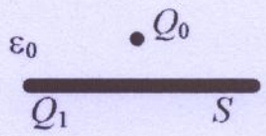
Elektrotehnika – teorijski deo ispita

3.8.2025.

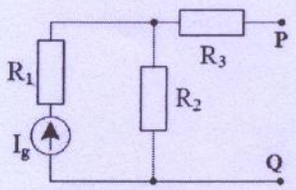
- ispit traje 45 minuta -

1. (6 poena) U vazduhu, iznad velike horizontalne ploče površine S nalektrisane količinom naelektrisanja $Q_1 = Q > 0$ nalazi se tačkasto naeletrisanje $Q_0 = -Q$. Odrediti intenzitet sile kojom ploča deluje na tačkasto naelektrisanje.

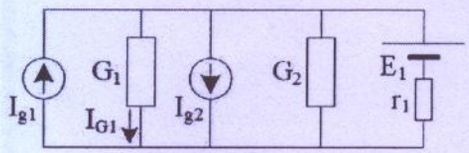


2. (6 poena) Dva tačkasta naelektrisanja naelektrisana jednakim količinama naelektrisanja $Q_1 = Q > 0$ i $Q_2 = -Q/2$ nalaze se u vazduhu na rastojanju d. Ako su vrednosti naelektrisanja Q i rastojanja d poznate, odrediti izraz za intenzitet vekrora električnog polja u tački koja se nalazi na zamišljenoj pravoj liniji koja povezuje ova dva naelektrisanja i udaljena je od prvog (Q_1) naelektrisanja $2d/3$, a od drugog $d/3$.

3. (6 poena) Za kolo na slici, između tačaka P i Q, nacrtati ekvivalentni Tevenenov generator i odrediti njegove parametre. Poznate su vrednosti parametara: I_g, R_1, R_2 i R_3.



4. (7 poena) U električnom kolu na slici poznate su vrednosti parametara: E_1, I_{g1}, I_{g2}, G_1, G_2 i r_1. Odrediti struju I_{G1}.



Studenti koji su položili 1. kolokvijum rade samo zadatke 5, 6, 7 i 8

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5. (6 poena) Kroz kalem induktivnosti L = 500 μH protiče naizmenična struja čiji je vremenski oblik $i(t) = 10\cos(1000t+3\pi/4)$ A.

a) Odrediti vremenski oblik napona na kalemu (usvojiti da napon i struja imaju usaglašen referentni smer).

b) Nacrtati fazorski dijagram napona i struje na kalemu.

6. (8 poena) Kroz potrošač nepoznate impedanse protiče struja intenziteta $i(t) = 10\cos(1000t+2\pi/3)$ A, dok napon na njegovim krajevima iznosi $u(t) = 40\cos(1000t+\pi/2)$ V. Odrediti kompleksnu impedansu potrošača, predstaviti ga rednom vezom dva elementa i odrediti parametre elemenata.

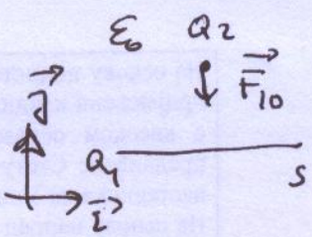
7. (5 poena) Trofazni potrošač, koji se sastoji od tri impedance $\overline{Z} = 4 - j3$ kΩ povezane u trougao, priključen je na simetričan sistem trofaznog napona 3×400 V. Odrediti intenzitet linijske struje.

8. (6 poena) Za kompleksnu impedansu $\overline{Z} = 12e^{j\pi/3}$ kΩ odrediti: rezistansu, reaktansu, konduktansu i susceptansu.

