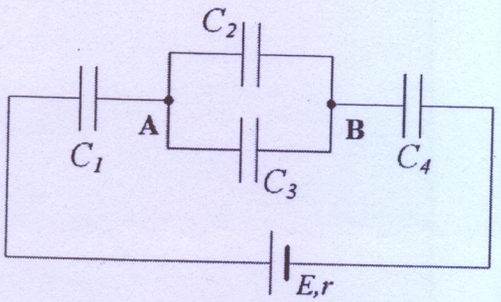
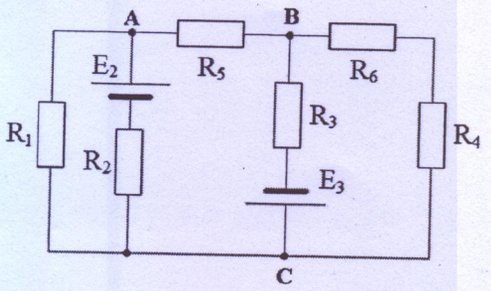
Elektrotehnika

pismeni ispit, 19. jul 2025.

1. U kolu na Slici 1 odrediti napon U_{AB} i elektrostatičku energiju kondenzatora C_1 i C_3. Parametri elemenata su: $E = 30\,\mathrm{V}$, $C_1 = C_2 = 20\,\mathrm{nF}$, $C_3 = 10\,\mathrm{nF}$, $C_4 = 60\,\mathrm{nF}$. Pre povezivanja u kolo, kondenzatori su bili neopterećeni. **(10 poena)**



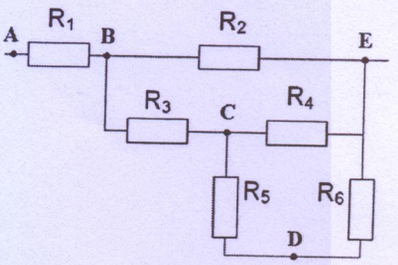
Slika 1



Slika 2

2. Poznate su sve otpornosti i ems u kolu prikazanom na Slici 2: $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 5\Omega$, $E_2 = 30\,\mathrm{V}, E_3 = 10\,\mathrm{V}$. Rešiti kolo primenom metode potencijala čvorova ili metode konturnih struja, a nakon toga odrediti snagu otpornika R_2. **(12 poena)**

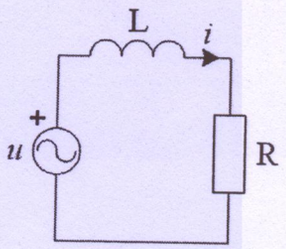
3. Na Slici 3 prikazana je grupa od šest otpornika poznatih otpornosti: $R_1 = R_2 = R_3 = R_5 = R_6 = R = 30\Omega$, $R_4 = 2R = 60\Omega$. Odrediti ekvivalentnu otpornost između tačaka B i C. **(8 poena)**



Slika 3

4. Na Slici 4 je prikazano kolo naizmenične struje koje se napaja naponskim generatorom $u(t) = 20\cos(\omega t - \pi/2)\,\mathrm{V}$, gde je $\omega = 100\,\mathrm{rad/s}$. Poznato je: $R = 10\Omega$, $L = 100\,\mathrm{mF}$.

a) Odrediti kompleksne izraze struje u kolu i napona na otporniku i kalemu; skicirati fazorski dijagram struje i napona na svim elementima. **(6 poena)**
b) Odrediti aktivnu, reaktivnu i prividnu snagu celokupnog potrošača. **(4 poena)**

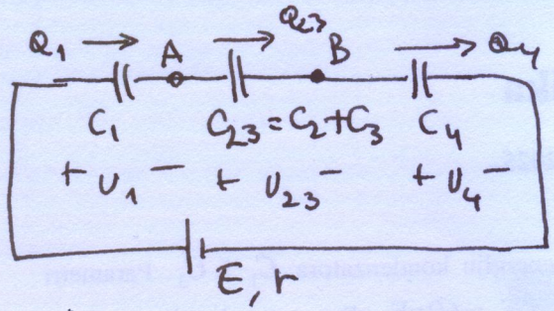


Slika 4

5. Na naponski generator $U = 400\,\mathrm{V}$, $f = 50\,\mathrm{Hz}$, priključena je paralelna veza pretežno induktivnog potrošača parametara $P_1 = 1\,\mathrm{kW}$, $\cos\varphi_1 = 1/\sqrt{2}$, i pretežno kapacitivnog potrošača parametara $S_2 = 5\,\mathrm{kVA}$ i $\cos\varphi_2 = 0.6$. Odrediti:

a) ukupnu aktivnu, reaktivnu i prividnu snagu paralelne veze potrošača; **(5 poena)**
b) efektivnu vrednost struje koju paralelna veza potrošača uzima iz mreže i kompleksnu impedansu celokupnog potrošača. **(5 poena)**

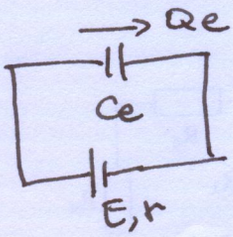
①



$C_{23} = C_2 + C_3 = 30\,\mu F$

$Q_1 = Q_{23} = Q_4 = Q_e = 300\,\mu C$

$U_{AB} = U_{23} = \frac{Q_{23}}{C_{23}} = \frac{300\mu}{30\mu} = 10\,V$



$Q_e = C_e \cdot E = 10\mu \cdot 30 = 300\,\mu C$

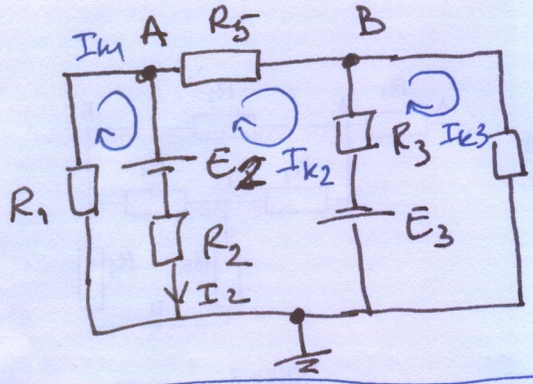
$$C_e = \frac{1}{\frac{1}{C_1} + \frac{1}{C_{23}} + \frac{1}{C_4}} = \frac{1}{\frac{1}{20\mu} + \frac{1}{30\mu} + \frac{1}{60\mu}} = \frac{1}{\frac{3+2+1}{60\mu}}$$

$$C_e = \frac{60\mu}{6} = 10\,\mu F$$

$$W_1 = \frac{1}{2}\frac{Q_1^2}{C_1} = \frac{(300\cdot 10^{-9})^2}{2\cdot 20\mu} = \frac{9\cdot 10^4 \cdot 10^{-18}}{4\cdot 10^{-8}} = \frac{9}{4}\cdot 10^{-6} = 2{,}25\,\mu J$$

$$W_2 = \frac{1}{2} C_3 U_3^2 = \frac{1}{2} 10\cdot 10^{-9} \cdot 10^2 = 0{,}5\,\mu J$$

②



MKS:

$$(R_1+R_2)I_{K1} - R_2 I_{K2} + 0\,I_{K3} = -E_2$$

$$-R_2 I_{K1} + (R_2+R_3+R_5)I_{K2} - R_3 I_{K3} = E_2 + E_3$$

$$0\cdot I_{K1} - R_3 I_{K2} + (R_3+R_4+R_6)I_{K3} = -E_3$$

$$10\,I_{K1} - 5\,I_{K2} = -30 \quad (1)$$

$$-5\,I_{K1} + 15\,I_{K2} - 5\,I_{K3} = 40 \quad (2)$$

$$-5\,I_{K2} + 15\,I_{K3} = -10 \quad (3)$$

$$(1) \Rightarrow I_{K1} = \frac{1}{2} I_{K2} - 3 \quad (4)$$

$$(4) \to (2) \Rightarrow -\frac{5}{2} I_{K2} - 15 + 15\,I_{K2} - 5\,I_{K3} = 40$$

$$-5\,I_{K3} = 25 - 12{,}5\,I_{K2} \;/:(-5)$$

$$I_{K3} = 2{,}5\,I_{K2} - 5 \quad (5)$$

MPČ:

$$A: \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5}\right)V_A - \frac{1}{R_5} V_B = \frac{E_2}{R_2}$$

$$B: -\frac{1}{R_5} V_B + \left(\frac{1}{R_5} + \frac{1}{R_3} + \frac{1}{R_4+R_6}\right) = -\frac{E_3}{R_3}$$

$$\frac{3}{5} V_A - \frac{1}{5} V_B = \frac{30}{5} \;/\cdot 25$$

$$-\frac{1}{5} V_A + \frac{5}{10} V_B = -\frac{10}{5} \;/\cdot 10$$

$$15\,V_A - 5\,V_B = 150$$

$$-2\,V_A + 5\,V_B = -20 \quad +$$

$$13\,V_A = 130$$

$$\boxed{V_A = 10\,V} \quad \boxed{V_B = 0\,V}$$

$$V_A = E_2 + R_2 I_2$$

$$I_2 = \frac{V_A - E_2}{R_2} = \frac{10-30}{5}$$

$$I_2 = -4\,A$$

$$P_{R2} = R_2 I_2^2 = 5(-4)^2$$

$$\boxed{P_{R2} = 80\,W}$$

MKS : (NASTAVAK)

$(5) \to (3) \Rightarrow \; -5 I_{K2} + 37{,}5 I_{K2} - 75 = -10$

$I_{K2} = 65/35{,}5 = 2\,A$

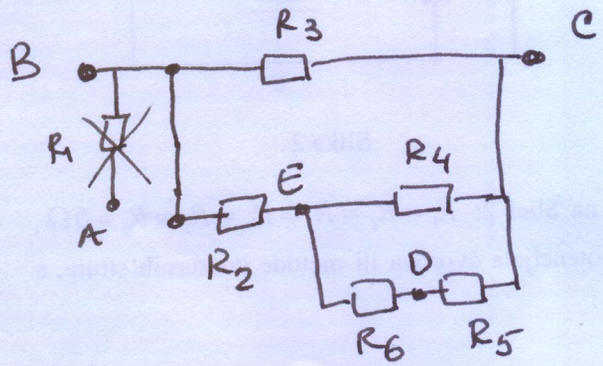
$I_{K3} = 2{,}5 \cdot 2 - 5 = 0\,A$

$I_{K1} = 0{,}5 \cdot 2 - 3 = -2\,A$

$I_2 = I_{K1} - I_{K2} = -2 - 2 = -4\,A$

$P_{R2} = R_2 I_2^2 = 80\,W$

3



$R_{BC} = R_3 \| \left(R_2 + R_4 \| (R_5 + R_6)\right)$

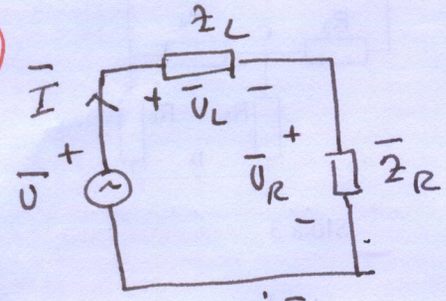
$R_{BC} = R \| \left(R + (2R) \| (2R)\right)$

$R_{BC} = R \| (R + R)$

$R_{BC} = R \| (2R) = \frac{R \cdot 2R}{R + 2R}$

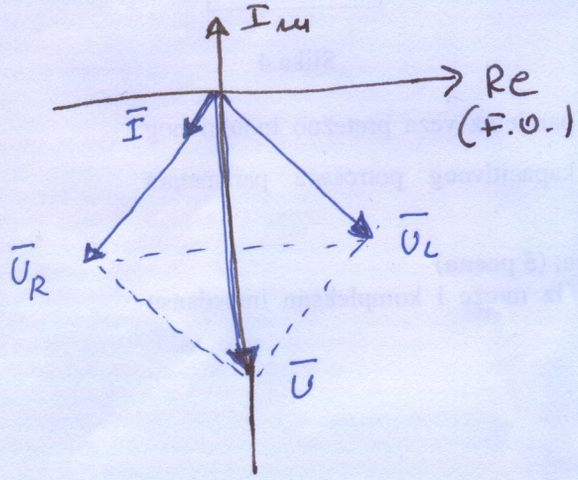
$R_{BC} = \frac{2R}{3} = 20\,\Omega$

4



$\bar{U} = \frac{20}{\sqrt{2}} e^{-j\bar{u}/2}\,V$

$\bar{U} = 10\sqrt{2}\, e^{-j\bar{u}/2}$



a) $\bar{Z}_L = j\omega L = j100 \cdot 100 \cdot 10^{-3} = j10\,\Omega = 10 e^{j\bar{u}/2}\,\Omega$

