pi/4$ $\quad \varphi \in [-\pi/2, \pi/2]$

$I_1 = \frac{S_1}{U_1} = \frac{10\sqrt{2}\,kVA}{400\,V} = 25\sqrt{2}\,A \approx 35\,A$

$I_2 = ?$ $\quad U_2 = 400\,V$, $R_2 = 8\,\Omega$, $X_2 = 6\,\Omega$

О.З. $U_2 = Z_2 \cdot I_2$ $\quad (\underline{U}_2 = \underline{Z}_2 \cdot \underline{I}_2)$

$Z_2 = |\underline{Z}_2| = |R_2 + jX_2| = \sqrt{R_2^2 + X_2^2}$

$Z_2 = \sqrt{8^2 + 6^2} = 10\,\Omega$

$I_2 = \frac{U_2}{Z_2} = 40\,A$

б) $P_{ук} = P_1 + P_2 = P_1 + R_2 I_2^2 = 10\,kW + 8\,\Omega \cdot (40\,A)^2 = \ldots$

$\ldots = 10\,kW + 12800\,W = 22{,}8\,kW$

$Q_{ук} = Q_1 + Q_2 = S_1 \sin\varphi_1 + X_2 I_2^2 = 10\sqrt{2}\,kVA \cdot \left(-\frac{1}{\sqrt{2}}\right) + 6\,\Omega \cdot (40\,A)^2 = \ldots$

$\varphi_1 = -\frac{\pi}{4}$

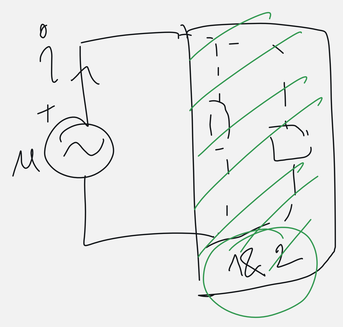
$\ldots = -10\,kVAr + 9600\,VAr = -0{,}4\,kVAr$

в) $I = ?$

~~$I = I_1 + I_2$~~

$\underline{I} = \underline{I}_1 + \underline{I}_2$, $\quad i(t) = i_1(t) + i_2(t)$

ІК3, ІІК3 НЕ ЗА ЕФ. ВР. u, i



$U = 400\,V$, $\quad I = ?$ $\quad U \cdot I = S_{ук}$

$S_{ук} = ?$

$S_{ук} = \sqrt{P_{ук}^2 + Q_{ук}^2} = \sqrt{(22{,}8\,kW)^2 + (-0{,}4\,kVAr)^2} =$

$= 10{,}008\,kVA \approx 22{,}8\,kVA$

$I = \frac{S_{ук}}{U} = \frac{22{,}8\,kVA}{400\,V} \approx 57\,A$

$\cos\varphi_1 = \frac{1}{\sqrt{2}} = 0{,}707$

$\cos\varphi_2 = \frac{R_2}{S_2} = \frac{R_2}{Z_2} = 0{,}8$

$\cos\varphi_{ук} = \frac{P_{ук}}{S_{ук}} = \frac{10\,kW}{10{,}008\,kVAr} = 0{,}9992$