1)

$$\vec{F}_{10} = \vec{K}_1 \cdot Q_0 = \frac{\sigma_1}{2\varepsilon_0}\vec{j}\cdot(-Q) = -\frac{Q^2}{2\varepsilon_0 S}\vec{j}$$

$$|\vec{F}_{10}| = \frac{Q^2}{2\varepsilon_0 S}$$

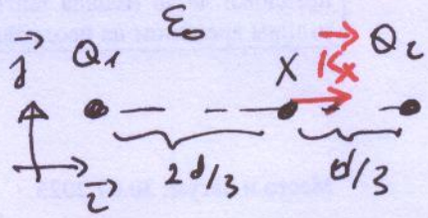


2)

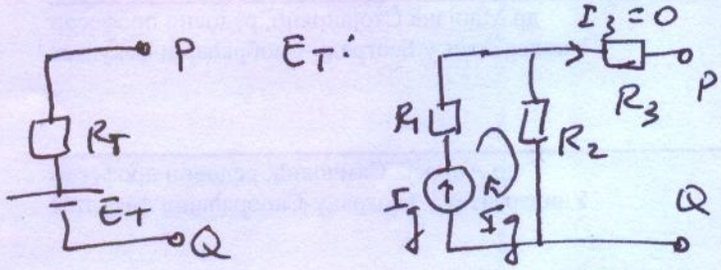
$$\vec{K}_x = \vec{K}_{1x} + \vec{K}_{2x} = \frac{Q_1\vec{r}_{01x}}{4\pi\varepsilon_0\left(\frac{2d}{3}\right)^2} + \frac{Q_2\vec{r}_{02x}}{4\pi\varepsilon_0\left(\frac{d}{3}\right)^2}$$

$$\vec{K}_x = \frac{9Q\,\vec{i}}{16\pi\varepsilon_0 d^2} + \frac{9(-Q/2)(-\vec{i})}{4\pi\varepsilon_0 d^2}$$

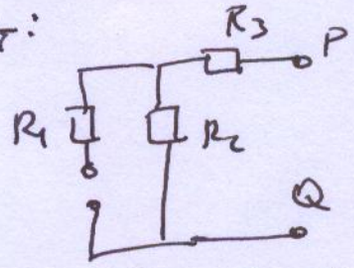
$$\vec{K}_x = \frac{9Q}{16\pi\varepsilon_0 d^2}\left(1+\frac{2}{2}\right)\vec{i} = \frac{2\cancel{7}Q}{16\pi\varepsilon_0 d^2}\vec{i}$$



3)



$$E_T = V_{PQ}^{ov} = R_2 I_g$$



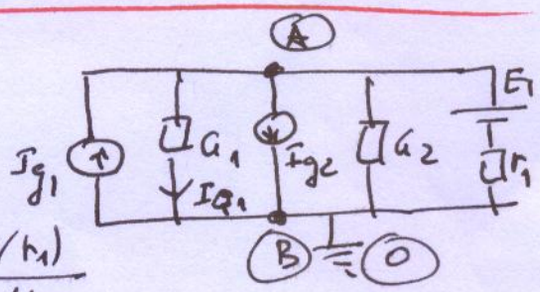
$$R_T = R_2 + R_3$$

4)

$$\left(G_1 + G_2 + \frac{1}{r_1}\right)V_A = I_{g1} - I_{g2} + \frac{E_1}{r_1}$$

$$V_A = \frac{I_{g1} - I_{g2} + E_1/r_1}{G_1 + G_2 + \frac{1}{r_1}}$$

$$I_{G1} = G_1 \cdot U_{AB} = G_1 V_A = \frac{G_1\left(I_{g1} - I_{g2} + E_1/r_1\right)}{G_1 + G_2 + 1/r_1}$$



5)

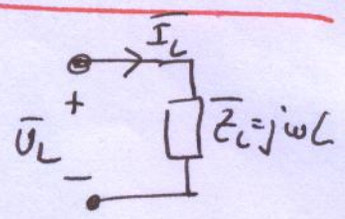
a) $\bar{I}_L = \frac{10}{\sqrt{2}} e^{j3\pi/4}\,A = 5\sqrt{2}\,e^{j3\pi/4}\,A$