$\bar{Z}_R = R = 10\,\Omega = 10 e^{j0}\,\Omega$

$\bar{I} = \frac{\bar{U}}{\bar{Z}_L + \bar{Z}_R} = \frac{10\sqrt{2} e^{-j\bar{u}/2}}{10 + j10} = \frac{10\sqrt{2} e^{-j\bar{u}/2}}{10\sqrt{2} e^{j\bar{u}/4}}$

$(\bar{Z}_e = \bar{Z}_R + \bar{Z}_L = 10 + j10\,\Omega = 10\sqrt{2}\, e^{j\bar{u}/4}\,\Omega)$

$\bar{I} = 1 \cdot e^{-j3\bar{u}/4}\,A$

$\bar{U}_L = \underbrace{10 e^{j\bar{u}/2}}_{\bar{Z}_L} \cdot \underbrace{1 \cdot e^{-j3\bar{u}/4}}_{\bar{I}} = 10 e^{-j\bar{u}/4}\,V$

$\bar{U}_R = \bar{Z}_R \cdot \bar{I} = 10 \cdot 1 \cdot e^{-j3\bar{u}/4} = 10 e^{-j3\bar{u}/4}\,V$

b) $\bar{S} = \bar{Z}_e \cdot I^2 = (10 + j10) \cdot 1^2 = 10 + j10\,VA$

$\bar{S} = 10\sqrt{2}\, e^{j\bar{u}/4}\,VA$

$P = 10\,W$

$Q = 10\,VAr$

$S = 10\sqrt{2}\,VA$

5

a) $P_1 = 1\,kW$, $\cos\varphi_1 = \frac{1}{\sqrt{2}}$ $\Rightarrow$

$$S_1 = \frac{P_1}{\cos\varphi_1} = \sqrt{2}\ kVA$$

$$\sin\varphi_1 = (+)\sqrt{1-\cos^2\varphi_1} = \sqrt{1-\frac{1}{2}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

(+) ↑ INDUKTIVAN POTROŠAČ

$$Q_1 = S_1 \sin\varphi_1 = 1\,kVAr$$

$S_2 = 5\,kVA$, $\cos\varphi_2 = 0{,}6$ $\Rightarrow$

$$P_2 = S_2 \cos\varphi_2 = 3\,kW$$

$$\sin\varphi_2 = (-)\sqrt{1-\cos^2\varphi_2} = -\sqrt{1-0{,}36} = -\sqrt{0{,}64} = -0{,}8$$

(−) ↑ KAPACITIVAN POTROŠAČ

$$Q_2 = S_2 \sin\varphi_2 = -4\,kVAr$$

$P_{UK} = P_1 + P_2 = 4\,kW$, $Q_{UK} = Q_1 + Q_2 = -3\,kVAr$ $\Rightarrow$

$$S_{UK} = \sqrt{P_{UK}^2 + Q_{UK}^2}$$

$$S_{UK} = \sqrt{4k^2 + (-3k)^2}$$

$$S_{UK} = \sqrt{25k^2} = 5\,kVA$$

b)

$$S_{UK} = U I \Rightarrow I = \frac{S_{UK}}{U} = \frac{5000}{400} = 12{,}5\,A$$

$$\bar{S}_{UK} = \bar{Z}_{UK} \cdot I^2 \Rightarrow \bar{Z}_{UK} = \frac{\bar{S}_{UK}}{I^2} = \frac{P_{UK}}{I^2} + j\frac{Q_{UK}}{I^2}$$

$$\bar{Z}_{UK} = 25{,}6 - j\,19{,}2\ \Omega$$