добре че $Q \downarrow$, $\cos\varphi \rightarrow 1$, $\varphi \rightarrow 90°$

$\underline{I} * \underline{I}_1 + \underline{I}_2$

~~57 A~~ $\neq 35\,A + 40\,A$

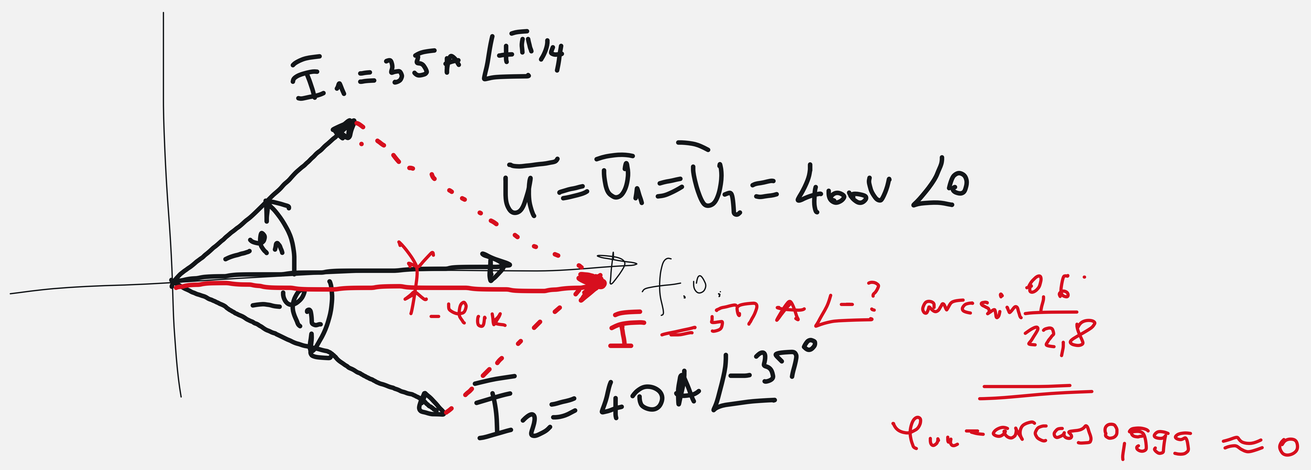
??

ф. 1.

$\underline{Z}_2 = R_2 + jX_2 = Z_2 e^{j\varphi_2}$

$\varphi_2 = \arctg\frac{6}{8} = 37°$

$\arctg\frac{X}{R}$



$\varphi_1 < 0$

$\theta_1 = \psi_1 + \varphi_1$

$\varphi_1 = \theta_1 - \psi_1$

$\psi_1 = \theta - \varphi_1 = -\varphi_1$

$\psi_2 = 0 - \varphi_2 = -\varphi_2$

$\psi_1 = \theta_1 - \varphi_1$

(5) ~~2 хо~~.