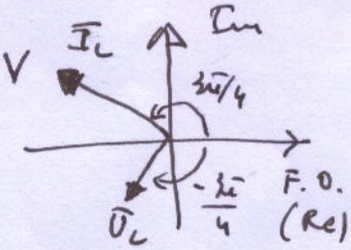
$$\bar{Z}_L = j\omega L = j\cdot 1000\cdot 500\cdot 10^{-6} = j0{,}5\,\Omega = 0{,}5\,e^{j\pi/2}\,\Omega$$

$$\bar{U}_L = \bar{Z}_L\bar{I}_L = 2{,}5\sqrt{2}\,e^{j5\pi/4}\,V = 2{,}5\sqrt{2}\,e^{-j3\pi/4}\,V$$

$$u_L(t) = U_m\cos(\omega t + \theta) = 2{,}5\cdot\sqrt{2}\cdot\sqrt{2}\cdot\cos(1000t - 3\pi/4)\,V$$

$$u_L(t) = 5\cos(1000t - 3\pi/4)\,V$$





6

$$\left.\begin{aligned}\bar{I} &= \frac{10}{\sqrt{2}} e^{j 2\pi/3}\ A \\ \bar{U} &= \frac{40}{\sqrt{2}} e^{j \pi/2}\ V\end{aligned}\right\} \Rightarrow \bar{Z} = \frac{\bar{U}}{\bar{I}} = \frac{\frac{40}{\sqrt{2}} e^{j\pi/2}}{\frac{10}{\sqrt{2}} e^{j 2\pi/3}} = 4 e^{-j\pi/6}\ \Omega$$

$$\bar{Z} = 4\cos(-\pi/6) + j 4 \sin(-\pi/6) = 4\frac{\sqrt{3}}{2} - j 4 \cdot \frac{1}{2} = (2\sqrt{3} - j2)\ \Omega$$

$$\bar{Z} = R + jX \Rightarrow R = 2\sqrt{3}\ \Omega$$

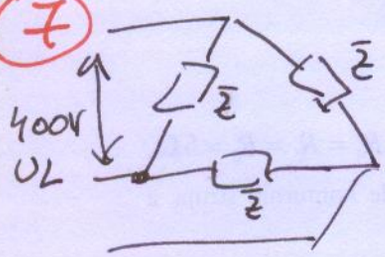
$X = -2\ \Omega < 0 \Rightarrow$ kondenzator

$R = 2\sqrt{3}\ \Omega$ $\quad C = 0{,}5\ mF$

$$X = -\frac{1}{\omega C} \Rightarrow C = \frac{-1}{\omega X} = \frac{-1}{1000 \cdot (-2)}$$

$$C = \frac{1}{2000} = 0{,}5\ mF$$

7



$U_L = 400\ V$

Δ: $U_F = U_L = 400\ V$

$$I_F = \frac{U_F}{Z} = \frac{400}{5000}$$

$I_F = 80\ mA$

$I_L = \sqrt{3}\, I_F = 80\sqrt{3}\ mA$

$\bar{Z} = (4 - j3)\ k\Omega$

$$Z = \sqrt{(4k\Omega)^2 + (-3k\Omega)^2}$$

$$Z = \sqrt{16 + 9}\ k\Omega$$

$$Z = \sqrt{25} = 5\ k\Omega$$

8

$$\bar{Z} = 12 e^{j\pi/3}\ k\Omega = 12\left[\cos(\pi/3) + j \sin(\pi/3)\right] = 12\left[\frac{1}{2} + j\frac{\sqrt{3}}{2}\right] k\Omega$$

$\left.\begin{aligned}\bar{Z} &= (6 + j6\sqrt{3})\ k\Omega \\ \bar{Z} &= R + jX\end{aligned}\right\} \Rightarrow$ Rezistansa $R = 6\ k\Omega$

Reaktanssa $X = 6\sqrt{3}\ k\Omega$

$$\bar{Y} = \frac{1}{\bar{Z}} = \frac{1}{12 e^{j\pi/3}} = \frac{1}{12} e^{-j\pi/3} = \frac{1}{12}\left[\cos(-\pi/3) + j \sin(-\pi/3)\right] mS$$

$$\bar{Y} = \frac{1}{12}\cdot\frac{1}{2} - j\frac{1}{12}\cdot\frac{\sqrt{3}}{2} = \left(\frac{1}{24} - j\frac{\sqrt{3}}{24}\right) mS$$

$\bar{Y} = G + jB \Rightarrow$ konduktanse: $G = \frac{1}{24}\ mS$

susceptanse: $B = -\frac{\sqrt{3}}{24}\ mS$