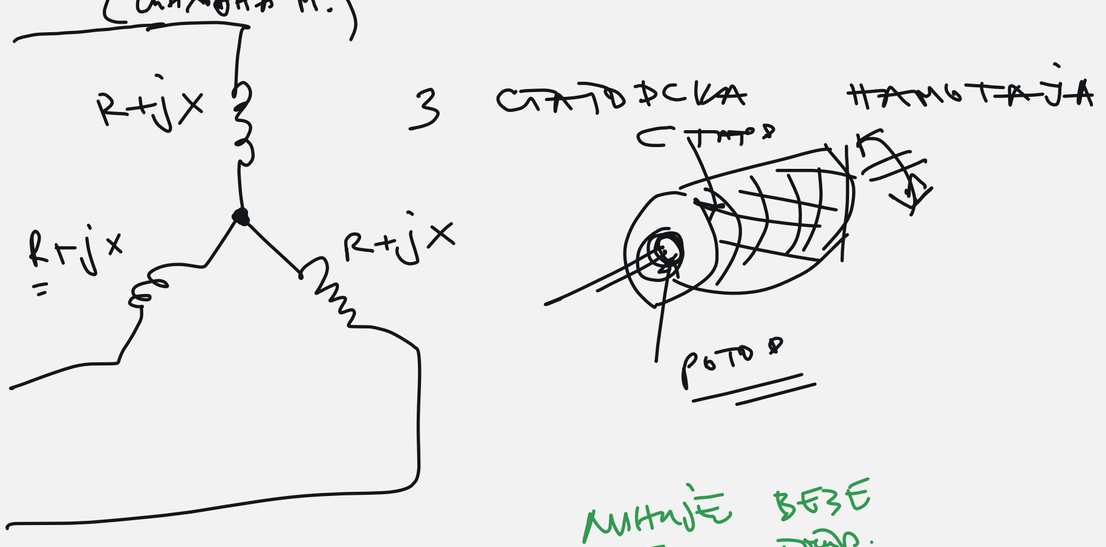
~~1~~ХЕ$\dot{S}$
2х$\dot{J}$S
2хNS

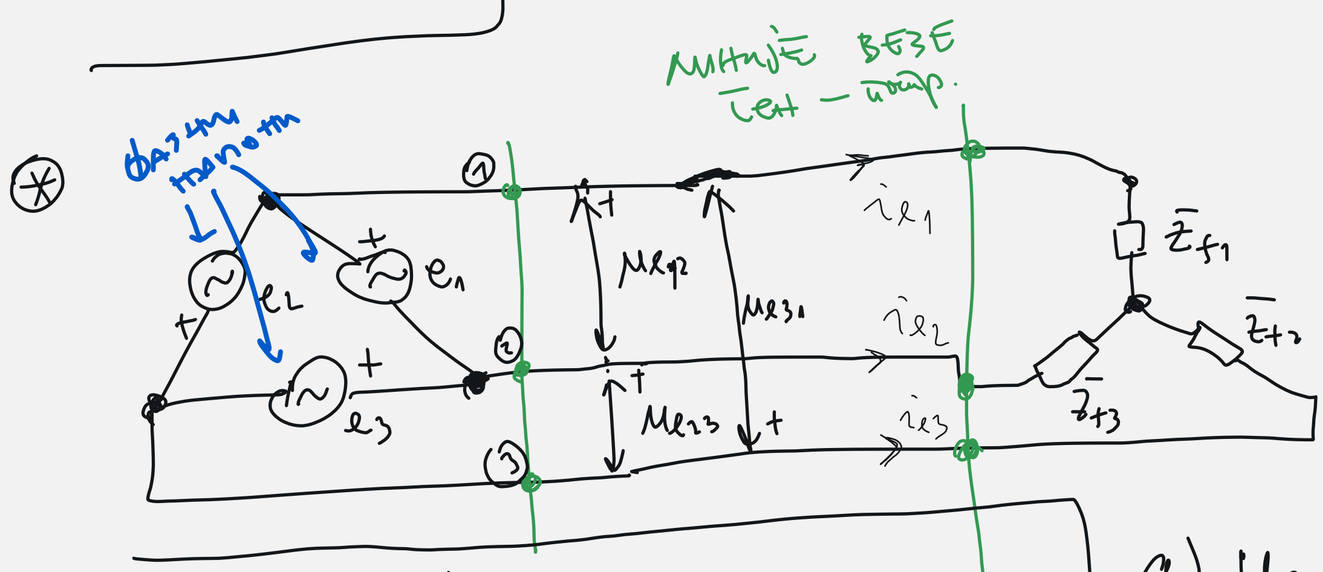
1ХЕS
1х$\dot{J}$S
3хNS

2х$\dot{J}$S
3хNS

3ф ТРОФАЗНИ СИСТЕМИ

Асинхронен мотор (машина) ← TESLA





Тел. "звезда у Δ" → $U_{l12} = e_1, U_{l23} = e_2, U_{l31} = e_3$

Товар. "звезда у Y"

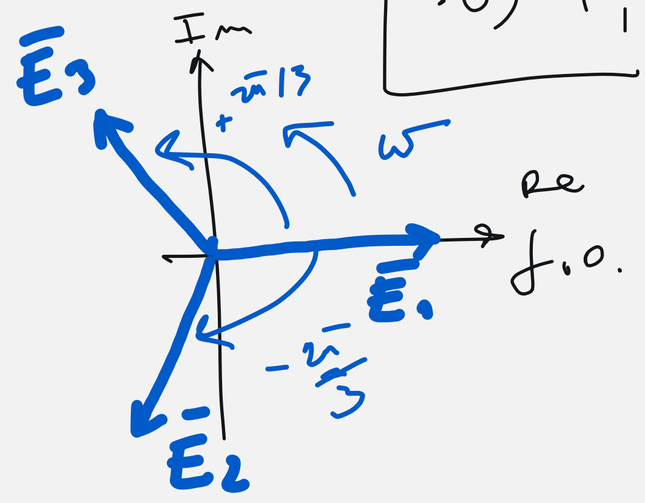
а) U_l, I_l

б) P, Q, S 3ф товар.

$e_1(t) = \sqrt{2} E \cos \omega t$

$e_2(t) = \sqrt{2} E \cos(\omega t - \frac{2\pi}{3})$

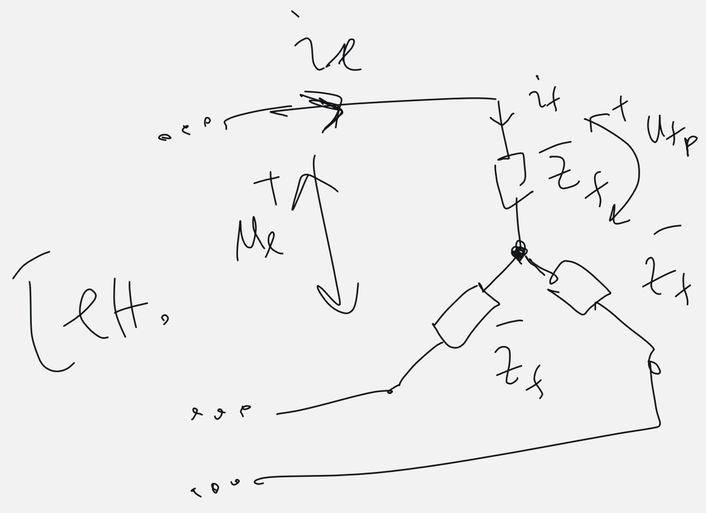
$e_3(t) = \sqrt{2} E \cos(\omega t - \frac{4\pi}{3})$ $+\frac{2\pi}{3}$



Тел. Δ ⇒ $U_l = U_{1gen} = 400V$

(Тел. Y ⇒ $U_l = \sqrt{3} \cdot U_{1gen}$

$\sqrt{3} \cdot 230V = 400V$)



$U_l = 400V$

товар. Y + симетричност

$U_{fp} = \frac{U_l}{\sqrt{3}} = \frac{400V}{\sqrt{3}} = 230V$

i_f → 0.3. $i_f = \frac{U_f}{Z_f}$

$I_f = \frac{U_f}{Z_f} = \frac{230V}{\sqrt{10^2 + 10^2}} = \frac{230V}{10\sqrt{2}\,\Omega} \approx \frac{23}{14} = 16{,}3A$

$Z_f = |\bar{Z}_f|$

Y ⇒ $I_l = I_f = 16A$

б) $P_{3\phi}, Q_{3\phi}, S_{3\phi}$?

$P_{3\phi} = 3 \cdot P_{1\phi} = 3 \cdot R_f \cdot I_f^2 = 3 \cdot 10\Omega \cdot (16A)^2 = 7800W = 7{,}8kW$

$Q_{3\phi} = 3 \cdot Q_{1\phi} = 3 \cdot X_f \cdot I_f^2 = \ldots = 7{,}8kVAr$

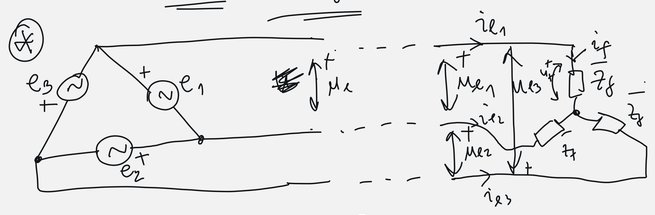
$S_{3\phi} = 3 \cdot S_{1\phi} = \sqrt{P_{3\phi}^2 + Q_{3\phi}^2} = 3 \cdot \sqrt{R^2 + X^2} I_f^2 = 3 Z_f I_f^2 = 3 U_f I_f$

$S_{3\phi} = 3 \cdot 230V \cdot 16{,}3A = 11270 = 11{,}3\ kVA$

($S_{3\phi} = \sqrt{P_{3\phi}^2 + Q_{3\phi}^2} = \sqrt{2} \cdot 7{,}8k = \ldots$)

Сл. нед. тест 13–17 ...

Tehn. ⊗ $U_e = U_{fg} = E$



$e_1(t) = 400 \cos \omega t$ V

e_2 ---

e_3 ---

a) $e_2(t)$ ... ? $e_3(t)$

b) $\cos \varphi_f$

v) $\underline{I}_e$

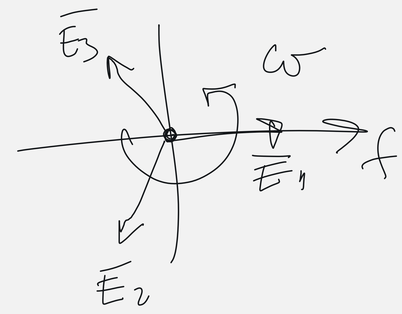
g) $\underline{Z}_f$

$P_{3\phi} = 1$ kW

$Q_{3\phi} = -1$ kVAr

a) $e_2(t) = 400 \cos(\omega t - \frac{2\pi}{3})$ V

$e_3(t) = 400 \cos(\omega t - \frac{4\pi}{3})$ V



---

a) U, PQS

$\underline{I} = ?$

$S_{3\phi} = 3 U_f I_f$ $(= \sqrt{3} U_e I_e)$

Λ, Δ

$S_{3\phi} = \sqrt{P_{3\phi}^2 + Q_{3\phi}^2} = \sqrt{2}$ kVA

$(\underline{S} = P + jQ)$

$U_f = ?$ MDOP ①

$U_f = \overset{\downarrow}{U_e}? = \frac{E}{\sqrt{3}} = \frac{400\text{V}}{\sqrt{3}} = 230\text{V}$

$I_f = \frac{S_{3\phi}}{3 U_f} = \frac{\sqrt{2}\,\text{kVA}}{3 \cdot 230\text{V}} = \frac{1410\,\text{VA}}{690\text{V}} = 2{,}04\,\text{A}$

MDOP ② $I_e - I_f = 2{,}04$A

$\left(I_e = \frac{S_{3\phi}}{\sqrt{3} U_e} = \frac{1410\,\text{VA}}{\sqrt{3} \cdot 400} = \dots \right)$

b) $\cos \varphi_p = \frac{P_{3\phi}}{S_{3\phi}} = \frac{1}{\sqrt{2}} = 0{,}71$

g) $\underline{Z}_f = ?$ $\underline{Z}_f = R_f + jX_f$

? ?

$P_{3\phi} = 3 \cdot R_f I_f^2$

~~I_e~~

$R_f = \frac{P_{3\phi}}{3 \cdot I_f^2} = \frac{1\text{kW}}{3 \cdot (2{,}04\text{A})^2} = 80{,}1\,\Omega$

$X_f = ?$

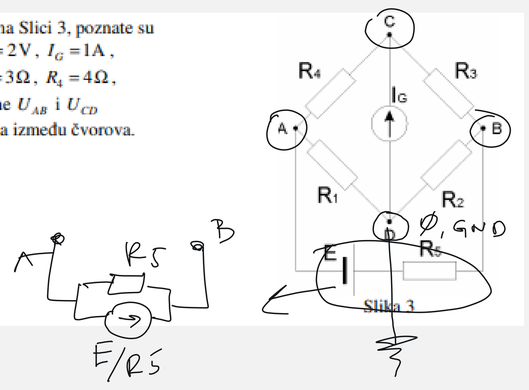
$Q_{3\phi} = 3 X_f I_f^2 \rightarrow X_f = \frac{Q_{3\phi}}{3 I_f^2} = \frac{-1\text{kVAr}}{3 \cdot 2{,}04\text{A}^2}$

$X_f = -80{,}1\,\Omega$

$\underline{Z}_f = R_f + jX_f = 80{,}1 - j80{,}1\,\Omega$

---

3. (zbirka II.12.) U kolu na Slici 3, poznate su vrednosti elemenata: $E = 2\text{V}$, $I_G = 1\text{A}$, $R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, $R_4 = 4\Omega$, $R_5 = 5\Omega$. Odrediti napone U_{AB} i U_{CD} primenom metode napona između čvorova.



MDP (MHY) $U_{AD} = V_A - \cancel{V_D}^{0} = V_A$

$V_A, V_B, V_C \dots$ (U_{AD}, U_{BD}, U_{CD})

pet. vor? ∅

makar closjegno!

Ⓐ $G_{AA} V_A - G_{AB} V_B - G_{AC} V_C = \sum I_g$

$\left(\frac{1}{R_4} + \frac{1}{R_1} + \frac{1}{R_5}\right) V_A - \frac{1}{R_5} V_B - \frac{1}{R_4} V_C = -\frac{E}{R_5}$

Ⓑ $\left(\frac{1}{R_3} + \frac{1}{R_2} + \frac{1}{R_5}\right) V_B - \frac{1}{R_5} V_A - \frac{1}{R_3} V_C = +\frac{E}{R_5}$

Ⓒ $\left(\frac{1}{R_4} + \frac{1}{R_3}\right) V_C - \frac{1}{R_4} V_A - \frac{1}{R_3} V_B = +I_g$

3X3 ...

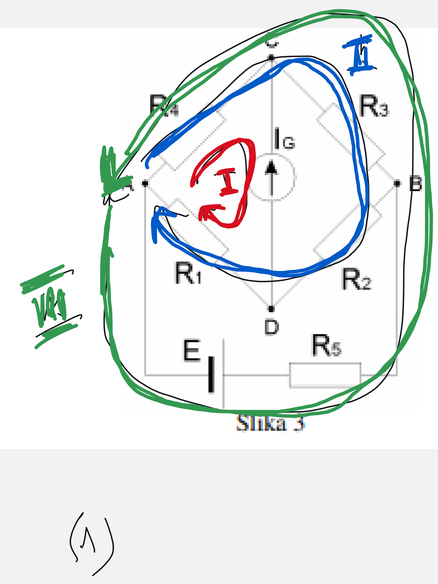
V_A, V_B, V_C u

$U_{AB} = V_A - V_B$ u

$U_{CD} = V_C - \cancel{V_D}^{0} = V_C$

---

3. (zbirka II.12.) U kolu na Slici 3, poznate su vrednosti elemenata: $E = 2\text{V}$, $I_G = 1\text{A}$, $R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, $R_4 = 4\Omega$, $R_5 = 5\Omega$. Odrediti napone U_{AB} i U_{CD} primenom metode napona između čvorova.



MKC 3X3

$M_G - M_C + 1 = 3$

→ ! ! !

CAMO 1 KOHT. kpo3

Ⓘ $I_I = -I_g$ (1)

Ⓘ $R_{21} I_I + R_{22} I_{II} + R_{23} I_{III} = \sum_{II} E$

$(R_4 + R_1)(-I_g) + (R_1 + R_2 + R_3 + R_4) I_{II} - (R_4 + R_3) I_{III} = 0$ (2)

Ⓘ $R_{31} I_I + R_{32} I_{II} + R_{33} I_{III} = \sum_{III} E$

$-R_4 I_I - (R_4 + R_3) I_{II} + (R_4 + R_3 + R_5) I_{III} = E$ (3)

3X3 ... I_I u

2X2

I_I, I_{II}, I_{III}